COS 217: Introduction to Programming Systems

Data Structures

PRINCETON UNIVERSITY

Goals of this Lecture

Help you learn (or refresh your memory) about:
- Common data structures: linked lists and hash tables

Why? Deep motivation:
- Common data structures serve as "high level building blocks"
- A mature programmer:
  - Rarely creates programs from scratch
  - Often creates programs using high level building blocks

Why? Shallow motivation:
- Provide background pertinent to Assignment 3
- ... especially for those who haven't taken COS 226
- ... especially for those who skipped COS 126

Symbol Table Data Structure

Goal: maintain a collection of key/value pairs

- For now, each key is a **string**; each value is an **int**
- Lookup by key, get value back
- Unknown number of key-value pairs

Examples

- (student name, class year)
  - ("Andrew Appel", 81), ("Jen Rexford", 91), ("JP Singh", 87)
- (baseball player, number)
  - ("Ruth", 3), ("Gehrig", 4), ("Mantle", 7)
- (variable name, value)
  - ("maxLength", 2000), ("i", 7), ("j", -10)

Agenda

Linked lists

Hash tables

Hash table issues

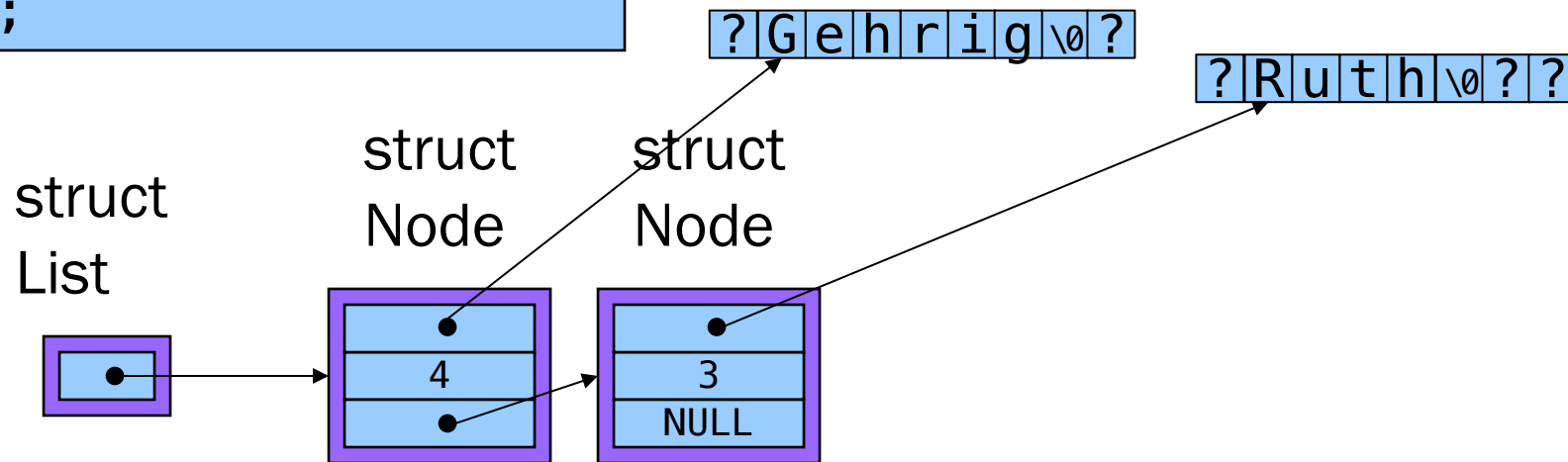Symbol table key ownership

Linked List Data Structure (for a Symbol Table)

```
struct Node {
    const char *key;
    int value;
    struct Node *next;
};

struct List {
    struct Node *first;
};
```

Your Assignment 3 data structures will be more general and perhaps more elaborate
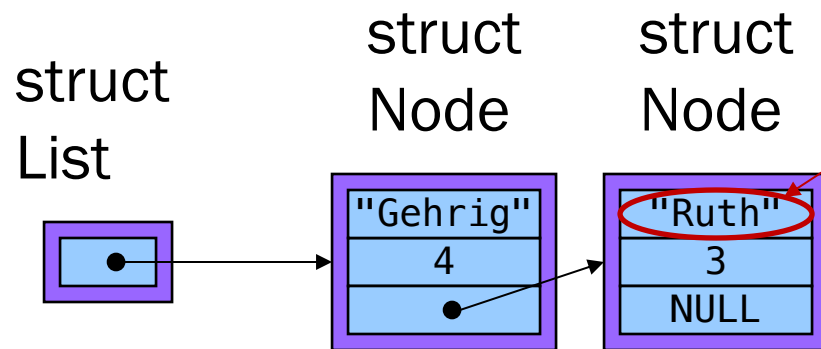
Linked List Data Structure

```
struct Node {
    const char *key;
    int value;
    struct Node *next;
};

struct List {
    struct Node *first;
};
```

Your Assignment 3 data structures will be more general and perhaps more elaborate

struct List

struct Node

struct Node

Really this is the address at which a string with contents "Ruth" resides

| "Gehrig" |
| 4 |
| • |

| "Ruth" |
| 3 |
| NULL |

Preview of A3/Lecture 13: Encapsulation (wrong!)

list.h

```
struct Node {const char* key; int value; struct Node *next;};
struct List {struct Node *first;};

struct List *new();
void insert(struct List *p, const char* key, int value);
void concat(struct List *p,
            struct List *q);
int nth_value(struct List *p, int n);
```

Nothing stops a client from doing this!

```
p->first = NULL;
```

If you put the representation here, then it's not an **abstract** data type, it's just a data type.

client.c

```
#include "list.h"

int f(void) {
  struct List *p, *q;
  p = new();
  q = new();
  insert(p,"six",6);
  insert(p,"sept",7);
  insert(q,"cinq",5);
  concat(p,q);
  concat(q,p);
  return nth_value(q,1);
}
```

list_linked.c

```
#include "list.h"

struct List *new()
{
  struct List *p;
  p = calloc(1, sizeof(*p));
  if(p == NULL) { cry(); return NULL; }
  return p;
}

void insert(struct List *p, const char* key, int value) {...}

void concat(struct List *p, struct List *q) { ... }

int nth_value(struct List *p, int n) { ... }
```

Preview of A3/Lecture 12: Encapsulation (right!)

list.h

```
struct List;
typedef struct List *List_T;

List_T new();
void insert(List_T p, const char* key, int value);
void concat(List_T p,
            List_T q);
int nth_value(List_T p, int n);
```

Now this code won't compile!

```
p->first = NULL;
```

Including only the declaration in header file **enforces** the abstraction: it keeps clients from accessing fields of the struct, allowing implementation to change

client.c

```
#include "list.h"

int f(void) {
  List_T p, q;
  p = new();
  q = new();
  insert(p,"six",6);
  insert(p,"sept",7);
  insert(q,"cinq",5);
  concat(p,q);
  concat(q,p);
  return nth_value(q,1);
}
```

list_linked.c

```
#include "list.h"

struct Node {const char *key; int value; struct Node *next;};
struct List {struct Node *first;};

struct List *new()
{
  struct List *p;
  p = calloc(1, sizeof(*p));
  if(p == NULL) {cry(); return NULL;}
  return p;
}

void insert(struct List *p, const char* key, int value) {...}
void concat(struct List *p, struct List *q) { ... }
int nth_value(struct List *p, int n) { ... }
```
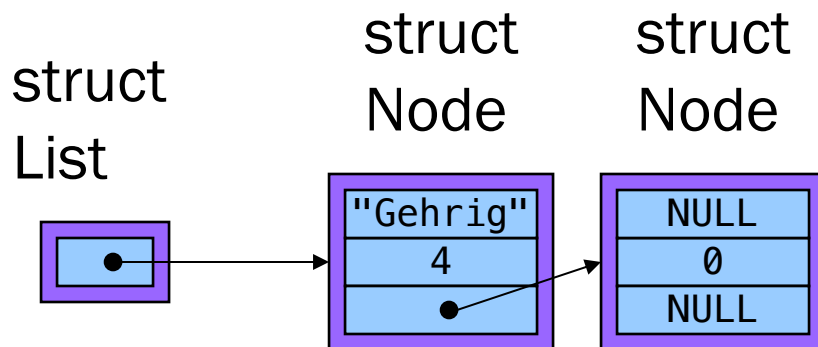
```
struct Node {
    const char *key;
    int value;
    struct Node *next;
};

struct List {
    struct Node *first;
};
```

```
struct List lineup;
struct Node g;
struct Node* r =
    calloc(1,sizeof(struct Node));
g.key = "Gehrig";
lineup.first = &g;
(*lineup.first).value = 4;
lineup.first->value = 4;
(*lineup.first).next = r;
lineup.first->next = r;
```

struct
List

struct
Node

struct
Node

	"Gehrig"		NULL
•	4		0
	•		NULL

Linked List Algorithms

Create
- Allocate `List` structure; set `first` to `NULL`
- Performance: $O(1)$ ⇒ fast

Add (no check for duplicate key required)
- Insert new node containing key/value pair at front of list
- Performance: $O(1)$ ⇒ fast

Add (check for duplicate key required)
- Traverse list to check for node with duplicate key
- Insert new node containing key/value pair into list
- Performance: $O(n)$ ⇒ slow

Linked List Algorithms

Search

- Traverse the list, looking for given key
- Stop when key found, or reach end
- Performance: ???

iClicker Question

Q: How fast is searching for a key in a linked list?

A. Always fast – O(1)

B. Always slow – O(n)

C. On average, fast

D. On average, slow

Not well specified:

Depends on order of inserts, queries, etc.

Best answer is D.

Linked List Algorithms

Search

- Traverse the list, looking for given key
- Stop when key found, or reach end
- Performance: $O(n) \Rightarrow$ slow

Free

- Free `Node` structures while traversing
- Free `List` structure
- Performance: $O(n) \Rightarrow$ slow

Agenda

Linked lists

Hash tables

Hash table issues

Symbol table key ownership

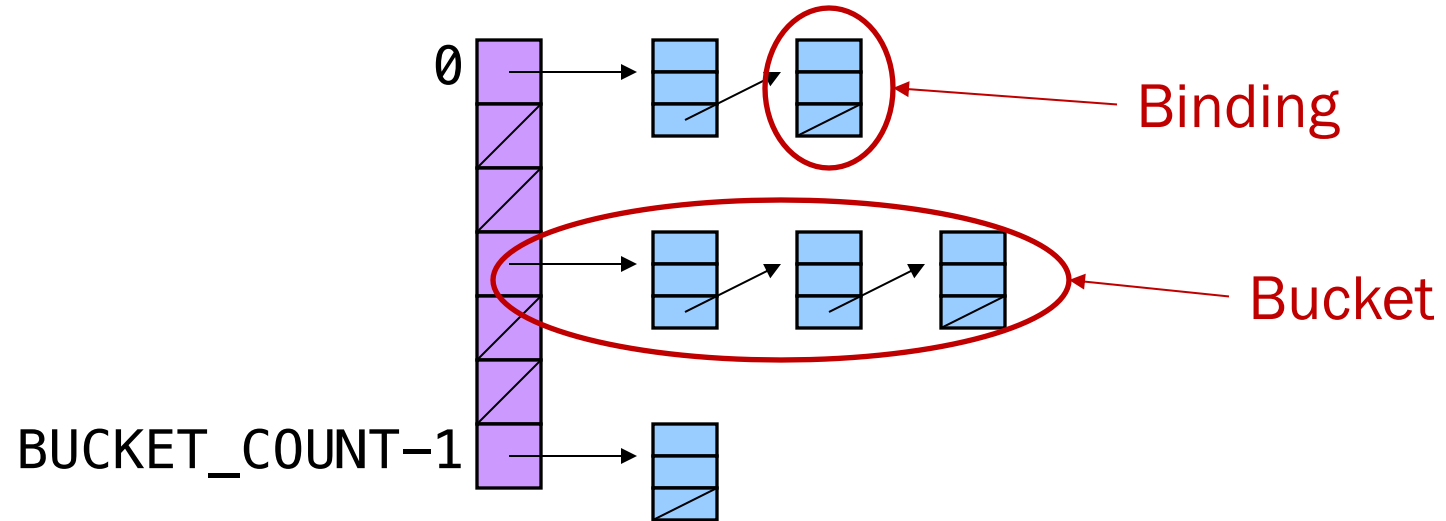# Hash Table Data Structure (For COS 226 nerds – hashing with separate chaining)

Array of linked lists

```
enum { BUCKET_COUNT = 1024 };

struct Binding {
   const char *key;
   int value;
   struct Binding *next;
};

struct Table {
   struct Binding *buckets[BUCKET_COUNT];
};
```

Really this is the address at which "Ruth" resides

struct Table

struct Binding

struct Binding

0	NULL
1	NULL
...	
23	●
...	
723	●
...	
806	NULL
...	
1023	NULL

"Ruth"
3
NULL

"Gehrig"
4
NULL

15
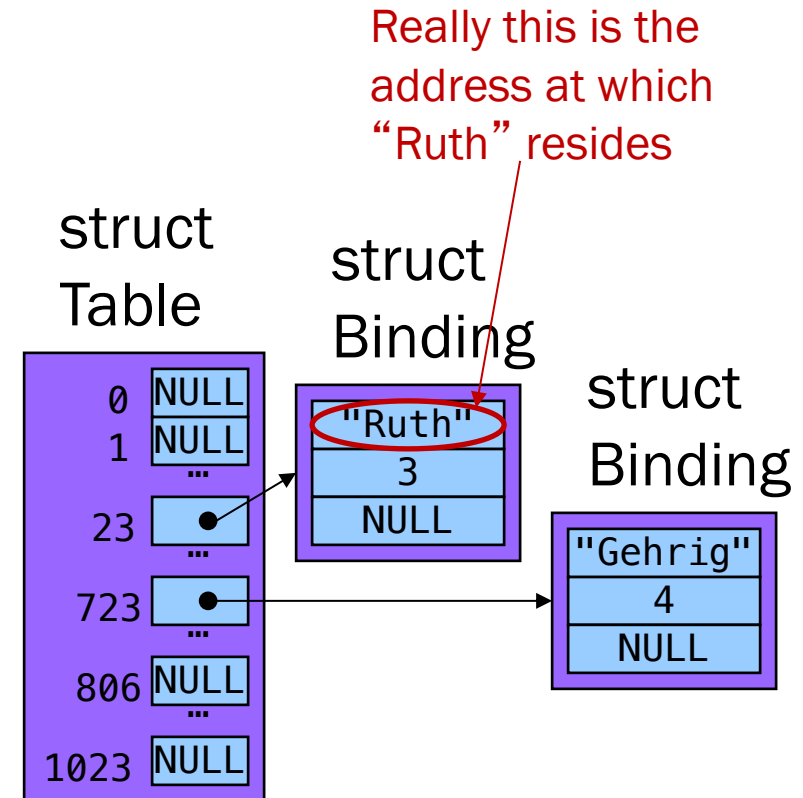
Hash Table Data Structure

Hash function maps given key to an integer

Mod integer by `BUCKET_COUNT` to determine proper bucket

Hash Table Example

Example: `BUCKET_COUNT` = 7

Add (if not already present) bindings with these keys:

- the, cat, in, the, hat

Hash Table Example (cont.)

First key: "the"

- hash("the") = 965156977; 965156977 % 7 = 1

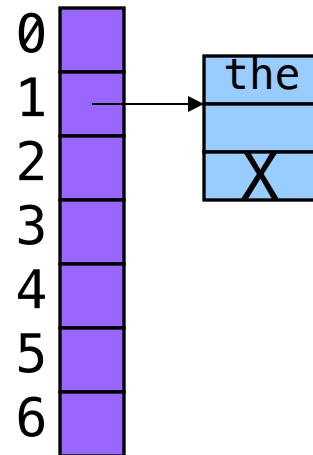Search `buckets[1]` for binding with key "the"; not found
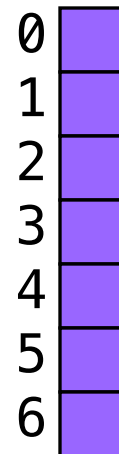
Add binding with key "the" and its value to `buckets[1]`

Hash Table Example (cont.)

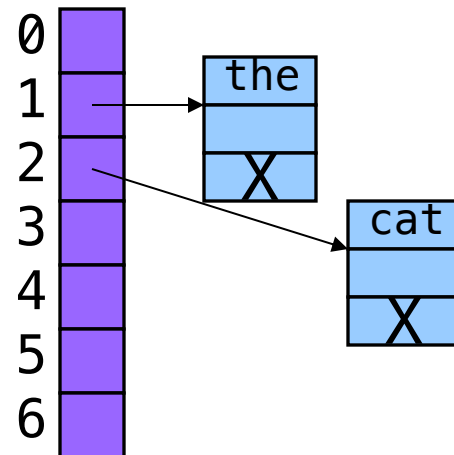Second key: "cat"
- hash("cat") = 3895848756; 3895848756 % 7 = 2

Search `buckets[2]` for binding with key "cat"; not found
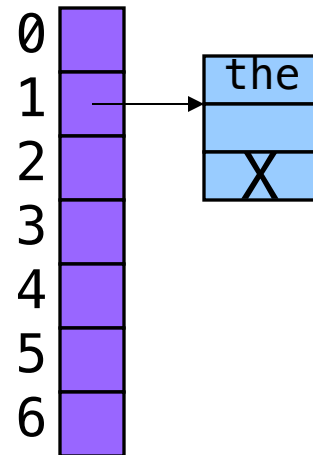
Add binding with key "cat" and its value to `buckets[2]`

Third key: "in"

- hash("in") = 6888005; 6888005% 7 = 5

Search `buckets[5]` for binding with key "in"; not found
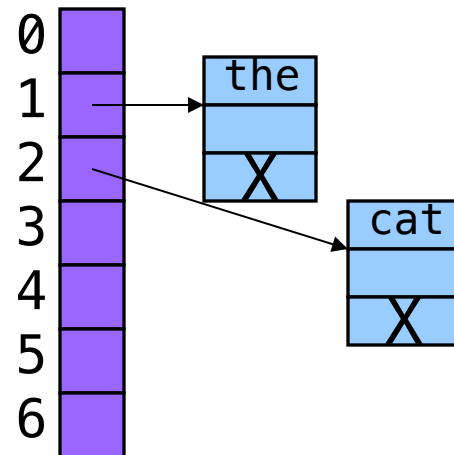
Add binding with key "in" and its value to `buckets[5]`

23
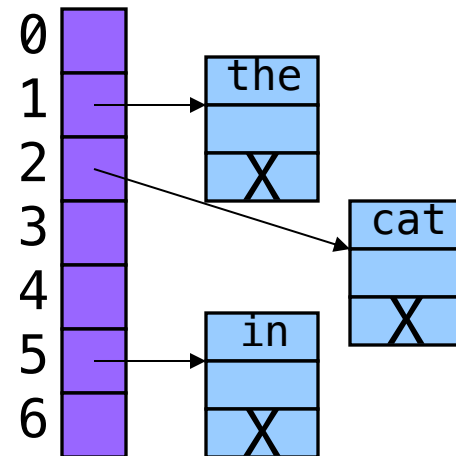
Fourth word: "the"
- hash("the") = 965156977; 965156977 % 7 = 1

Search `buckets[1]` for binding with key "the"; found it!
- Don't change hash table

```
0
1 ──→ the
2 ──┐    X
3   └──→ cat
4          X
5 ──→ in
6      X
```
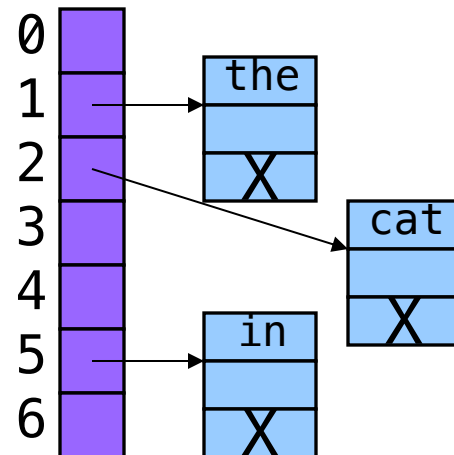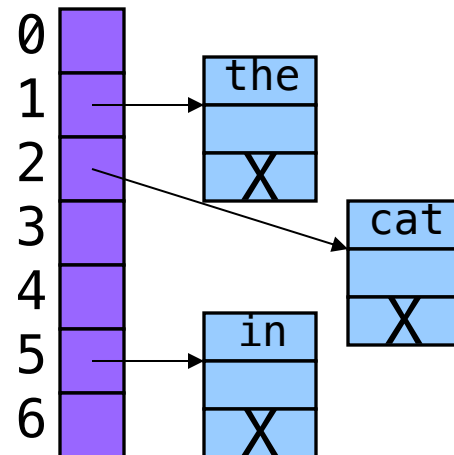
Fifth key: "hat"

- hash("hat") = 865559739; 865559739 % 7 = 2
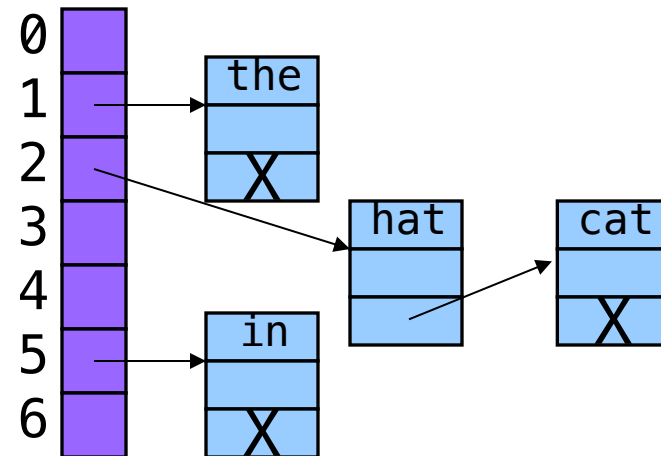
Search `buckets[2]` for binding with key "hat"; not found

Add binding with key "hat" and its value to `buckets[2]`
- At front or back?

Hash Table Algorithms

Create

- Allocate `Table` structure; set each bucket to `NULL`
- Performance: O(1) ⇒ fast

Add

- Hash the given key
- Mod by `BUCKET_COUNT` to determine proper bucket
- Traverse proper bucket to make sure no duplicate key
- Insert new binding containing key/value pair into proper bucket
- Performance: ???

Q: How fast is adding a key to a hash table?

A. Always fast

B. Usually fast, but depends on how many keys are in the table

C. Usually fast, but depends on how many keys hash to the same bucket

D. Usually slow

E. Always slow

C

If bindings are spread across buckets, this is fast (though B is a concern).

Worst case: everything hashes to the same bucket – O(n)

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Hash Table Algorithms

Search

- Hash the given key
- Mod by `BUCKET_COUNT` to determine proper bucket
- Traverse proper bucket, looking for binding with given key
- Stop when key found, or reach end
- Performance: Usually O(1) ⇒ fast

Free

- Traverse each bucket, freeing bindings
- Free `Table` structure
- Performance: O(n) ⇒ slow

Agenda

Linked lists

Hash tables

Hash table issues

Symbol table key ownership

Many!
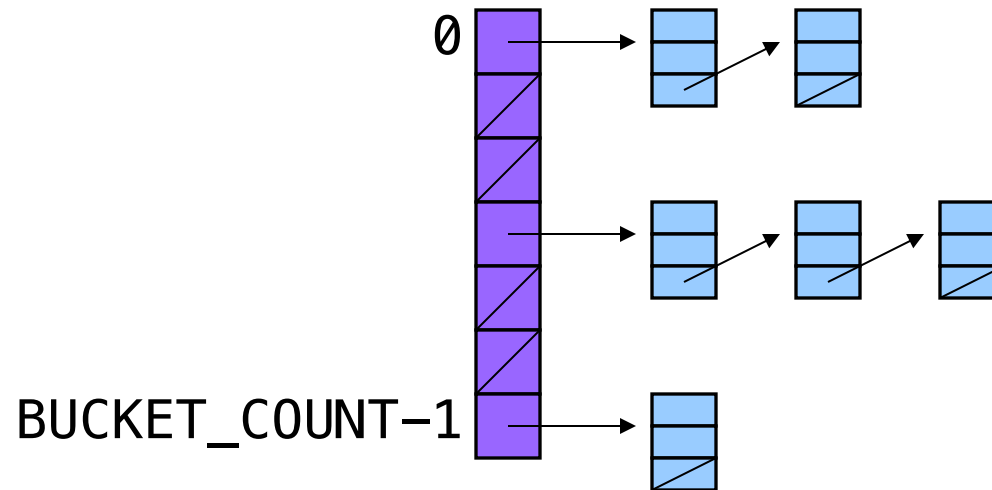- Too few ⇒ large buckets ⇒ slow add, slow search
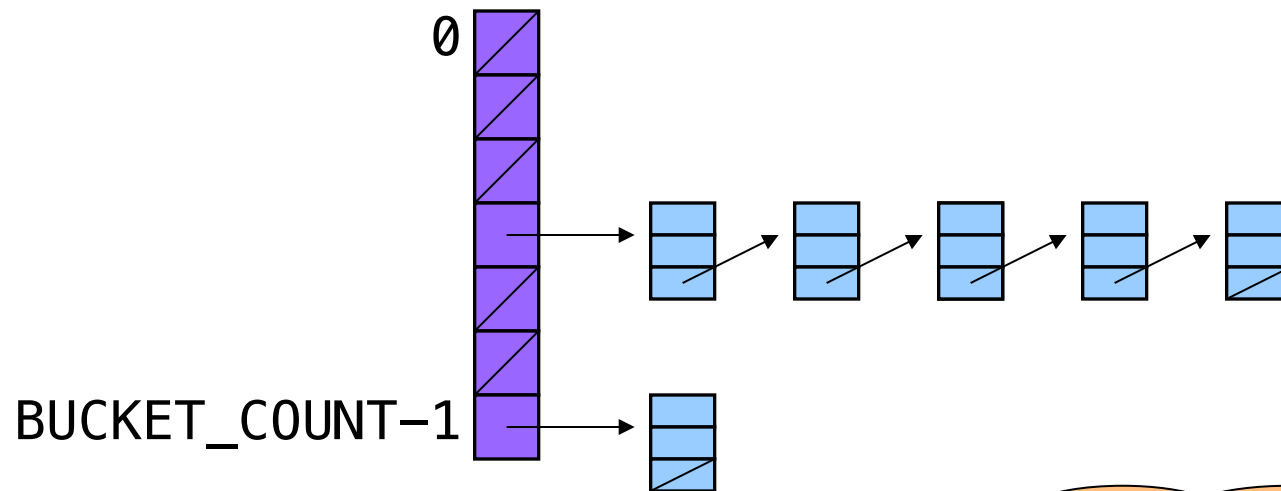
But not too many!
- Too many ⇒ memory is wasted

This is OK:

What Hash Function?

Should distribute bindings across the buckets well

- Distribute bindings over the range `0, 1, …, BUCKET_COUNT–1`
- Distribute bindings *evenly* to avoid very long buckets

This is not so good:

What would be the worst possible hash function?

How to Hash Strings?

Simple hash schemes don't distribute the keys evenly

- Number of characters, mod `BUCKET_COUNT`
- Sum the numeric codes of all characters, mod `BUCKET_COUNT`
- …

A reasonably good hash function:

- Weighted sum of characters s_i in the string s
  - $(\Sigma\ a^i s_i)\ \text{mod}\ \texttt{BUCKET\_COUNT}$
- Best if a and `BUCKET_COUNT` are relatively prime (i.e., their GCD is 1)
  - e.g., $a = 65599$, `BUCKET_COUNT` = 1024

A bit of math, and translation to code, yields:

```
size_t hash(const char *s, size_t bucketCount)
{
    enum { HASH_MULT = 65599 };
    size_t i;
    size_t h = 0;
    for (i = 0; s[i] != '\0'; i++)
        h = h * HASH_MULT + (size_t)s[i];
    return h % bucketCount;
}
```

Agenda

Linked lists

Hash tables

Hash table issues

Symbol table key ownership

How to Protect Keys?

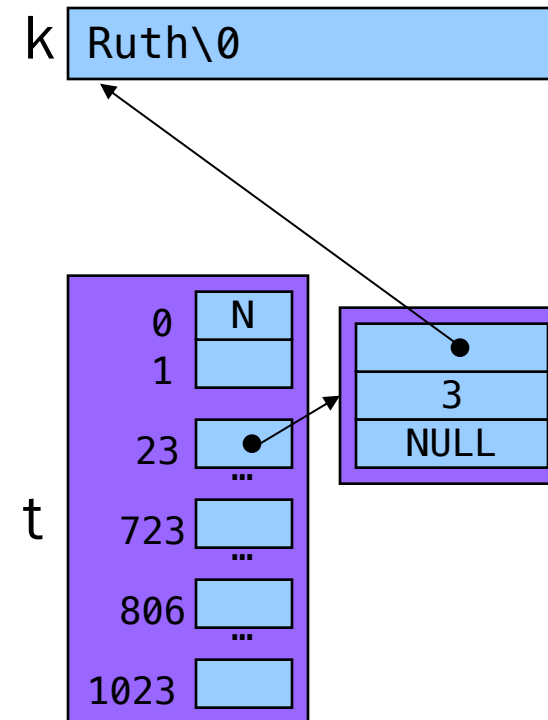Suppose a hash table function `Table_add()` contains this code:

```
void Table_add(struct Table *t, const char *key, int value)
{   …
    struct Binding *p =
        (struct Binding*)malloc(sizeof(struct Binding));
    p->key = key;

    …
}
```

How to Protect Keys?

Problem: Consider this calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
```
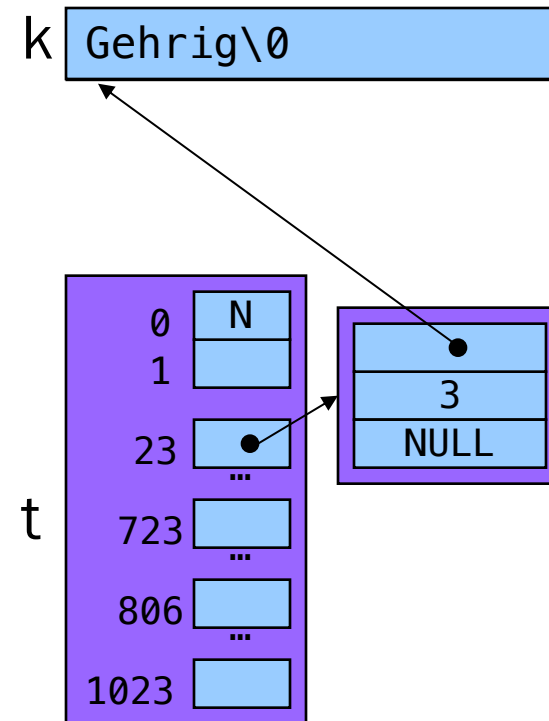
Problem: Consider this calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

k is REALLY &k[0]!

What happens if the client searches t for "Ruth"? For "Gehrig"?

k `Gehrig\0`

0	N
1	
23	•
723	
806	
1023	

t

| 3 |
| NULL |

Solution: Table_add() saves a defensive copy of the given key

```
void Table_add(struct Table *t, const char *key, int value)
{   …
    struct Binding *p =
        (struct Binding*)malloc(sizeof(struct Binding));
    p->key = (const char*)malloc(strlen(key) + 1);
    strcpy((char*)p->key, key);
    …
}
```
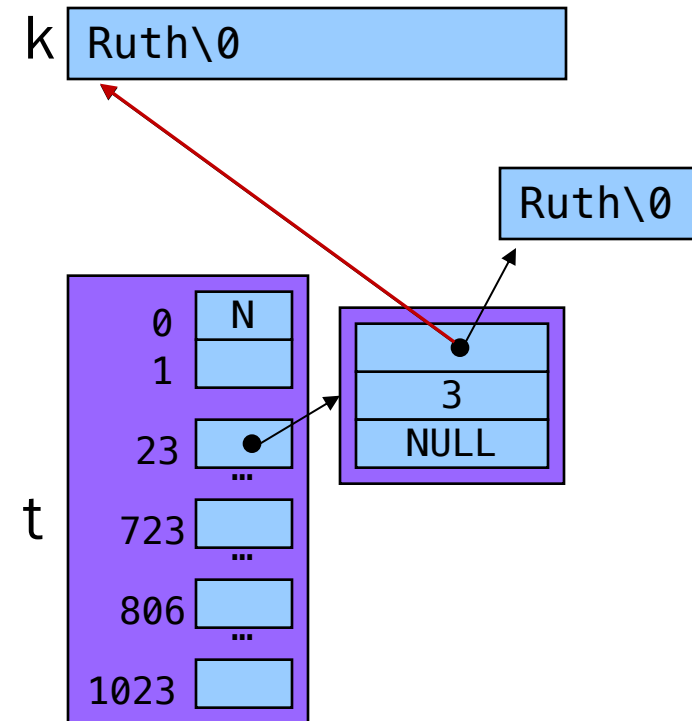
Why add 1?

What is missing from this code that you should have in yours?

Now consider same calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
```

k | Ruth\0

Ruth\0

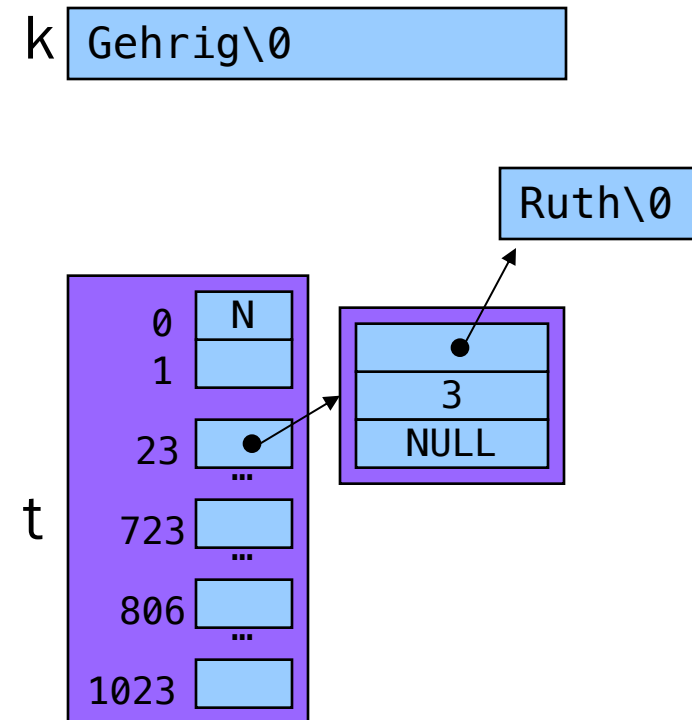t

0	N
1	
23	•
723	…
806	…
1023	

•
3
NULL

40

Now consider same calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

k | Gehrig\0

Hash table is
not corrupted!

Ruth\0

t

0	N
1	
23	•
723	
806	
1023	

•
3
NULL

Who Owns the Keys?

Then the hash table **owns** its keys
- That is, the hash table allocated the memory in which its keys reside
- `Table_remove()` function must also free the memory in which the key resides, not just the binding containing the key

Summary

Common data structures and associated algorithms
- Linked list
  - (Maybe) fast add
  - Slow search
- Hash table
  - (Potentially) fast add
  - (Potentially) fast search
  - Very common

Hash table issues
- (Initial) Bucket array size
- Hashing algorithms

Symbol table concerns
- Key ownership