COS320: Compiling Techniques

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April 11, 2024

Static Single Assignment form

SSA

- Each `%uid` appears on the left-hand-side of at most one assignment in a CFG

```
if (x < 0) {
  y := y - x;
} else {
  y := y + x;
}
return y
```

$\longrightarrow$

```
if (x₀ < 0) {
  y₁ := y₀ - x₀;
} else {
  y₂ := y₀ + x₀;
}
y₃ := φ(y₁, y₂)
return y₃
```

  - Recall: $y_3 := \phi(y_1, y_2)$ picks either y_1 or y_2 (whichever one corresponds to the branch that is actually taken) and stores it in y_3
- Well-formedness condition: uids must be defined before they are used.
  - Formal definition to follow!

Register allocation

- SSA form reduces register pressure
  - Each variable x is replaced by potentially many "subscripted" variables x_1, x_2, x_3,...
    - (At least) one for each definition of of x
  - Each x_i can potentially be stored in a different memory location

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- Interference graphs for SSA programs are *chordal* (every cycle contains a chord)
  - Chordal graphs can be colored optimally in polytime
  - (*But* optimal translation out of SSA form is intractable)

Dead assignment elimination

Simple algorithm for eliminating assignment[1] instructions that are never used:

while *some* $\%x$ *has no uses* **do**
  Remove definition of $\%x$ from CFG;

- SSA conversion $\Rightarrow$ more assignments are eliminated

```
x := 0
x := 1
return 2 * x
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[1] does *not* eliminate dead *stores*

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x_0 := 0
x_1 := 1
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```
x1 := 1
return 2 * x1
```

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Recall: constant propagation

- The goal of constant propagation: determine at each instruction I a *constant environment*
  - A constant environment is a symbol table mapping each variable x to one of:
    - an integer n (indicating that x's value is n whenever the program is at I)
    - $\top$ (indicating that x might take more than one value at I)
    - $\bot$ (indicating that x may take no values at run-time – I is unreachable)
- Say that the assignment **IN**, **OUT** is conservative if
  1. **IN**$[s]$ assigns each variable $\top$
  2. For each node *bb* $\in N$,
  $$\textbf{OUT}[bb] \sqsupseteq post_{CP}(bb, \textbf{IN}[bb])$$
  3. For each edge *src* $\rightarrow$ *dst* $\in E$,
  $$\textbf{IN}[dst] \sqsupseteq \textbf{OUT}[src]$$

(Dense) constant propagation performance

- Memory requirements: $\Theta(|N| \cdot |\mathit{Var}|)$
  - Constant environment has size $\Theta(|\mathit{Var}|)$, need to track $\Theta(1)$ per node
- Time requirements: $\Theta(|E| \cdot |\mathit{Var}|) = \Theta(|N| \cdot |\mathit{Var}|)$
  - Processing a single node takes $\Theta(1)$ time
  - Each edge is processed $\Theta(|\mathit{Var}|)$ times
    - Height of the abstract domain (length of longest strictly ascending sequence): $|\mathit{Var}| + 1$
- Can we do better?

Sparse constant propagation

- Idea: SSA connects variable *definitions* directly to their *uses*
  - Don't need to store the value of *every* variable at *every* program point
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- Can think of SSA as a graph, where edges correspond to *data flow* rather than *control flow*
  - Define *rhs*$(\%x)$ to be the right hand side of the **unique** assignment to $\%x$
  - Define *succ*$(\%x) = \{\%y :$ *rhs*$(\%y)$ *reads* $\%x\}$

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- Local specification for constant propagation:
  - *scp* is the smallest function *Uid* $\rightarrow \mathbb{Z} \cup \{\top, \bot\}$ such that
    - If G contains no assignments to *%x*, then *scp*$(\%x) = \top$
    - For each instruction ***%x = e***, *scp*$(\%x) = eval(e,$ *scp*$)$
    - For each instruction ***%x =ϕ(%y,%z)***, *scp*$(\%x) =$ *scp*$(\%y) \sqcup$ *scp*$(\%z)$

Worklist algorithm

$$\textit{scp}(\%x) = \begin{cases} \bot & \text{if } \%x \text{ has an assignment} \\ \top & \text{otherwise} \end{cases}$$

$\textit{work} \leftarrow \{\%x \in \textit{Uid} : \%x \text{ is defined}\}$;

while $\textit{work} \neq \emptyset$ **do**

  Pick some $\%x$ from work;

  $\textit{work} \leftarrow \textit{work} \setminus \{\%x\}$;

  if $\textit{rhs}(\%x) = \phi(\%y, \%z)$ **then**

    $v \leftarrow \textit{scp}(\%y) \sqcup \textit{scp}(\%z)$

  else

    $v \leftarrow \textit{eval}(\textit{rhs}(\%x), \textit{scp})$

  if $v \neq \textit{scp}(\%x)$ **then**

    $\textit{scp}(\%x) \leftarrow v$;

    $\textit{work} \leftarrow \textit{work} \cup \textit{succ}(\%x)$

Computational complexity of constant propagation

	Dense	Sparse
Memory	$\Theta(\lvert N \rvert \cdot \lvert \mathit{Var} \rvert)$	$\Theta(\lvert N \rvert) = \Theta(\lvert \mathit{Var} \rvert)$
Time	$\Theta(\lvert N \rvert \cdot \lvert \mathit{Var} \rvert)$	$\Theta(\lvert N \rvert) = \Theta(\lvert \mathit{Var} \rvert)$

- *However*, observe that we only find constants for uids, not stack slots.
  - Again, advantageous to use uids to represent variable whenever possible

Computing SSA

(High-level) SSA conversion

- Replace each definition $x = e$ with a $x_i = e$ for some unique subscript i
- Replace each *use* of a variable y with y_i, where the ith definition of y is the unique reaching definition

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- Replace each *use* of a variable y with y_i, where the ith definition of y is the unique reaching definition
- If multiple definitions reach a single use, then they must be merged using a ϕ (phi) statement

```
y := 0;
while (x >= 0) {
  x := x - 1;
  y := y + x;
}
return y
```

$\rightarrow$

```
y0 := 0;
while (true) {
  x2 = φ(x0, x1)
  y2 = φ(y0, y1)
  if (x2 < 0) break;
  x1 := x2 - 1;
  y1 := y2 + x1;
}
return y2
```

Placing ϕ statements

- Easy, inefficient solution: place a ϕ statement for each variable locaction at each *join point*
  - A *join point* is a node in the CFG with more than one predecessor

[2] The entry node of the CFG is considered to be an implicit definition of every variable

Placing ϕ statements

- Easy, inefficient solution: place a ϕ statement for each variable locaction at each *join point*
  - A *join point* is a node in the CFG with more than one predecessor
- Better solution: place a ϕ statement for variable x at location n exactly when the following **path convergence criterion** holds: there exist a pair of non-empty paths P_1, P_2 ending at n such that
  1. The start node of both P_1 and P_2 defines x[2]
  2. The only node shared by P_1 and P_2 is n
- The path convergence criterion can be implemented using the concept of *iterated dominance frontiers*

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Dominance

- Let $G = (N, E, s)$ be a control flow graph
- We say that a node $d \in N$ *dominates* a node $n \in N$ if every path from s to n contains d
  - Every node dominates itself
  - d *strictly dominates* n if d is not n
  - d *immediately dominates* n if d strictly dominates n and but does not strictly dominate any strict dominator of n.

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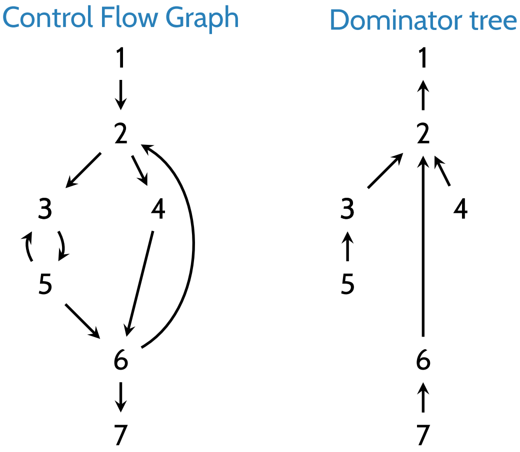
- Observe: dominance is a partial order on N
  - Every node dominates itself (reflexive)
  - If n_1 dominates n_2 and n_2 dominates n_3 then n_1 dominates n_3 (transitive)
  - If n_1 dominates n_2 and n_2 dominates n_1 then n_1 must be n_2 (anti-symmetric)

If we draw an edge from every node to its immediate dominator, we get a data structure called the *dominator tree*.

- (Essentially the Haase diagram of the dominated-by order)



Dominance and SSA

- SSA well-formedness criteria
  - If $\%x$ is used in a non-ϕ statement in block n, then the definition of $\%x$ must dominate n
  - If $\%x$ is the ith argument of a ϕ function in a block n, then the definition of $\%x$ must dominate the ith predecessor of n.

Dominator analysis

- Let $G = (N, E, s)$ be a control flow graph.
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- *Local specification*: *dom* is the largest (equiv. least in superset order) function such that
  - $dom(s) = \{s\}$
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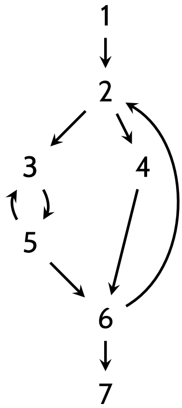
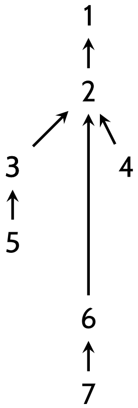
- Can be solved using dataflow analysis techniques
  - In practice: nearly linear time algorithm due to Lengauer & Tarjan

- Recall: If $\%x$ is the ith argument of a ϕ function in a block n, then the definition of $\%x$ must dominate the ith predecessor of n.
- The *dominance frontier* of a node n is the set of all nodes m such that n dominates a *predecessor* of m, but does not strictly dominate m itself.
    - $DF(n) = \{m : (\exists p \in Pred(m).n \in dom(p)) \wedge (m = n \vee n \notin dom(m))\}$
- Whenever a node n contains a definition of some uid $\%x$, then any node m in the dominance frontier of n needs a ϕ function for $\%x$.

Control Flow Graph



Dominator tree



- $DF(1) = \emptyset$

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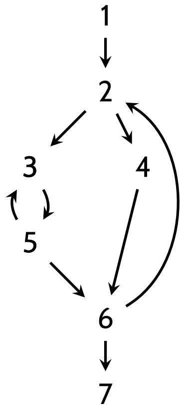

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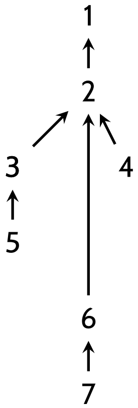
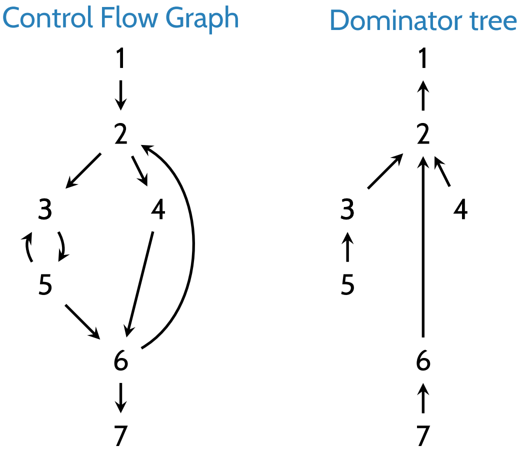


- $DF(1) = \emptyset$
- $DF(2) = \{2\}$



- $DF(1) = \emptyset$
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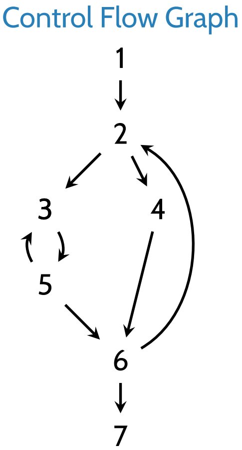
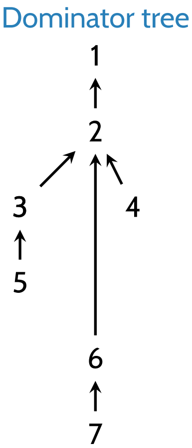
- $DF(3) = \{3, 6\}$





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- $DF(5) = \{3, 6\}$
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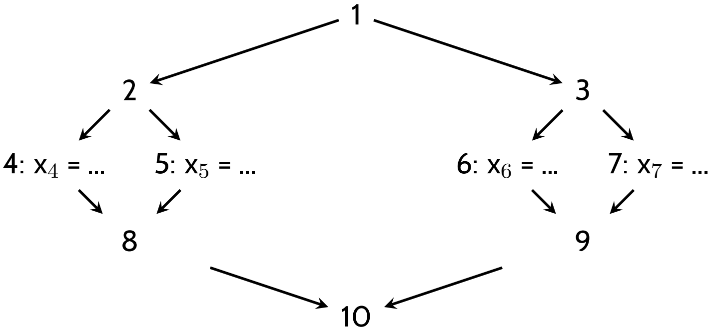
Dominance frontier is not enough!

- Whenever a node n contains a definition of some uid $\%x$, then any node m in the dominance frontier of n needs a ϕ statement for $\%x$.
- *But*, that is not the only place where ϕ statements are needed



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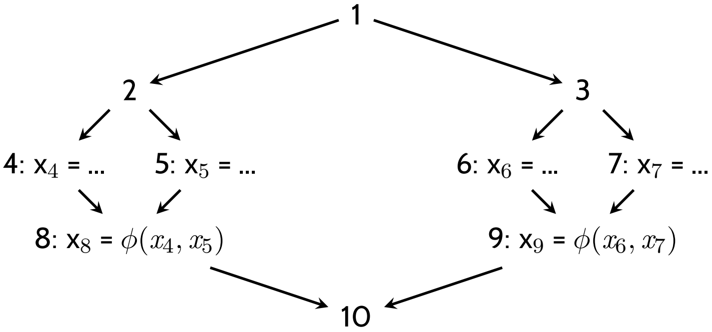

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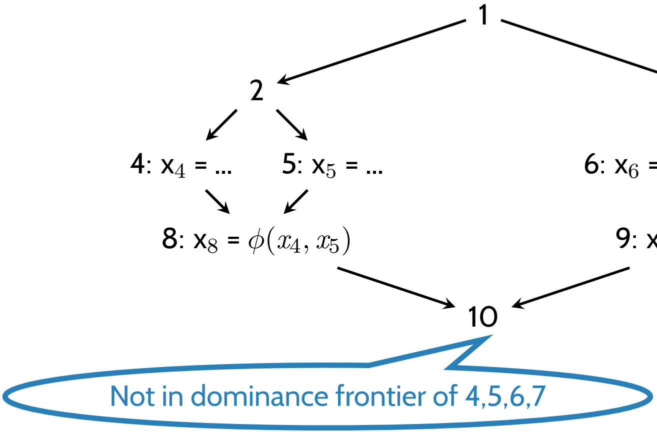

Placing ϕ statements

- Extend dominance frontier to sets of nodes by letting $DF(M) = \bigcup_{m \in M} DF(m)$
- Define the *iterated dominance frontier IDF*$(M) = \bigcup_i IDF_i(M)$, where
  - $IDF_0(M) = DF(M)$
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- For any node x, let *Def*(x) be the set of nodes that define x
- *Finally,* we can characterize ϕ statement placement:

Insert a ϕ statement for x at every node in $IDF($**Def**$(x))$

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- Using a graph coalescing register allocator, often possible to eliminate the resulting move instructions

SSA overview

- SSA can make analysis and optimization
  - simpler
  - more efficient
  - more accurate
- at the cost of
  - having to compute SSA / maintain SSA invariants
  - complicating code generation
- Most imperative compilers use SSA: LLVM, gcc, HotSpot, mono, v8, spidermonkey, go, ...
- *Dominance* is the key idea needed to efficiently transform into SSA
  - Will also make an appearence next week when we talk about loop optimizations