Lecture 14 & 15: Edge Degree Constrained Subgraph

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# 1 Preliminary

**Definition 1** (EDCS)**.** *For unweighted graph $G = (V, E)$, parameter $\beta \geq \beta^- \geq 0$, subgraph $H = (V, E_H)$ is an $(\beta, \beta^-)$-EDCS of $G$ if:*

- *for $e \in H$, $w(e) \leq \beta$;*
- *for $e \notin H$, $w(e) \geq \beta^-$.*

*where $w(e) = \deg_H(u) + \deg_H(v)$ for $e = (u, v)$.*

**Lemma 1.** *For every graph $G$, an EDCS exists for $\beta \geq \beta^- + 1$.*

*Proof.* Consider the following algorithm: We start with an arvitrary subgraph $H$, and iteratively fix a violated edge until no such edges.

Define the potential function

$$\Phi(H) = (\beta - \frac{1}{2}) \cdot \sum_{v \in V} \deg_H(v) - \sum_{e \in H} w(e).$$

Suppose edge $e \in E$ violates EDCS conditions:

- If $w(e) \geq \beta + 1$: $\Delta\Phi \geq (\beta - \frac{1}{2}) \cdot (-2) + (\beta - 1 + \beta + 1) \geq 1$;
- If $w(e) \leq \beta^- - 1 \leq \beta - 2$: $\Delta\Phi \geq (\beta - \frac{1}{2}) \cdot 2 - (\beta - 2 + \beta) \geq 1$.

Therefore, every fix would increase $\Phi$ by at least 1, the algorithm terminates in polynomial iterations since $|\Phi(H)| \leq \text{poly}(n, \beta)$ for every subgraph $H$. □

# 2 Approximate Maximum Matching via EDCS [AB19]

For graph $G = (V, E)$, let $N_G(A)$ be the set of neighbors of vertices $A \subseteq V$ in $G$, and let $\mu(G)$ be the maximum matching size of $G$.

**Theorem 2.** *For any $(\beta, \beta^-)$-EDCS $H$ of $G$, with $\beta^- \geq (1 - O(\epsilon)) \cdot \beta$ and $\beta \geq O(\epsilon^{-3})$, we have $\mu(G) \leq (3/2 + \epsilon) \cdot \mu(H)$.*

We first show the theorem for the case when $G$ is a bipartite graph, and then extend it to general graphs.

### 2.1 Approximate Matching on Bipartite Graphs

**Lemma 3** (Hall's Theorem). *Let $G = (L, R, E)$ be any bipartite graph with $|L| = |R| = n$. Then,*

$$\max_{A \subseteq L} (|A| - |N(A)|) = n - \mu(G).$$

*We refer to such set $A$ as a witness set.*

**Lemma 4.** *For any $(\beta, \beta^-)$-EDCS $H$ of bipartite graph $G = (L, R, E)$, we have $\mu(G) \leq (1/2 + \beta/\beta^-) \cdot \mu(H)$.*

*Proof.* Suppose $|L| = |R| = n$. Let $H = (L, R, E_H)$ be a $(\beta, \beta^-)$-EDCS of $G$ with $\beta$ and $\beta^-$ satisfying the conditions in the theorem 2, and our goal is to show that

$$\mu(G) \leq (3/2 + \epsilon) \cdot \mu(H).$$

We apply Hall's theorem (Theorem 3) to $H$, and consider the witness set $S \subseteq L$ and $T = N_H(S) \subseteq R$, we have $\mu(H) = |\bar{S}| + |T| = n - |S| + |T|$.

Let $M_G \subseteq L \times R$ be the maximum matching of $G$, and let $M' = M_G \cap (S \times \bar{T})$. We must have

$$|M'| \geq \mu(G) - \mu(H)$$

because otherwise it raises the contradiction that

$$\begin{aligned}\mu(G) - n \leq |N_G(S)| - |S| &\leq |N_H(S)| + |M'| - |S| \\ &< (|S| - n + \mu(H)) + (\mu(G) + \mu(H)) - |S| \\ &= \mu(G) - n\end{aligned}$$

Let $V' = V(M') \subseteq (S \cup \bar{T})$ be the endpoints of the matching $M'$, and $E' = E_H \cap (V' \times N_H(V'))$. By the fact that $H$ is a $(\beta, \beta^-)$-EDCS and $M' \cap H = \varnothing$, we have

$$|E'| = \sum_{v' \in V'} \deg_H(v') = \sum_{(u,v) \in M'} (\deg_H(u) + \deg_H(v)) \geq \beta^- \cdot |M'| = \frac{\beta^-}{2} \cdot |V'|$$

On the other hand,

$$\begin{aligned}\beta \cdot |E'| &\geq \sum_{(u,v) \in E'} (\deg_{E'}(u) + \deg_{E'}(v)) \\ &= \sum_{v \in V'} (\deg_{E'}(v))^2 + \sum_{v \in N_H(V')} (\deg_{E'}(v))^2 \\ &\geq |V'| \cdot \left( \frac{1}{|V'|} \sum_{v \in V'} \deg_{E'}(v) \right)^2 + |N_H(V')| \cdot \left( \frac{1}{|N_H(V')|} \sum_{v \in N_H(V')} \deg_{E'}(v) \right)^2 \qquad (*) \\ &\geq \frac{|E'|^2}{|V'|} + \frac{|E'|^2}{|N_H(V')|} \\ &\geq (\frac{\beta^-}{2} + \frac{|E'|}{|N_H(V')|}) \cdot |E'|\end{aligned}$$

Inequality (*) is due to the Cauchy-Schwarz inequality due to the fact that $\sum_{v \in V'} \deg_{E'}(v) = \sum_{v \in N_H(V')} \deg_{E'}(v) = |E'|$.

Therefore, $|E'| \leq (\beta - \frac{\beta^-}{2})|N_H(V')|$. Note that $|V'| \geq 2(\mu(G) - \mu(H))$ and $|N_H(V')| \leq \mu(H)$, we have

$$\beta^- \cdot (\mu(G) - \mu(H)) \leq |E'| \leq (\beta - \frac{\beta^-}{2}) \cdot \mu(H)$$

As a result,

$$\mu(G) \leq (1/2 + \beta/\beta^-) \cdot \mu(H)$$

□

## 2.2 Approximate Matching on General Graphs

In this section, we will show that we can generalize the result to general graph $G = (V, E)$ with a maximum matching $M_G$, and a $(\beta, \beta^-)$-EDCS $H = (V, E_H)$.

**Construction** To utilize results in the bipartite case, we apply the following randomized construction:

- Bipartition $V = L \cup R$ uniformly at random such that for all $(u, v) \in M_G$, $u$ and $v$ belong to different sides.
- Let $G' = (L, R, E \cap (L \times R))$ and $H' = (L, R, E_H \cap (L \times R))$.

It immediately follows from the construction that $\mu(G) = \mu(G')$ and $\mu(H') \leq \mu(H)$.

We want to show that a good partition exists, i.e., we can sample that partition with strictly positive probability using the following Lovasz Local Lemma.

**Lemma 5** (Lovasz Local Lemma). *For events $E_1, \cdots, E_n$, $\Pr \bigcap \bar{E}_i > 0$ if the following conditions hold:*

- $\Pr E_i \leq p$;
- *$E_i$ is independent of all but $d$ events;*
- $d \cdot p \leq 0.1$.

**Lemma 6.** *$H'$ is a $((1 + O(\epsilon))\beta/2, (1 - O(\epsilon))\beta^-/2)$-EDCS of $G'$ with probability $> 0$.*

*Proof of Lemma 6.* For any fixed $v \in V$, the number of its neighbors in subgraph $H'$ can be modeled as: for every $u \in N_H(v)$, it is added to $N_{H'}(v)$ with probability $1/2$ *independently.* Moreover, we have $\mathbb{E}[\deg_{H'}(v)] = \deg_H(v)/2$ ($+1$ if $v$ is in $M_G$). Therefore, by Chernoff Bound, we have

$$\Pr |\deg_{H'}(v) - \deg_H(v)/2| \geq \epsilon \cdot \beta + 1 \leq \exp(-\Omega(\epsilon^2 \beta)) \leq 1/\operatorname{poly}(\beta)$$

By Lovasz Local Lemma with $d \leq \beta^4$ □

Finally, we are ready to prove the main theorem.

**Theorem 2.** *For any* $(\beta, \beta^-)$*-EDCS* $H$ *of* $G$*, with* $\beta^- \geq (1 - O(\epsilon)) \cdot \beta$ *and* $\beta \geq O(\epsilon^{-3})$*, we have* $\mu(G) \leq (3/2 + \epsilon) \cdot \mu(H)$*.*

*Proof of Theorem 2.* By Lemma 6, there exists a partition $V = L \cup R$ such that $H' = (L, R, E_H \cap (L \times R))$ is a $((1 + O(\epsilon))\beta/2, (1 - O(\epsilon))\beta^-/2)$-EDCS of the bipartite graph $G' = (L, R, E \cap (L \times R))$. Since $G'$ is a bipartite graph, by Lemma 4, we have

$$\begin{aligned}\mu(G) = \mu(G') &\leq (1/2 + (1 + O(\epsilon))\beta/(1 - O(\epsilon))\beta^-)\mu(H')\\ &\leq (3/2 + \epsilon)\mu(H')\\ &\leq (3/2 + \epsilon)\mu(H)\end{aligned}$$

□

### 2.3 Application: One-Way Communication for Approximate Matching

In this section, we demonstrate an application of EDCS in the one-way communication model for approximate matching.

**Setting** Alice and Bob are given graphs $G_A = (V, E_A)$ and $G_B = (V, E_B)$, respectively. Alice is allowed to send a small message to Bob, and the goal is that Bob can find a large matching in the union graph $G = (V, E_A \cup E_B)$ given the message from Alice.

**Protocol** Alice computes an $(\beta, \beta - 1)$-EDCS $H_A$ of $G_A$ and send it to Bob, the message size is only $O(n)$. Bob can then compute a maximum matching in $H_A \cup G_B$ and output it.

**Claim** $\mu(G) \leq (3/2 + \epsilon) \cdot \mu(H_A \cup G_B)$.

*Proof.* We fix a maximum matching $M$ in $G$. Define $M_B = M \cap E_B$, $G' = (V, E_A \cup M_B)$, and $H' = (V, E_H \cup M')$, where $M' = \{e \in M_B \mid w_{H_A}(e) \leq \beta\}$. Now we show that $H'$ is a $(\beta + 2, \beta - 1)$-EDCS of $G'$. Note that $\deg_{H'}(v) - \deg_H(v) \in \{0, 1\}$.

- For any $(u, v) \in H'$: we have $\deg_{H'}(u) + \deg_{H'}(v) \leq \deg_H(u) + \deg_H(v) + 2$.

  If $(u, v) \in H$, then $\deg_H(u) + \deg_H(v) \leq \beta$ by the properties of EDCS $H$. Otherwise, by definition of $M'$, we have $\deg_H(u) + \deg_H(v) \leq \beta$ as well.

- For any $(u, v) \notin H'$: we have $\deg_{H'}(u) + \deg_{H'}(v) \geq \deg_H(u) + \deg_H(v)$.

  If $(u, v) \in G \setminus H$, there is $\deg_H(u) + \deg_H(v) \geq \beta - 1$ by properties of EDCS $H$. Otherwise, $\deg_H(u) + \deg_H(v) > \beta$ by the definition of $M'$.

Finally, since $\mu(G') = \mu(G)$, we have

$$\mu(G) = \mu(G') \leq (3/2 + \epsilon) \cdot \mu(H') \leq (3/2 + \epsilon) \cdot \mu(H_A \cup G_B).$$

□

# 3 Maintaining an EDCS in General Graphs [GSSU22]

In this section, we introduce a **deterministic** algorithm with **worst-case** update-time $O_\epsilon(m^{1/4})$ for maintaining an $(3/2+\epsilon)$-MCM in general graphs using EDCS.

**Theorem 7.** *For any dynamic graph $G$ subject to edge updates and for any $\epsilon < 1/2$, one can maintain a $(3/2+\epsilon)$-MCM for $G$ with a deterministic worst-case update time of $O(m^{1/4}\epsilon^{-5/2} + \epsilon^{-6})$.*

**Related Works.** In the following summirization of related works, we focus on the following key properties: the approximation factor, the update time, *whether the analysis is worst-case(W) or amortized(A)*, and *whether the algorithm is deterministic(D) or randomized(R)*.

| | MCM | Update Time | W/A | D/R |
|---|---|---|---|---|
| [BGS11] | 2 | $O(\log n)$ | A | R |
| [Sol16] | 2 | $O(1)$ | A | R |
| [BFH19] | 2 | $O(\text{poly}\log n)$ | W | R |
| [NS13] | 3/2 | $O(\sqrt{m})$ | W | D |
| [BS16] | $3/2+\epsilon$ | $O_\epsilon(m^{1/4})$ | A | D |
| [GSSU22] | $3/2+\epsilon$ | $O_\epsilon(m^{1/4})$ | W | D |

Table 1: Related works. Result of [GSSU22] is stated as Theorem 7.

**High-level idea** Approximate MCM $\Leftrightarrow$ EDCS

In the following discussion, let $G=(V,E)$ be an undirected, unweighted dynamic graph with $n=|V|$ and $m=|E|$. We will dynamically maintain a subgraph $H=(V,E_H)$. Let $\Delta$ be an upper bound of the maximum degree in the dynamic graph. For simplicity, let $\beta^- = (1-\epsilon)\beta$.

## 3.1 Part I: Update Time $O_\epsilon(\Delta)$

In this section, we show that an $(\beta,\beta^-)$-EDCS can be maintained with a worst-case update time $O_\epsilon(\Delta)$. We define the following two types of edges:

- "Full" edge $e=(u,v)$ if $e \in H$ and $w(u,v)=\beta$;
- "Deficient" edge $e=(u,v)$ if $e \notin H$ and $w(u,v)=\beta^-$.

Furthermore, a vertex $v$ is said to be "increase-safe" (or respectively, "decrease-safe") if it has no incident full (or respectively, deficient) edges.

**Insertion.** When inserting edge $e=(u,v)$ into $G$, there are two cases:

- If $w(u,v) \geq \beta^-$: then $e$ will not be added to $H$. In this case, none of the weights will change, and the properties of EDCS will continue be satisfied.

- Otherwise, we add $e$ to $H$. In this case, both $d_H(u)$ and $d_H(v)$ will increase by 1, which may lead incident edges to violate the EDCS properties. We focus on $v \in V$ first:
  - If $v$ is increase-safe, by definition, then we can safely increase $d_H(v)$ by 1. No further action is needed.
  - If $v$ is not increase-safe: Let $(v, p_1) \in H$ be the incident full edge, and we remove $(v, p_1)$ from $H$. As a result, $d_H(v)$ will remain unchanged (compared to the time before insertion).

    Such removal may also lead to violation of EDCS properties for $p_1$ since $d_H(p_1)$ will decrease by 1. Similarly, if $p_1$ is decrease-safe, no further action is needed. Otherwise, we find deficient edge $(p_1, p_2)$ and add it to $H$. We repeat such process, updating full/deficient edges until we reach an increase-safe/decrease-safe vertex, like finding an alternative path.

**Deletion.** When deleting edge $e = (u, v)$ from $G$, the process is symmetric to insertion. We alternatively delete and add edges until no EDCS properties are violated.

**Analysis.** It remains to show that such process will terminate quickly, i.e., such alternative path will not be too long.

**Lemma 8.** *For any alternative path $P$ of full and deficient edges, we have $|P| \leq 2/\epsilon$.*

*Proof.* Consider the path $(p_1, p_2, p_3, \cdots, p_l)$ that begins with a full edge, with $d_H(p_1) + d_H(p_2) \geq \beta$. Then $(p_2, p_3)$ is a deficient edge with $d_H(p_2) + d_H(p_3) \leq \beta^-$. Therefore, we have

$$d_H(p_3) \leq \beta^- - d_H(p_2) \leq d_H(p_1) + (\beta^- - \beta) \leq d_H(p_1) - \epsilon\beta.$$

By induction, we have $d_H(p_l) \leq d_H(p_1) - \frac{l\epsilon\beta}{2}$. However, $d_H(p_l) \geq 0$ and $d_H(p_1) \leq \beta$. Therefore, we must have $l \leq 2/\epsilon$. □

**Update Time.** In the implementation, we maintain for every vertex its adjacent edges, full adjacent edges, and deficient adjacent edges. These lists should be updated in $O(\Delta)$ when the degree of the vertex changes. By the algorithm above, regardless of the length of the alternative path, at most 2 vertices will change in their degrees. Therefore, the list update time is $O(\Delta)$. Finally, we have showed that the worst-case update time is $O(\Delta + 1/\epsilon)$.

### 3.2 Part II: Improved Update Time $O_\epsilon(\sqrt{\Delta})$

In this section, we improve the above algorithm to allow a better update time – instead of updating all $O(\Delta)$ neighbors, we only update $\frac{10\Delta}{\epsilon\beta}$ out of its $O(\Delta)$ neighbors in a cyclic manner. It immediately follows that the new update time is $O(\frac{\Delta}{\epsilon\beta} + 1/\epsilon)$.

For $(v, w) \in E$, let $\tilde{d}_H^w(v)$ be the estimated degree of $v$ by its neighbor $w$. The maximum error in estimation is the maximum number of batches a neighbor will wait until get notified, i.e.,

$$Dis(H) = \max_{v,w}\{d_H(v) - \tilde{d}_H^w(v)\} \leq \Delta / \frac{10\Delta}{\epsilon\beta} = \frac{\epsilon\beta}{10}.$$

Since the update is incomplete, we need to analyze the consequences of not having accurate degree information in the following two aspects: change of length of alternative path, and the changed properties of EDCS.

**Length of Alternative Path.** We then show that the maximum length of an alternative path remains $O(1/\epsilon)$ using the estimated degree information. For path $(p_1, p_2, p_3, \cdots, p_l)$, similarly to the proof of Lemma 8, we have

$$\begin{aligned}\tilde{d}_H^{p_2}(p_3) \leq \beta^- - d_H(p_2) &\leq \beta^- - \tilde{d}_H^{p_1}(p_2) + Dis(H) \\ &\leq \beta^- - \beta + d_H(p_1) + Dis(H) \\ &\leq d_H(p_1) - \epsilon\beta + \frac{\epsilon\beta}{10} = d_H(p_1) - \frac{9\epsilon\beta}{10}.\end{aligned}$$

Similarly, note that $d_H(p_1) \leq \beta$ and $d_H(p_l) \geq 0$, we have $l \leq \frac{5}{2\epsilon}$.

**EDCS Properties.** We than argue that $H$ will still hold weaker EDCS properties with the estimated degree information.

**Lemma 9.** *The modified algorithm maintains a $(\gamma, (1-2\epsilon)\gamma)$-EDCS with $\gamma = \beta(1+\epsilon/10)$.*

*Proof.* For every edge $(u, v) \in H$, consider vertex $u$ that knows its exact degree but only have estimation for its neighbors: the estimated weight

$$w(u, v) = d_H(u) + \tilde{d}_H^u(v) \leq d_H(u) + d_H(v) + Dis(H) \leq \beta + \frac{\beta\epsilon}{10} = \gamma.$$

On the other hand, for some edge $(u, v) \notin H$, we have

$$w(u, v) = d_H(u) + \tilde{d}_H^u(v) \geq d_H(u) + d_H(v) - Dis(H) \geq (1-\epsilon)\beta - \frac{\beta\epsilon}{10} \geq \gamma(1-2\epsilon).$$

Therefore, the maintained $H$ is a $(\gamma, (1-2\epsilon)\gamma)$-EDCS. □

Furthermore, we can verify that $(\gamma, (1-2\epsilon)\gamma)$ satisfies the conditions in Theorem 2 to maintain a $(3/2+\epsilon)$-approximate MCM.

**Maintaining the MCM.** By Gupta-Peng [GP13] algorithm, a $(1+\epsilon)$-MCM can be maintained on top of $H$ within $O(\beta/\epsilon^2)$ time. Therefore, the combination yields a $(1+\epsilon)(3/2+\epsilon) \leq (3/2+\frac{7}{2}\epsilon)$ approximation. Putting all together, we have a deterministic algorithm with worst-case update time

$$O(\frac{\Delta}{\epsilon\beta} + \frac{1}{\epsilon} \cdot \frac{\beta}{\epsilon^2}) = O(\frac{\Delta}{\epsilon\beta} + \frac{\beta}{\epsilon^3}) = O(\frac{\sqrt{\Delta}}{\epsilon^2} + \epsilon^{-6}).$$

The last equality is achieved by setting $\beta$ to $\epsilon\sqrt{\Delta}$ if $\epsilon\sqrt{\Delta} \geq O(\epsilon^{-3})$ (by the requirement of Theorem 2), or to $O(\epsilon^{-3})$ otherwise.

### 3.3 Part III: Improved Update Time $O_\epsilon(\sqrt{\alpha})$

**Definition 2** (Arboricity). *For graph $G$, the arboricity $\alpha(G)$ is the minimum number of forests into which the edges of $G$ can be partitioned.*

Let $\alpha$ be the upper bound of the arboricity of the dynamic graph. In this section, we improve the update time to $O_\epsilon(\sqrt{\alpha})$.

One important property of arboricity is that $\alpha(G) \leq \sqrt{m}$ for an $m$-edge graph $G$. Therefore, an $O_\epsilon(\sqrt{\alpha})$ update time immediately implies an $O_\epsilon(m^{1/4})$ update time.

**Sparsification.** The matching sparsification algorithm by Solomon [Sol18] is as follows: For a parameter $\eta$, for every vertex $v \in V$, we mark up to $\eta$ arbitrary incident edges. An edge will be added to the sparsified graph if it is marked twice. It is shown that by setting $\eta = 5(5/\epsilon + 1)2\alpha$, the resulting graph $G'$ is a $(1-\epsilon)$-approximate matching sparsifier for $G$.

**Maintaining Sparsifier.** The idea is to maintain a sparsifier $G'$ of the dynamic graph $G$, and feed $G'$ to the algorithm in previous sections. In fact, the following lemma is proven.

**Lemma 10.** *One can dynamically maintain a $(1+\epsilon)$-approximate sparcifier $G'$ for dynamic graph $G$ with constant worst-case update time and recourse bound, where $\Delta(G') \leq O(\sqrt{m}/\epsilon)$.*

Finally, we are able to show the main Theorem 7. The overall approximation ratio is $(3/2 + \epsilon)$, and the worst-case update time is

$$O(\frac{\sqrt{\Delta}}{\epsilon^2} + \epsilon^{-6}) = O(m^{1/4} \cdot \epsilon^{-5/2} + \epsilon^{-6}).$$

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