Computational Complexity: A Modern Approach

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`Comments welcome!`

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I am turning lecture notes from my graduate complexity course into a book. Some chapters are more finished than others. References and attributions are largely missing.

About this book

Computational complexity theory has developed rapidly in the past three decades. The list of surprising and fundamental results proved in the 1990s alone could fill a book: these include new probabilistic definitions of classical complexity classes (IP = PSPACE and the PCP Theorems) and their implications for the field of approximation algorithms; Shor's algorithm to factor integers using a quantum computer; an understanding of why current approaches to the famous P versus NP will not be successful; a theory of derandomization and pseudorandomness based upon computational hardness; and beautiful constructions of pseudorandom objects such as extractors and expanders.

This book is a graduate level text that aims to describe such recent achievements of complexity theory in the context of the classical results. It assumes undergraduate level knowledge of the theory of computation (e.g., from Sipser's book *Theory of Computation*). It is divided into three parts: (Part A) **Basic complexity classes:** This part covers broadly the same ground —but more quickly— as Papadimitriou's text from the early 1990s. (Part B) **Lowerbounds for concrete computational models:** This concerns lowerbounds on resources required to solve algorithmic tasks on concrete models such as circuits, decision trees, etc. Such models may seem at first sight very different from the Turing machine used in Part A, but looking deeper one finds interesting interconnections. (Part C) **Advanced topics:** This constitutes the latter half of the book and is largely devoted to the abovementioned developments from the 1990s (a couple of results date from the late 1980s).

Any successful text in this area must simultaneously cater to many audiences, and the book is carefully designed with that goal. The book chapters can largely be read in isolation except for the 100 pages in Part A. While writing the book I am keeping the following classes of readers in mind:

- *Physicists, mathematicians, and other scientists.* This group has become increasingly interested in computational complexity theory, especially because of high-profile results such as Shor's algorithm and the recent deterministic test for primality. This intellectually sophisticated group will generally be interested in the 150-page synopsis of basic complexity theory in Part A. Progressing on to Parts B and C, they can read individual chapters and find almost everything they need to understand current research.

- *Computer scientists (e.g., algorithms designers) who do not work in complexity theory per se.* They may use the book for selfstudy or even to teach a graduate course as a way of learning the latest results. Such a course would probably include many topics from Part A and then a sprinkling from the rest of the book. I plan to include detailed course-plans they can follow (e.g., if they plan to teach an advanced result such as the PCP Theorem, they may wish to prepare the students by teaching other results first).

- *All those —professors or students— who do research in complexity theory or plan to do so.* They will largely know Part A and will use the book for Parts B and C. They would typically use Part C in a seminar or reading course. The coverage of advanced topics here is detailed enough to make this possible. For example, teaching all results related to the PCP Theorems in this book takes me at least 12 lectures of 1.5 hours each (though I will include sample teaching plans for introducing this topic in fewer lectures) and the chapter on Derandomization and Extractors takes me another 4 to 5 lectures.

The use of examples and exercises throughout the book is intended to make it suitable both for selfstudy and as a text. The appendix will include probabilistic and algebraic facts needed in the book.

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