**Precept Outline**

- Review of Lectures 17 and 18:
  - Minimum Spanning Trees
  - Shortest Paths
  - Algorithm Design

**Relevant Book Sections**

- Book chapters: 4.3 and 4.4

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### A. Review: MSTs and Shortest Paths

Your preceptor will briefly review key points of this week's lectures.

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### B. Dorm Rooms and Routers

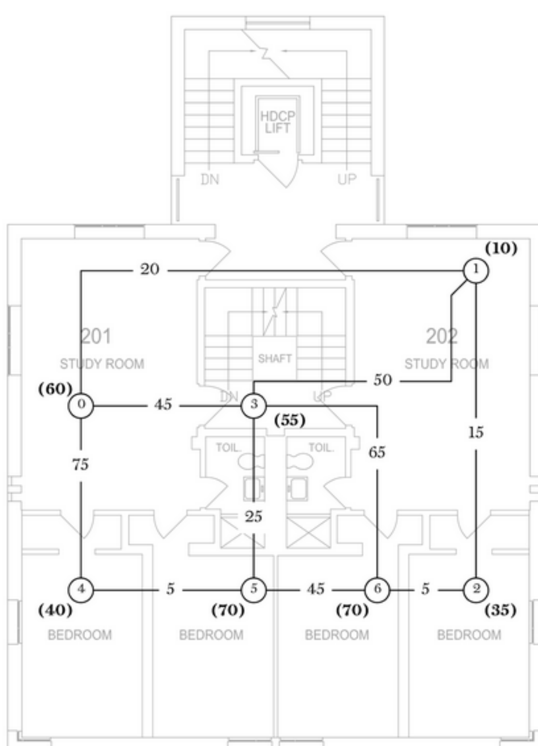
A college has just unveiled a brand-new dorm facility with $n$ rooms. They need to make sure all of them have an internet connection (of course), and are looking for the most cost-effective way to do so. Room number $i$ has internet access if either of the following is true:

- There is a router installed in room $i$.
- Room $i$ is connected by some fiber path to a room $j$ which has internet access.

Installing a router in room $i$ costs $r_i > 0$, and putting down fiber between rooms $i$ and $j$ costs $f_{ij} > 0$.

The goal of this problem is to determine in which rooms to install a router, and in which pair of rooms to connect together with fiber, so as to minimize the total cost.

Formulate this as a *minimum spanning tree problem*: define a graph $G = (V, E)$ with vertices $V = \{1, 2, \ldots, n\}$ and edges/edge weights that depend on $r_i$ and $f_{ij}$. You may use the example below to test your formulation.



*This instance contains 7 dorm rooms and 10 possible connections. The router installation costs are indicated in bold and parentheses; the fiber costs are given on the edges.*

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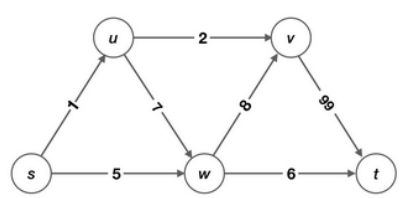
### C. Shortest Teleport Path

*This problem was taken and slightly adapted from the Fall 2014 Final exam*

Given an edge-weighted digraph $G$ with non-negative edge weights, a source vertex $s$ and a destination vertex $t$, find a shortest path from $s$ to $t$ where you are permitted to teleport across one edge for free. That is, the weight of a path is the sum of the weights of all but the largest edge weights in the path.

For example, in the edge-weighted digraph below, the shortest path from $s$ to $t$ is $s \to u \to t$ (with weight 11) but the shortest teleport path is $s \to u \to v \to t$ (with weight $1 + 2 + 99 - 99 = 3$).




A full solution should run in $O(E \log V)$ time and $O(V)$ extra space.

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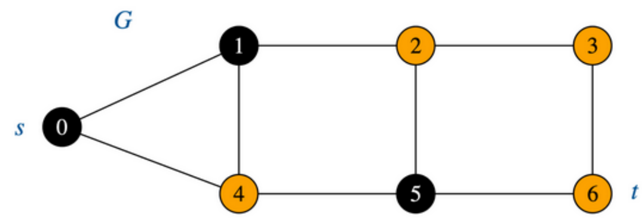
### D. Shortest Tiger Path

*This problem was taken and slightly adapted from the Spring 2023 Final exam*

Consider a graph $G$ in which each vertex is colored black or orange. A *tiger path* is a path that contains exactly one edge whose endpoints have opposite colors.

Our goal is to solve the *shortest tiger path problem*: given an undirected unweighted graph $G$ and two vertices $s$ and $t$, find a tiger path between $s$ and $t$ that uses the fewest edges (or report that no such path exists).

For example, the shortest path between $s = 0$ and $t = 6$ in the graph below is $0 \to 4 \to 5 \to 6$, but it is not a tiger path; the shortest tiger path is $0 \to 1 \to 2 \to 3 \to 6$.




Formulate the shortest tiger path problem as a traditional (unweighted) shortest path problem in a directed graph. Specifically, define a digraph $G'$, source $s$, and destination $t$ such that the length of the shortest path from $s$ to $t$ in $G'$ is always equal to the length of the shortest tiger path between $s$ and $t$ in $G$. For simplicity, you may assume that $s$ is black and $t$ is orange.

For full credit, the number of vertices in $G'$ must be $\Theta(V)$ and the number of edges must be $\Theta(E)$, where $V$ and $E'$ are the number of vertices and edges in $G'$, respectively.