Hash Tables with External Chaining

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Key-value store

Maintain a collection of key/value pairs

- Each key is a **string**; each value is an **int**
- Unknown number of key-value pairs

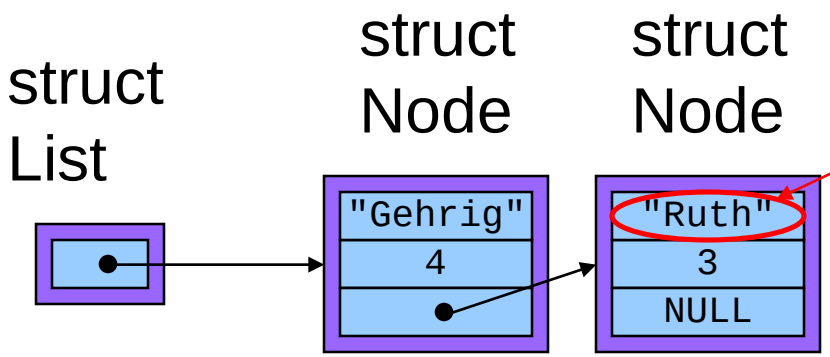
Examples

- (student name, grade)
  - (“john smith”, 84), (“jane doe”, 93), (“bill clinton”, 81)
- (baseball player, number)
  - (“Ruth”, 3), (“Gehrig”, 4), (“Mantle”, 7)
- (variable name, value)
  - (“maxLength”, 2000), (“i”, 7), (“j”, -10)

Linked List Data Structure

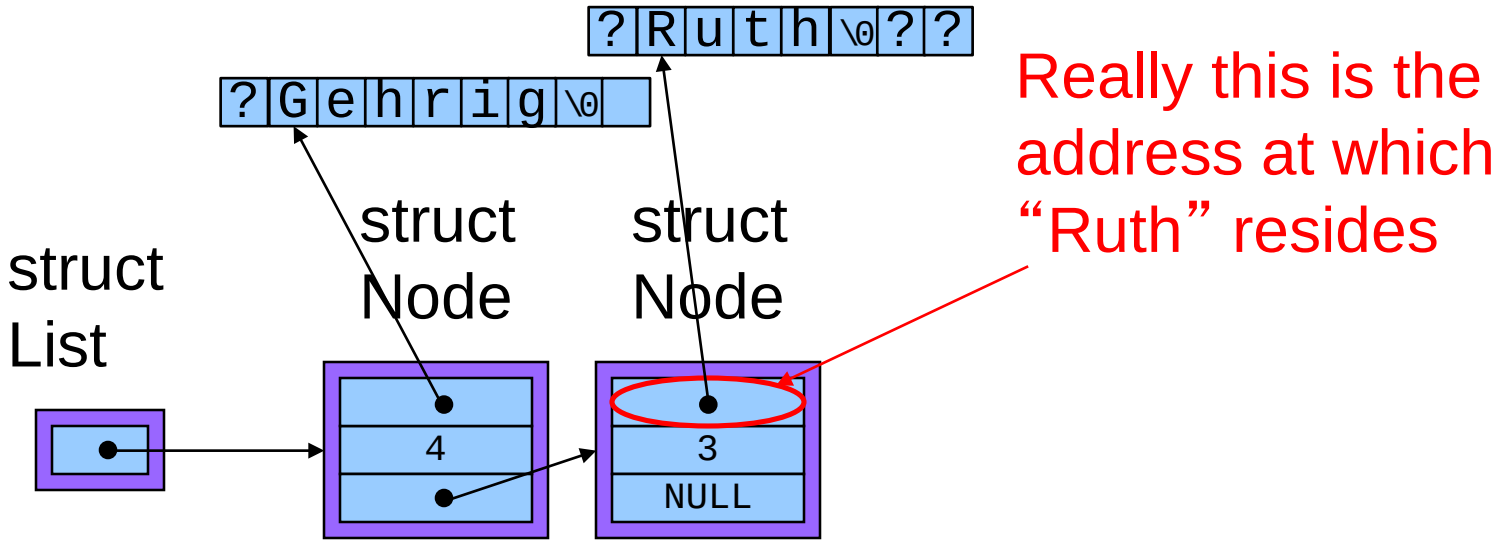
```
struct Node
{  const char *key;
   int value;
   struct Node *next;
};

struct List
{  struct Node *first;
};
```



Really this is the address at which "Ruth" resides

Linked List Data Structure



Linked List Algorithms

Create

- Allocate **`List`** structure; set **`first`** to **`NULL`**
- Performance: O(1) ⇒ fast

Add (no check for duplicate key required)

- Insert new node containing key/value pair at front of list
- Performance: O(1) ⇒ fast

Add (check for duplicate key required)

- Traverse list to check for node with duplicate key
- Insert new node containing key/value pair into list
- Performance: O(n) ⇒ slow

Linked List Algorithms

Search

- Traverse the list, looking for given key
- Stop when key found, or reach end
- Performance: O(n) ⇒ slow

Free

- Free **`Node`** structures while traversing
- Free **`List`** structure
- Performance: O(n) ⇒ slow

Would it be better to keep the nodes sorted by key?

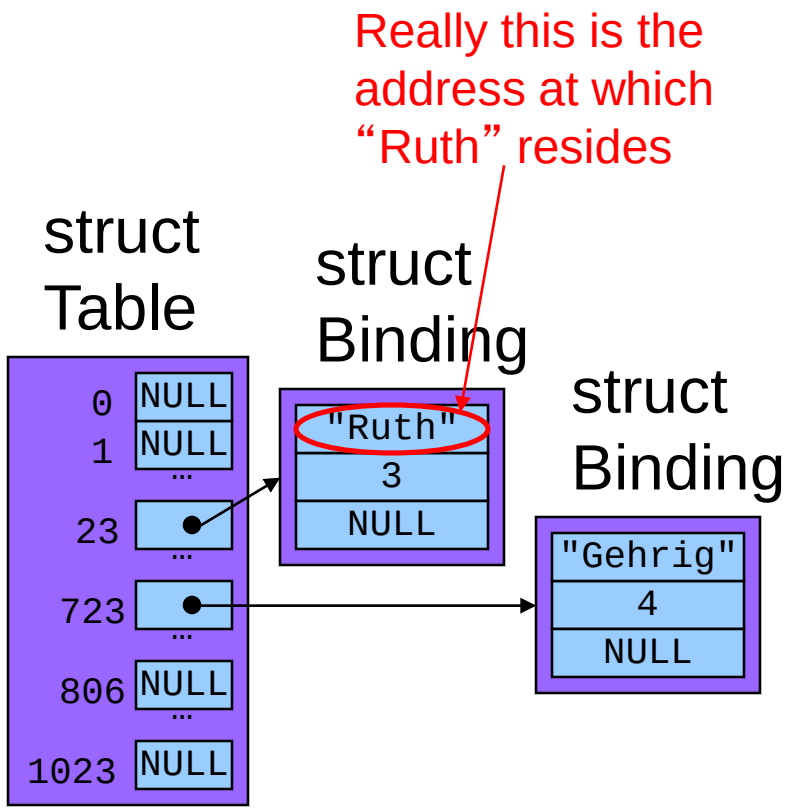
Hash Table Data Structure

Array of linked lists

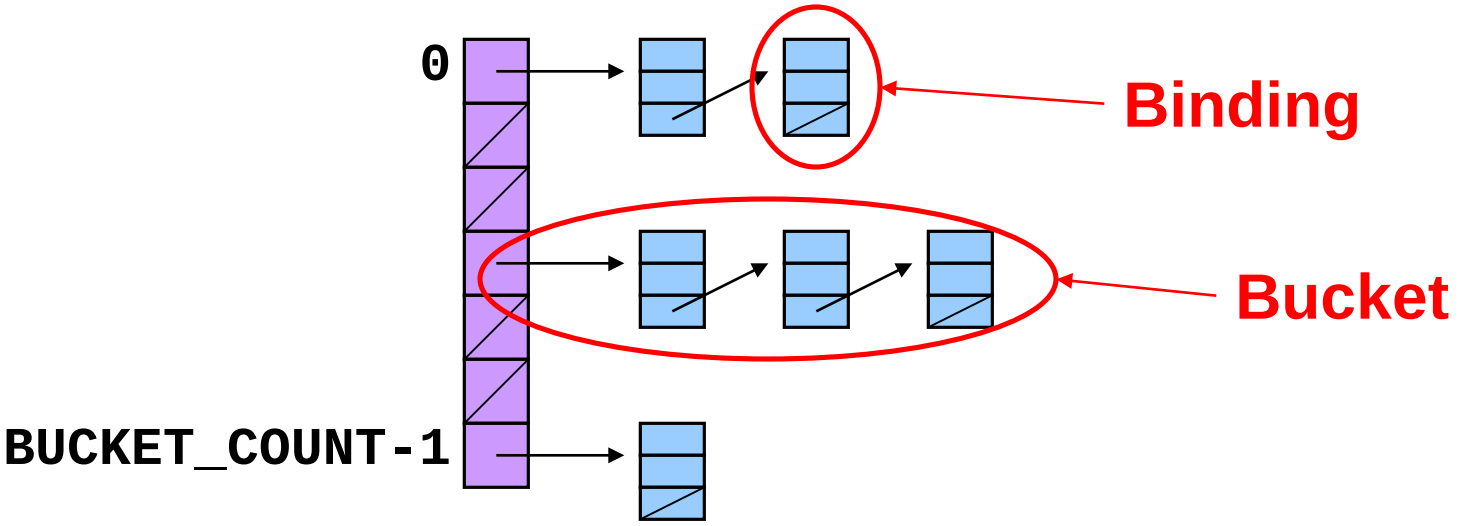
```
enum {BUCKET_COUNT = 1024};

struct Binding
{  const char *key;
   int value;
   struct Binding *next;
};

struct Table
{  struct Binding *buckets[BUCKET_COUNT];
};
```



Hash Table Data Structure



Hash function maps given key to an integer

Mod integer by **`BUCKET_COUNT`** to determine proper bucket

Hash Table Example

Example: **`BUCKET_COUNT`** = 7

Add (if not already present) bindings with these keys:

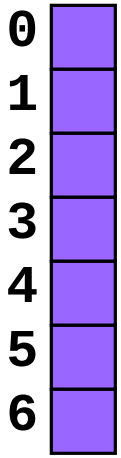
- the, cat, in, the, hat

Hash Table Example (cont.)

First key: “the”

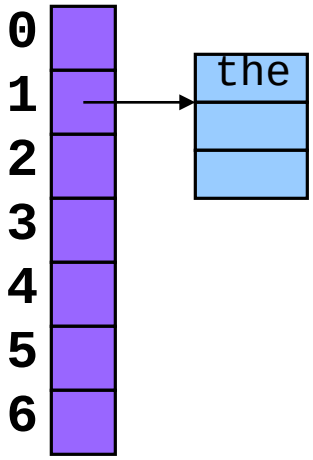
- hash(“the”) = 965156977; 965156977 % 7 = 1

Search **`buckets[1]`** for binding with key “the”; not found



Hash Table Example (cont.)

Add binding with key “the” and its value to **`buckets[1]`**

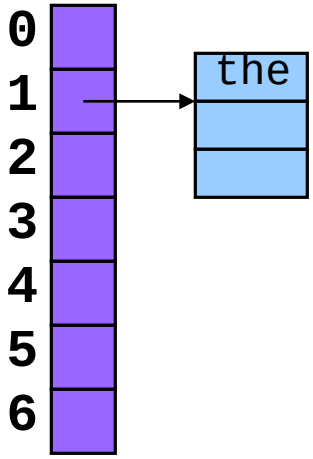


Hash Table Example (cont.)

Second key: “cat”

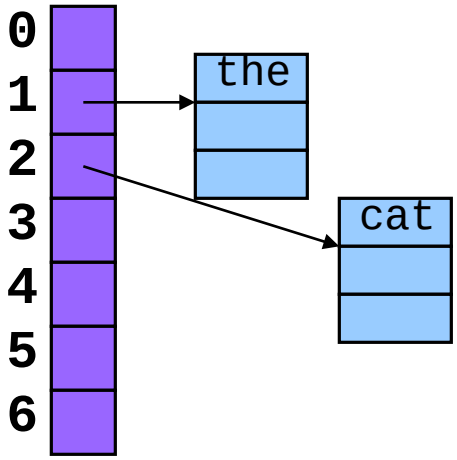
- hash(“cat”) = 3895848756; 3895848756 % 7 = 2

Search **`buckets[2]`** for binding with key “cat”; not found



Hash Table Example (cont.)

Add binding with key “cat” and its value to **`buckets[2]`**

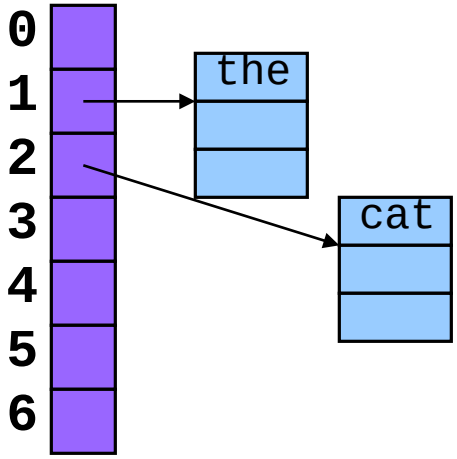


Hash Table Example (cont.)

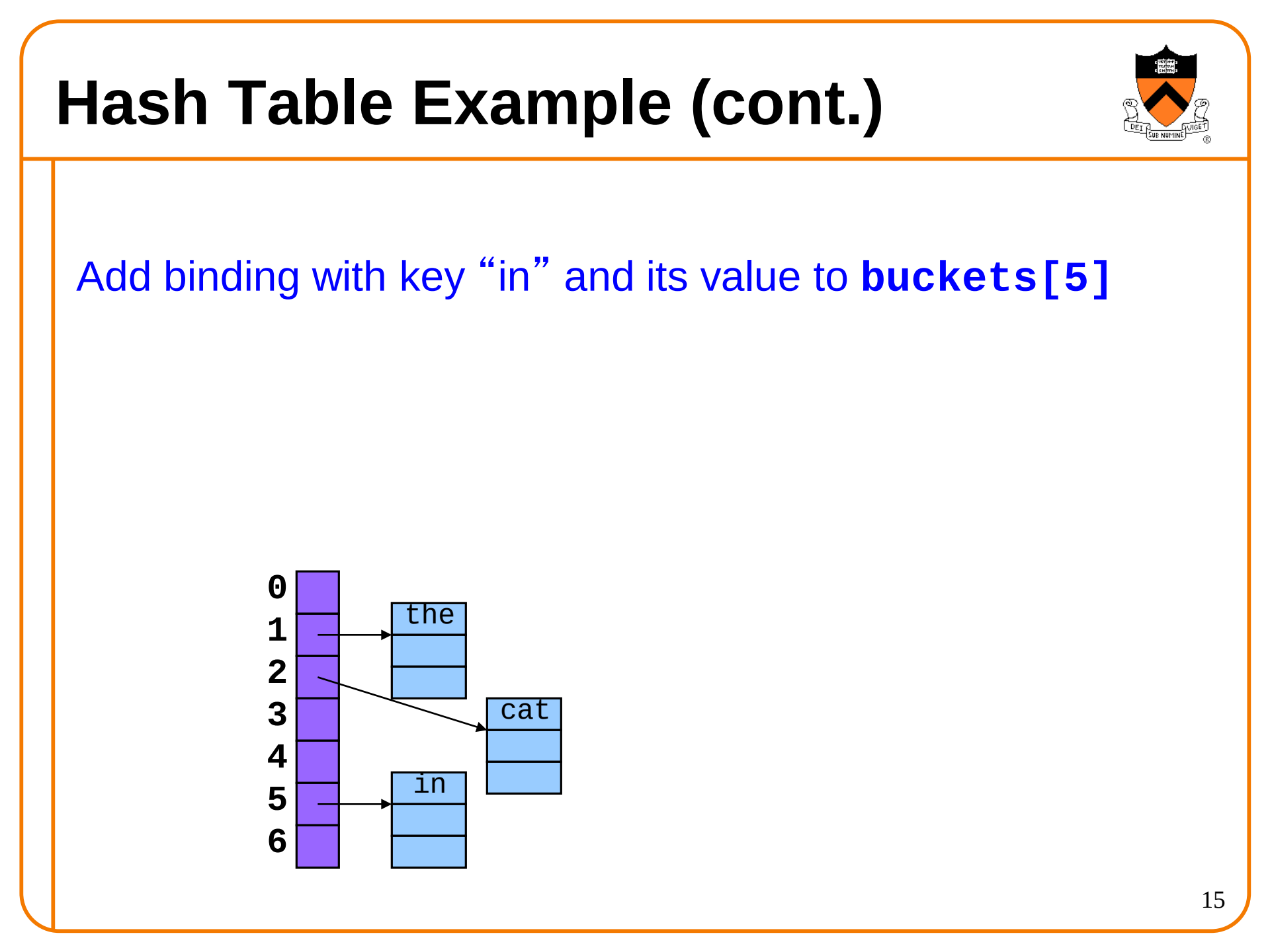
Third key: “in”

- hash(“in”) = 6888005; 6888005% 7 = 5

Search **`buckets[5]`** for binding with key “in”; not found



Hash Table Example (cont.)



Add binding with key “in” and its value to **`buckets[5]`**

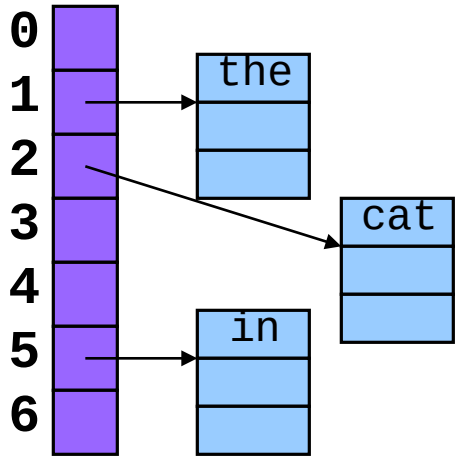
Hash Table Example (cont.)

Fourth word: “the”

- hash(“the”) = 965156977; 965156977 % 7 = 1

Search **`buckets[1]`** for binding with key “the”; found it!

- Don’t change hash table

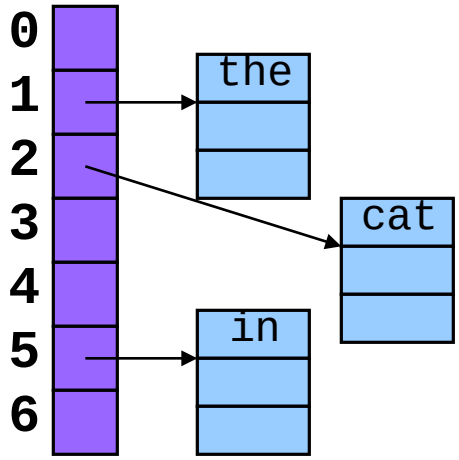


Hash Table Example (cont.)

Fifth key: “hat”

- hash(“hat”) = 865559739; 865559739 % 7 = 2

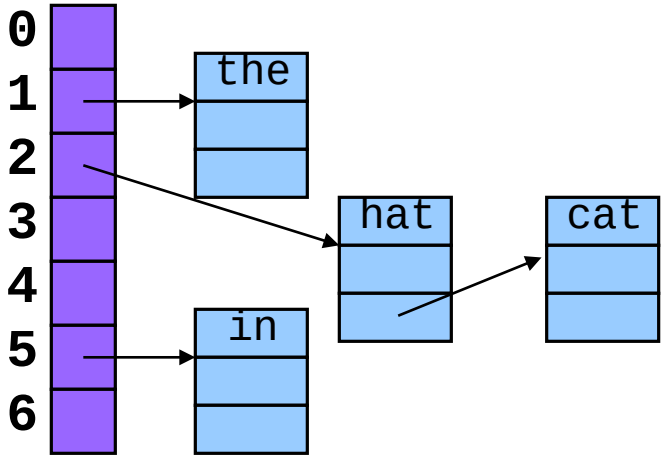
Search **`buckets[2]`** for binding with key “hat”; not found



Hash Table Example (cont.)

Add binding with key "hat" and its value to **`buckets[2]`**

- At front or back? Doesn't matter
- Inserting at the front is easier, so add at the front



Hash Table Algorithms

Create

- Allocate **`Table`** structure; set each bucket to **`NULL`**
- Performance: O(1) ⇒ fast

Add

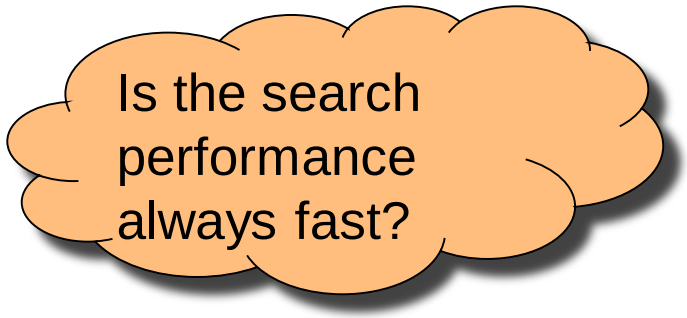
- Hash the given key
- Mod by **`BUCKET_COUNT`** to determine proper bucket
- Traverse proper bucket to make sure no duplicate key
- Insert new binding containing key/value pair into proper bucket
- Performance: O(1) ⇒ fast

Is the add performance always fast?

Hash Table Algorithms

Search

- Hash the given key
- Mod by **`BUCKET_COUNT`** to determine proper bucket
- Traverse proper bucket, looking for binding with given key
- Stop when key found, or reach end
- Performance: O(1) ⇒ fast



Free

- Traverse each bucket, freeing bindings
- Free **`Table`** structure
- Performance: O(n) ⇒ slow

How Many Buckets?

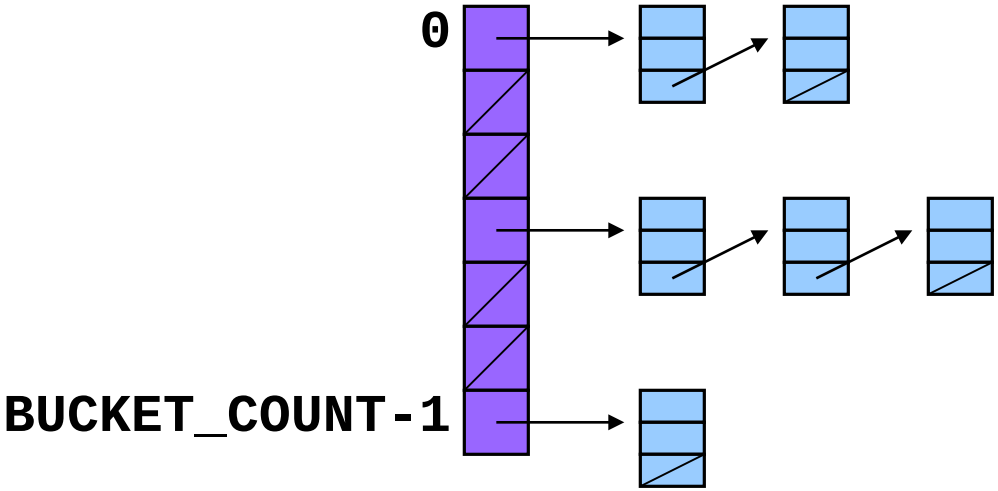
Many!

- Too few ⇒ large buckets ⇒ slow add, slow search

But not too many!

- Too many ⇒ memory is wasted

This is OK:

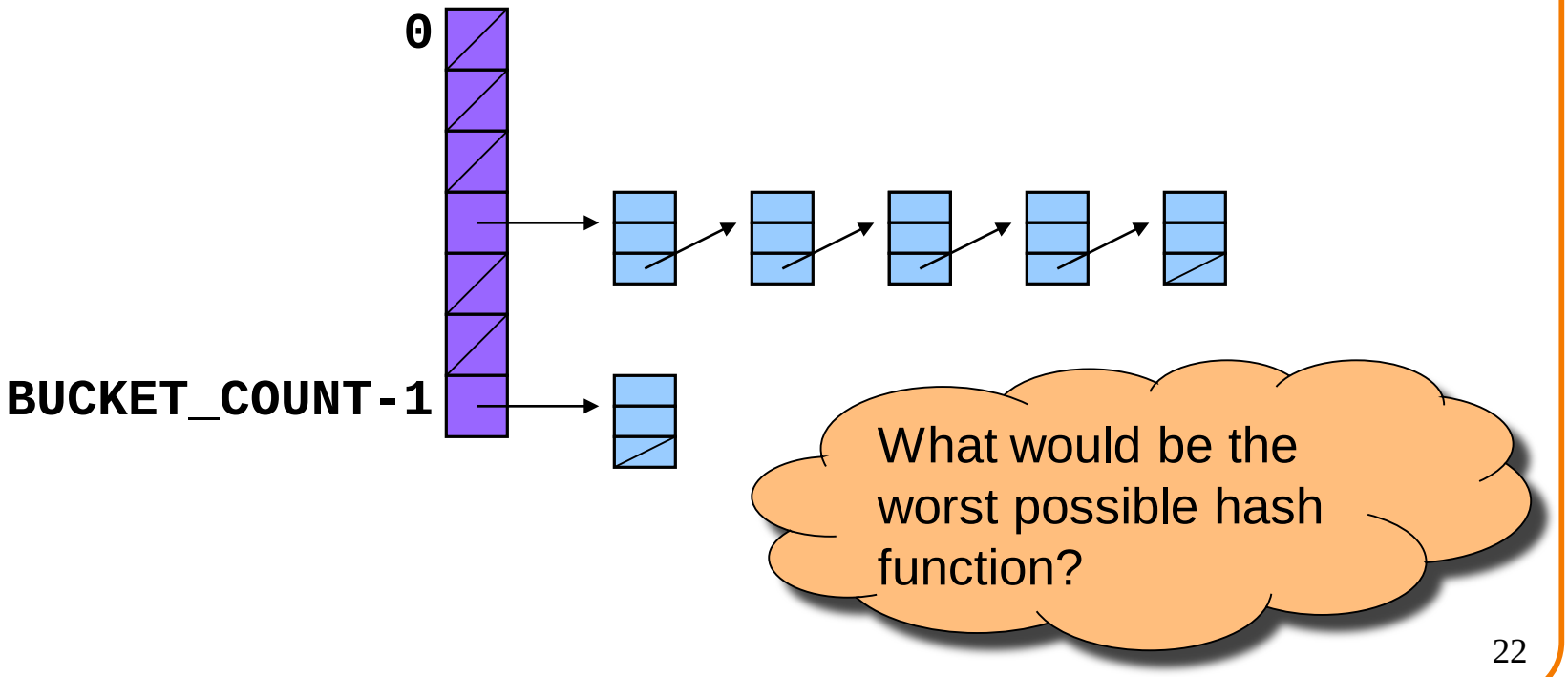


What Hash Function?

Should distribute bindings across the buckets well

- Distribute bindings over the range **`0, 1, …, BUCKET_COUNT-1`**
- Distribute bindings *evenly* to avoid very long buckets

This is not so good:



How to Hash Strings?

Simple hash schemes don't distribute the keys evenly enough

- Number of characters, mod **BUCKET_COUNT**
- Sum the numeric codes of all characters, mod **BUCKET_COUNT**
- …

A reasonably good hash function:

- Weighted sum of characters s_i in the string **s**
  - $(\Sigma\ a^i s_i)$ **mod BUCKET_COUNT**
- Best if **a** and **BUCKET_COUNT** are relatively prime
  - E.g., **a** = 65599, **BUCKET_COUNT** = 1024
- Even better if **BUCKET_COUNT** is prime.

Why?

Footnote [A. Appel]: I originally designed this homework so that BUCKET_COUNT is a prime number. In 2016 I wondered, "wouldn't it work just as well if a and BUCKET_COUNT are just relatively prime? Measurements show no: using a prime number of buckets leads to more even distribution of bucket contents."

How to Hash Strings?

Potentially expensive to compute $\Sigma\ a^i s_i$

So let's do some algebra ("Horner's rule")

- (by example, for string **s** of length 5, **a**=65599):

$h = \Sigma 65599^i * s_i$

$h = 65599^0 * s_0 + 65599^1 * s_1 + 65599^2 * s_2 + 65599^3 * s_3 + 65599^4 * s_4$

Direction of traversal of s doesn't matter, so…

$h = 65599^0 * s_4 + 65599^1 * s_3 + 65599^2 * s_2 + 65599^3 * s_1 + 65599^4 * s_0$

$h = 65599^4 * s_0 + 65599^3 * s_1 + 65599^2 * s_2 + 65599^1 * s_3 + 65599^0 * s_4$

$h = (((((s_0) * 65599 + s_1) * 65599 + s_2) * 65599 + s_3) * 65599) + s_4$

How to Hash Strings?

Yielding this function

```
size_t hash(const char *s, size_t bucketCount)
{  size_t i;
   size_t h = 0;
   for (i=0; s[i]!='\0'; i++)
      h = h * 65599 + (size_t)s[i];
   return h % bucketCount;
}
```

How to Protect Keys?

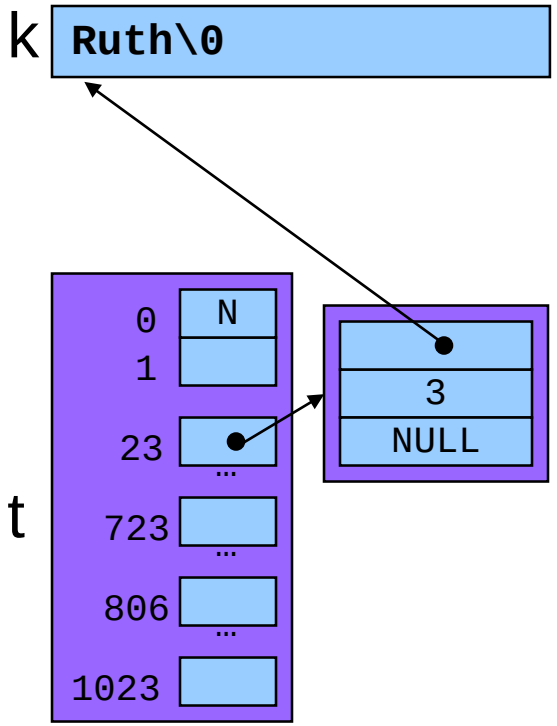
Suppose **`Table_add()`** function contains this code:

```
void Table_add(struct Table *t, const char *key, int value)
{  …
   struct Binding *p =
      (struct Binding*)malloc(sizeof(struct Binding));
   p->key = key;
   …
}
```

How to Protect Keys?

Problem: Consider this calling code:

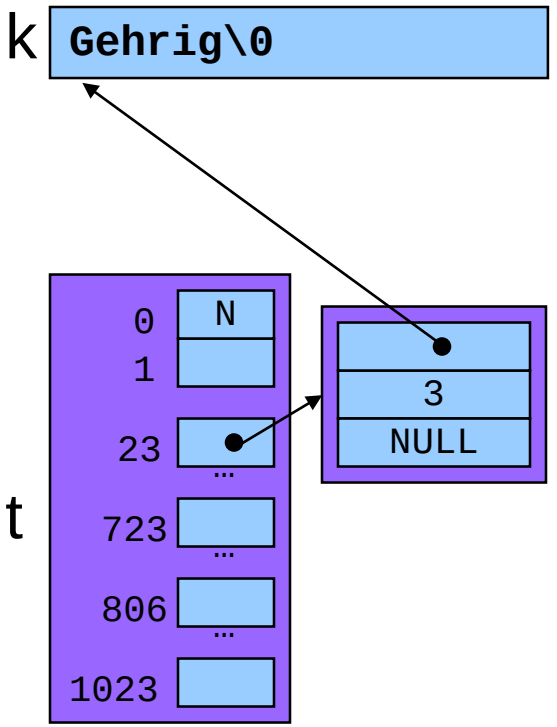
```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
```



How to Protect Keys?

Problem: Consider this calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```



What happens if the client searches t for "Ruth"? For Gehrig?

How to Protect Keys?

Solution: **`Table_add()`** saves a **defensive copy** of the given key

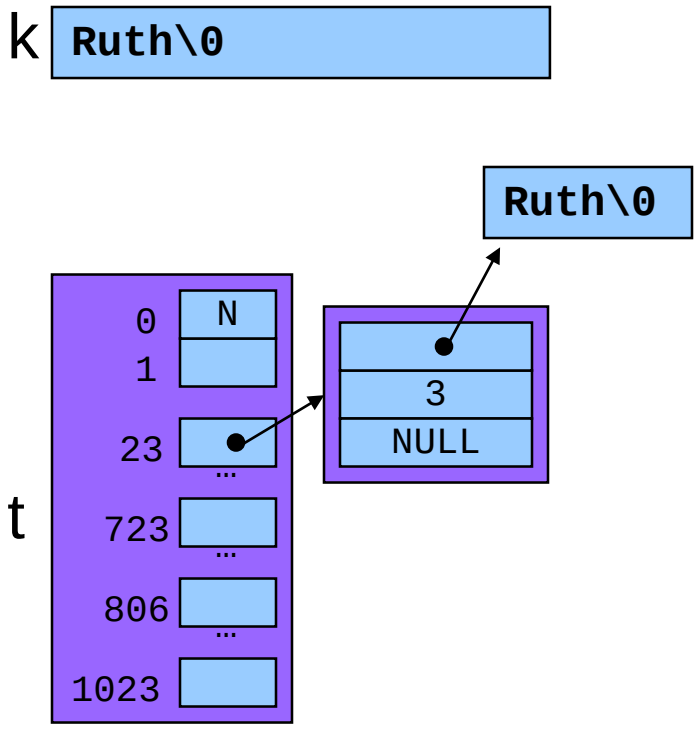
```
void Table_add(struct Table *t, const char *key, int value)
{  …
   struct Binding *p =
      (struct Binding*)malloc(sizeof(struct Binding));
   p->key = (const char*)malloc(strlen(key) + 1);
   strcpy((char*)p->key, key);
   …
}
```

Why add 1?

How to Protect Keys?

Now consider same calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
```

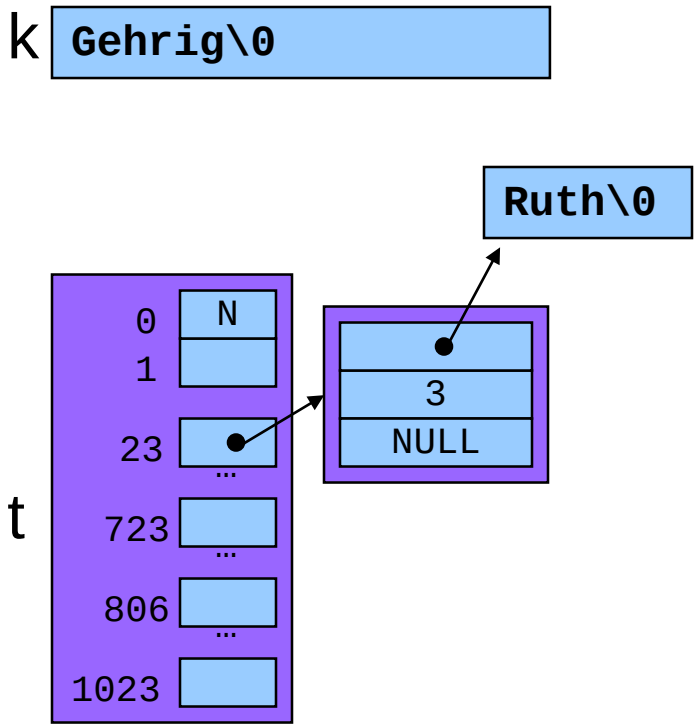


How to Protect Keys?

Now consider same calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

Hash table is not corrupted



Who Owns the Keys?

Then the hash table **owns** its keys

- That is, the hash table owns the memory in which its keys reside
- **`Hash_free()`** function must free the memory in which the key resides

Summary

Common data structures and associated algorithms

- Linked list
  - (Maybe) fast add
  - Slow search
- Hash table
  - (Potentially) fast add
  - (Potentially) fast search
  - Very common

Hash table issues

- Hashing algorithms
- Defensive copies
- Key ownership