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1.2 BUILT-IN DATA TYPES

- ▶ *strings*
- ▶ *integers*
- ▶ *floating-point numbers*
- ▶ *booleans*
- ▶ *type conversion*

Built-in data types

A **data type (type)** is a set of values and a set of operations on those values.

| type | set of values | example values | examples of operations |
|----------------|--------------------------------|-------------------------------------|---|
| <i>int</i> | <i>integers</i> | 17 -12345 | <i>add, subtract, multiply, divide, compare, equality</i> |
| <i>double</i> | <i>floating-point numbers</i> | 2.5 -0.125 | <i>add, subtract, multiply, divide, compare, equality</i> |
| <i>boolean</i> | <i>truth values</i> | true false | <i>and, or, not, equality</i> |
| <i>String</i> | <i>sequences of characters</i> | "Hello, World" "COS 126 is fun!" | <i>concatenate</i> |

Java's built-in data types
(that we use regularly in this course)

Programming terminology

Program. Sequence of statements. ← *for now*

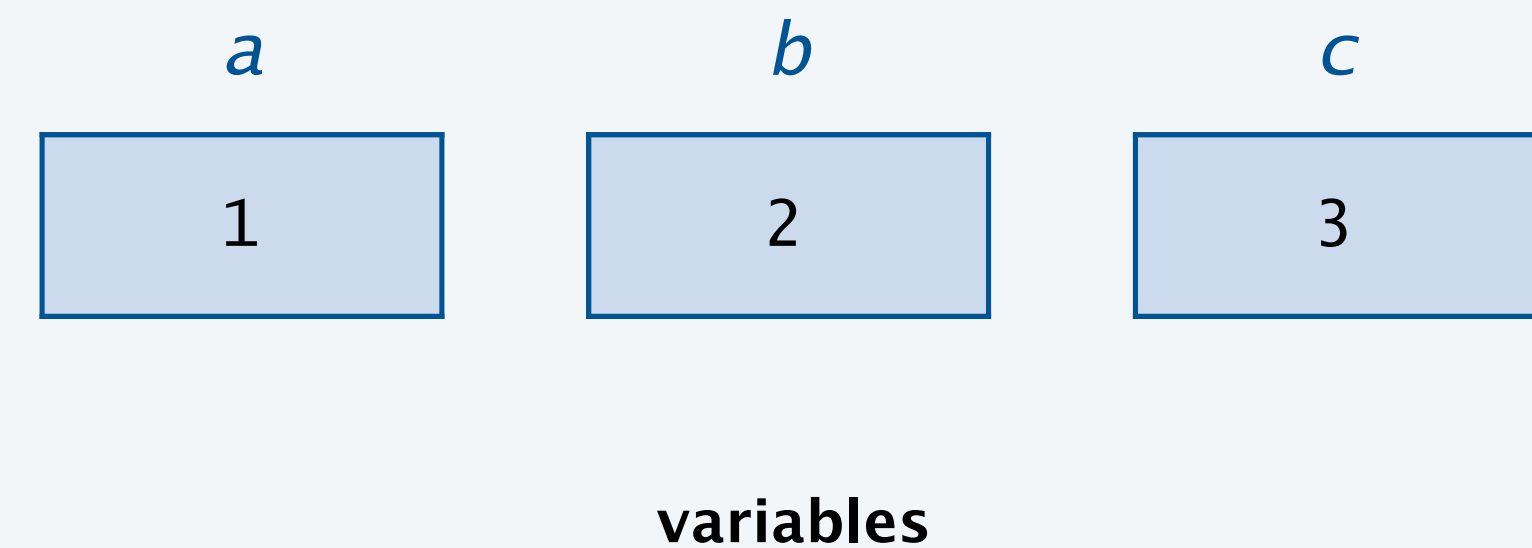
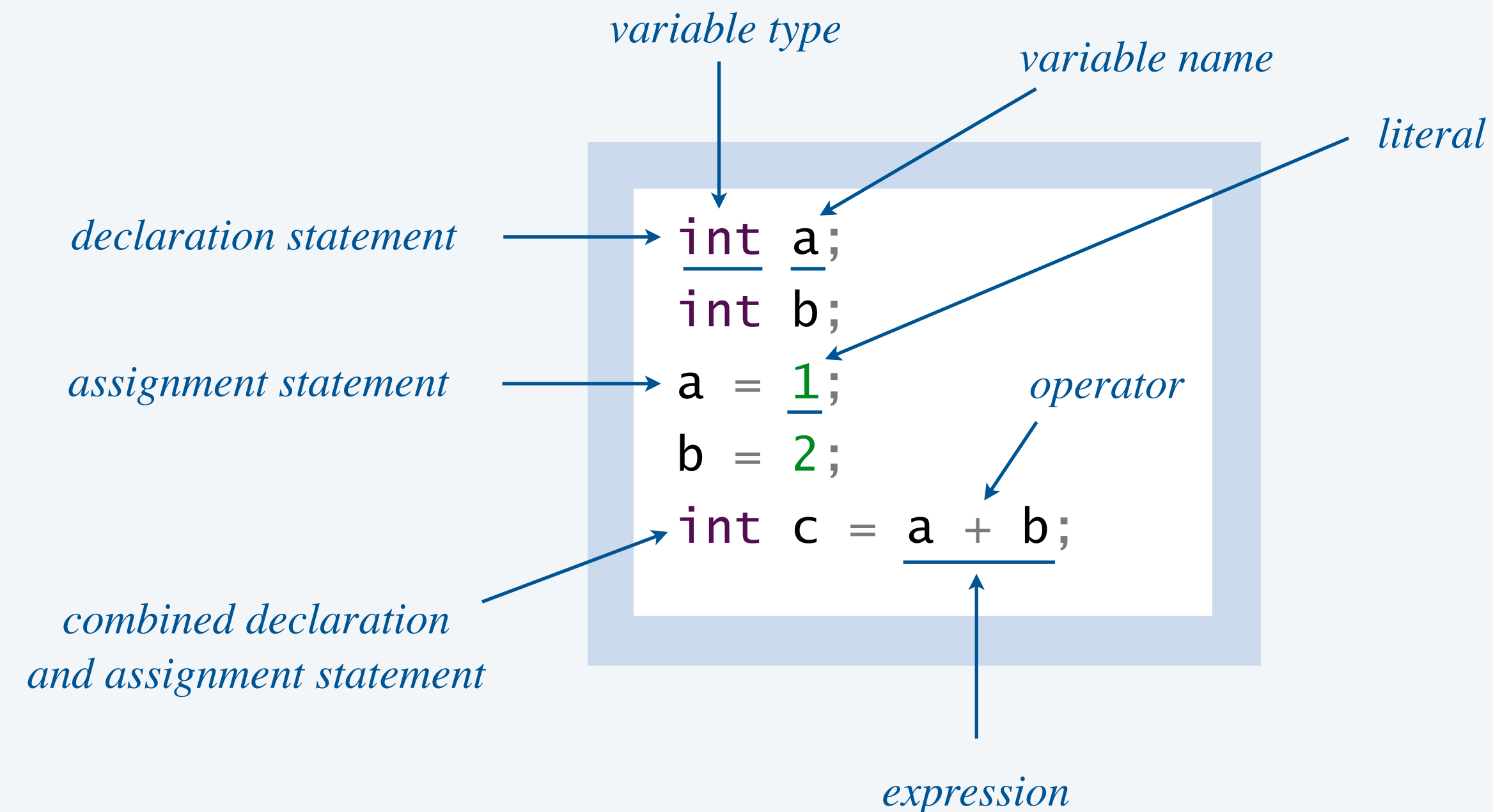
Declaration statement. Associates a variable with a name and type.

Variable. A place to store a data-type value.

Assignment statement. Stores a value in a variable.

Literal. Programming-language representation of a data-type value.

Expression. A combination of variable names, literals, operators, etc. that evaluates to a value.

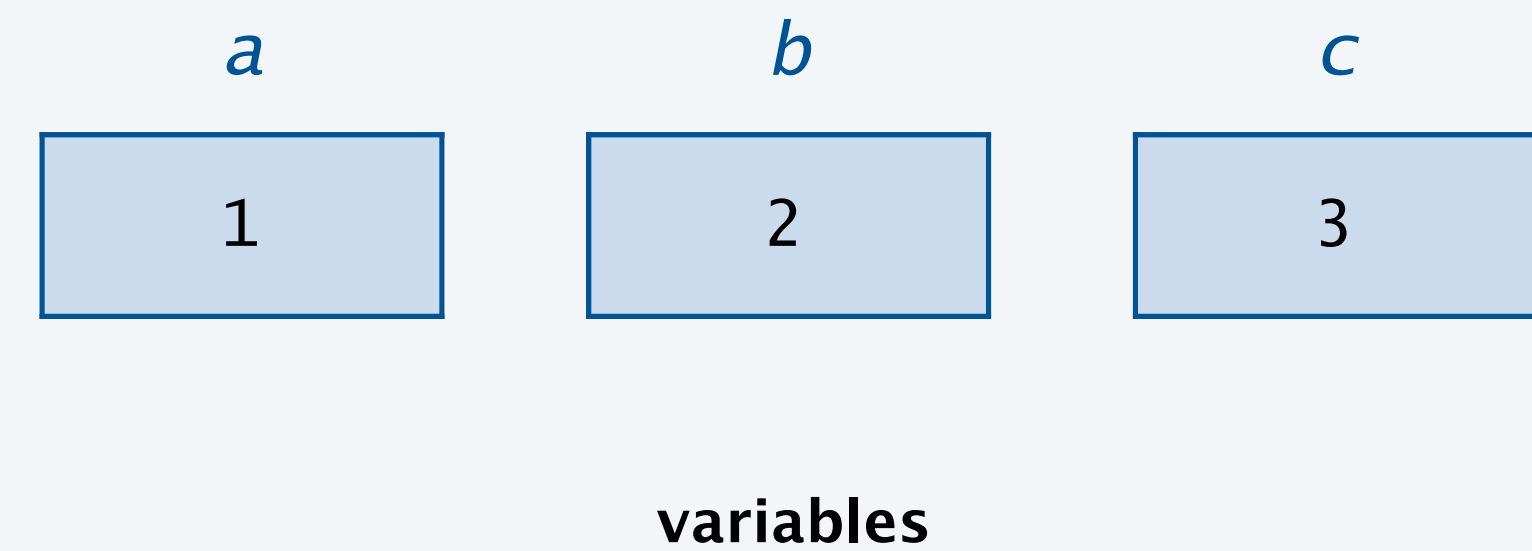
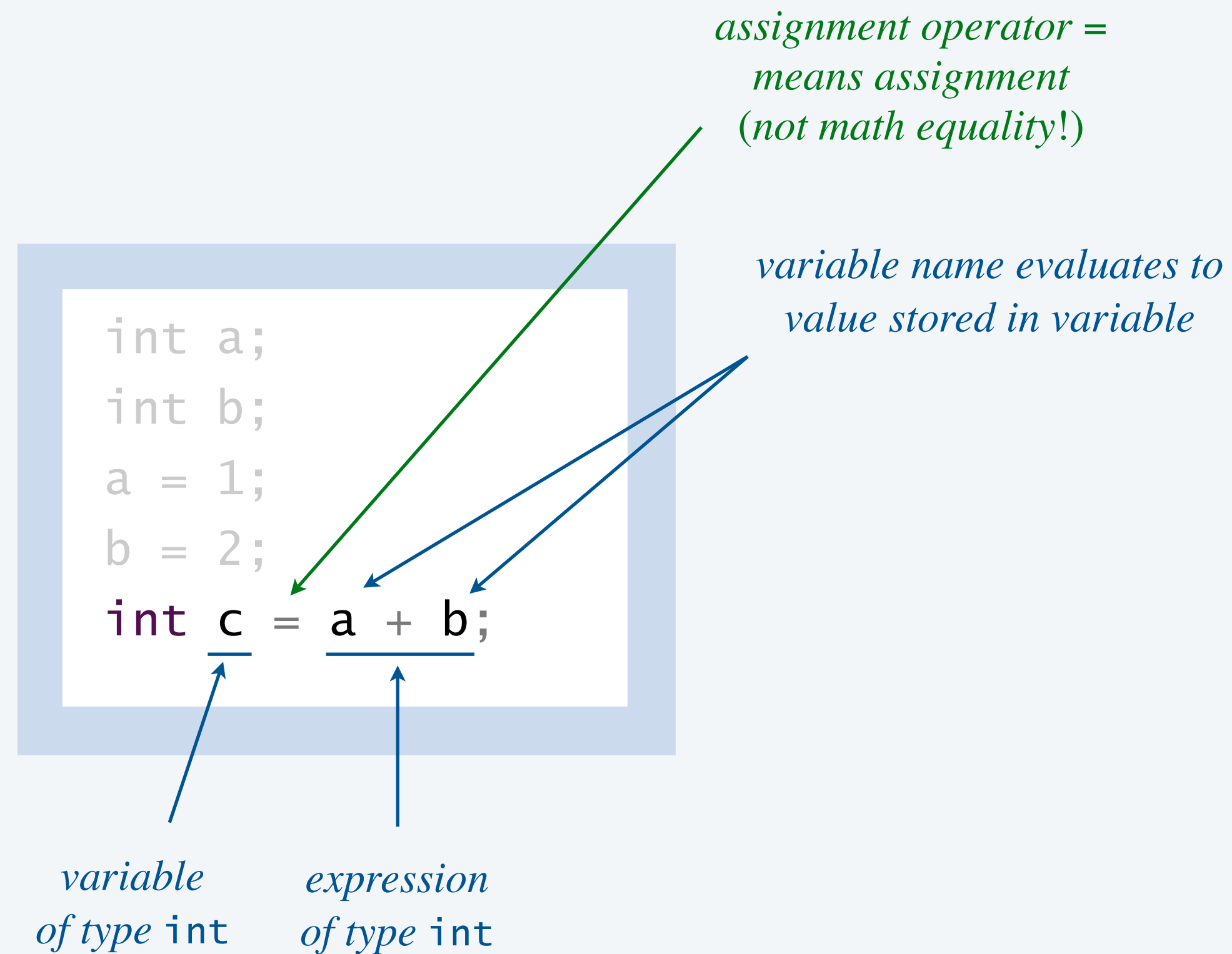


Assignment statements

Q. How does an assignment statement work?

A. Java evaluates the **expression on the RHS** and assigns that value to the **variable on the LHS**.

↑
expression type must be compatible with variable type



Valid and invalid assignment statements

Q. Which of these independent code fragments are valid?

| statements | compiles? | remark |
|---------------------------------------|-----------|---|
| <pre>int a = 1; 123 = a;</pre> | 😞 | <i>LHS is not a variable (= does not mean math equality)</i> |
| <pre>double a = 2.5; int b = a;</pre> | 😞 | <i>RHS type is incompatible with LHS type</i> |
| <pre>String s = 123;</pre> | 😞 | <i>RHS type is incompatible with LHS type</i> |
| <pre>int b = 2; int a = 3 * b;</pre> | 😍 | <i>RHS can be an expression</i> |
| <pre>int a = 3; a = 2 * a;</pre> | 😍 | <i>a variable can be reassigned (that's why it's called a variable!)</i> |
| <pre>int a = 2 * a;</pre> | 😞 | <i>a variable must be assigned a value before it can be used in an expression</i> |

Tracing the execution of a program



Q. What does this code fragment do?

A. Let's **trace** the variables during execution of the code. ← *table of variable values*

```
int a = 100;  
int b = 126;  
int temp = a;  
a = b;  
b = temp;
```

*this idiom exchanges
the values stored in the
variables a and b*

| | <i>a</i> | <i>b</i> | <i>temp</i> |
|-------------------------------|-------------------|-------------------|-------------------|
| <i>start of code fragment</i> | <i>undeclared</i> | <i>undeclared</i> | <i>undeclared</i> |
| <code>int a = 100;</code> | 100 | <i>undeclared</i> | <i>undeclared</i> |
| <code>int b = 126;</code> | 100 | 126 | <i>undeclared</i> |
| <code>int temp = a;</code> | 100 | 126 | 100 |
| <code>a = b;</code> | 126 | 126 | 100 |
| <code>b = temp;</code> | 126 | 100 | 100 |

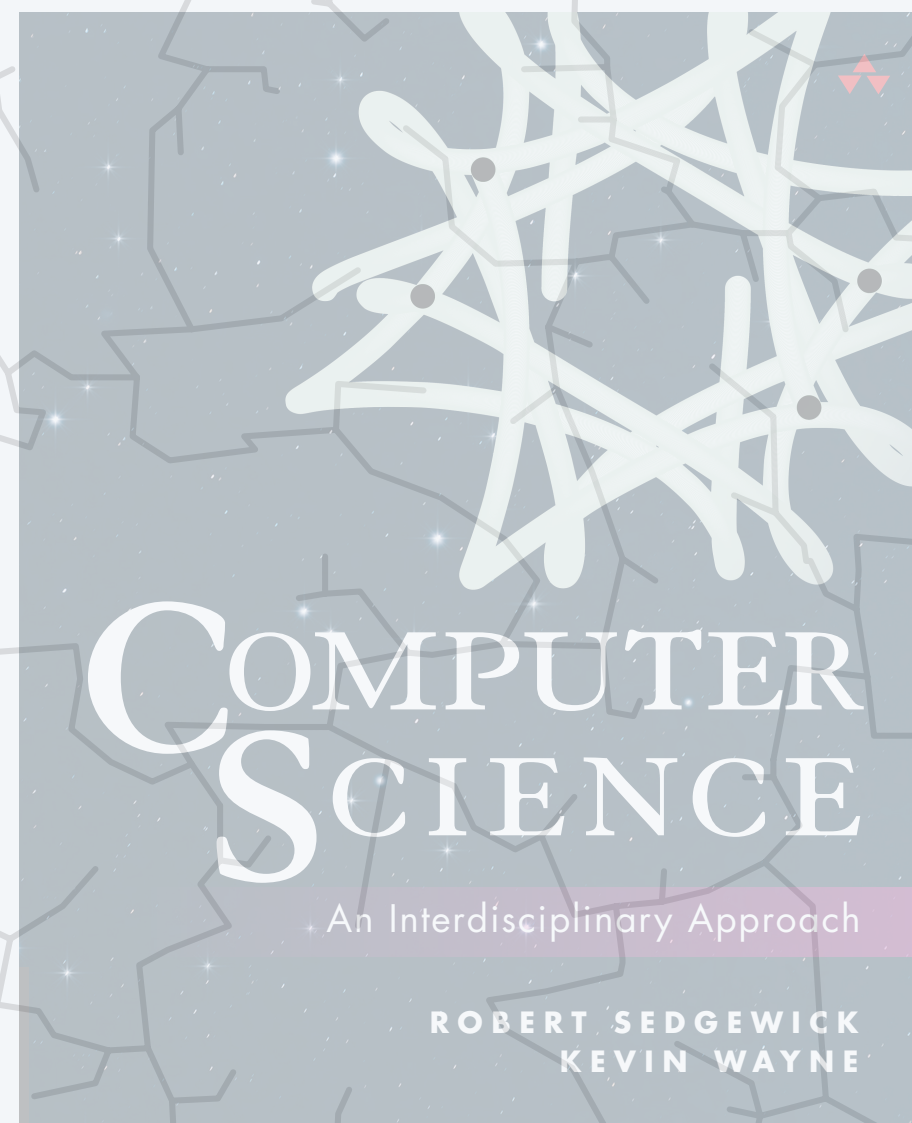
**trace of variables
(after each statement)**



What are the values stored in the variables *a* and *b* after the code fragment is executed?

- A. 100 and 126.
- B. 126 and 100.
- C. 226 and 126.
- D. -26 and -26.
- E. Compile-time error.

```
int a = 100;  
int b = 126;  
a = a + b;  
b = a - b;  
a = a - b;
```



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BUILT-IN DATA TYPES

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- ▶ *floating-point numbers*
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- ▶ *type conversion*

The *String* data type

Typical usage. Program input and output; text processing.

| | |
|------------------|--------------------------------|
| values | <i>sequences of characters</i> |
| example literals | "Hi" "1234" "Nǐ hǎo" "💩💩💩" |
| operation | <i>concatenation</i> |
| operator | + |

| expression | value | remark |
|--------------------|---------------|--|
| "My " + "Precious" | "My Precious" | <i>spaces within a string literal matter</i> |
| "1234" + "99" | "123499" | <i>strings are not integers</i> |
| "A" + "B" + "C" | "ABC" | <i>can concatenate several strings together, in one expression</i> |
| "ᠠᠯᠤᠰ " + "ᠠᠵᠤᠳᠤ!" | "ᠠᠯᠤᠰ ᠠᠵᠤᠳᠤ!" | <i>Unicode supported</i> |

Command-line arguments are strings

Command-line arguments. The variables `args[0]`, `args[1]`, `args[2]`, ... are of type `String`. Java initializes them automatically to corresponding values.

*we'll revisit notation
in Section 1.4 (arrays)*

```
public class CommandLineArguments {  
    public static void main(String[] args) {  
        String a = args[0];  
        String b = args[1];  
        String c = args[2];  
        String result = a + "-" + b + "-" + c;  
        System.out.println(result);  
    }  
}
```

```
~/cos126/datatypes> java CommandLineArguments A B C  
A-B-C
```

args[0]

```
~/cos126/datatypes> java CommandLineArguments do re mi  
do-re-mi
```

*arguments delimited
by whitespace*

```
~/cos126/datatypes> java CommandLineArguments  
Exception in thread "main"  
java.lang.ArrayIndexOutOfBoundsException:  
Index 0 out of bounds for length 0 at  
CommandLineArguments.main(CommandLineArguments.java:3)
```

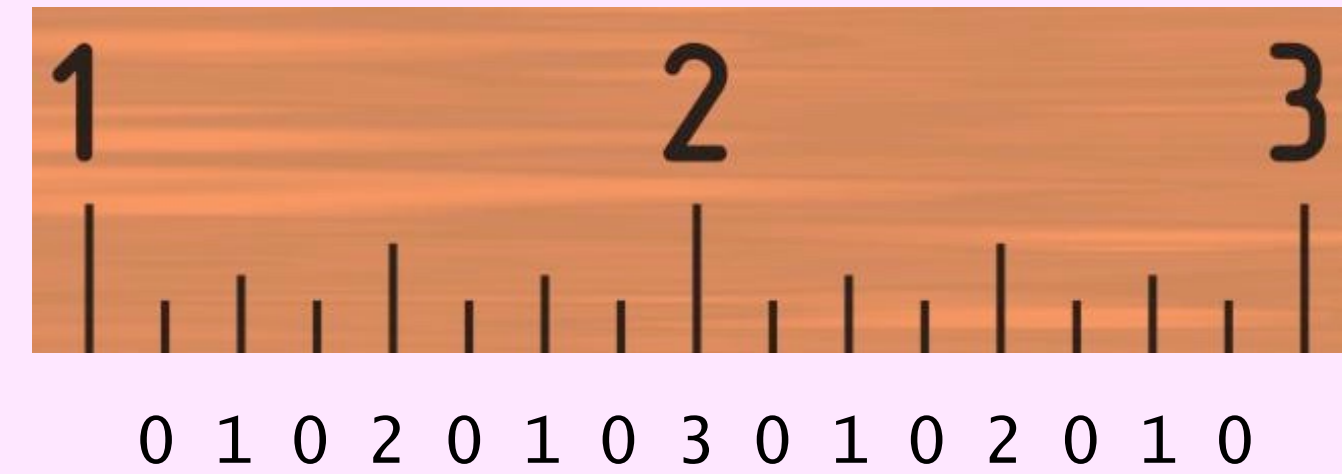
*line number
of error*

Ruler function



```
public class Ruler {  
    public static void main(String[] args) {  
        String ruler0 = "0";  
        String ruler1 = ruler0 + " 1 " + ruler0;  
        String ruler2 = ruler1 + " 2 " + ruler1;  
        String ruler3 = ruler2 + " 3 " + ruler2;  
        System.out.println(ruler3);  
    }  
}
```

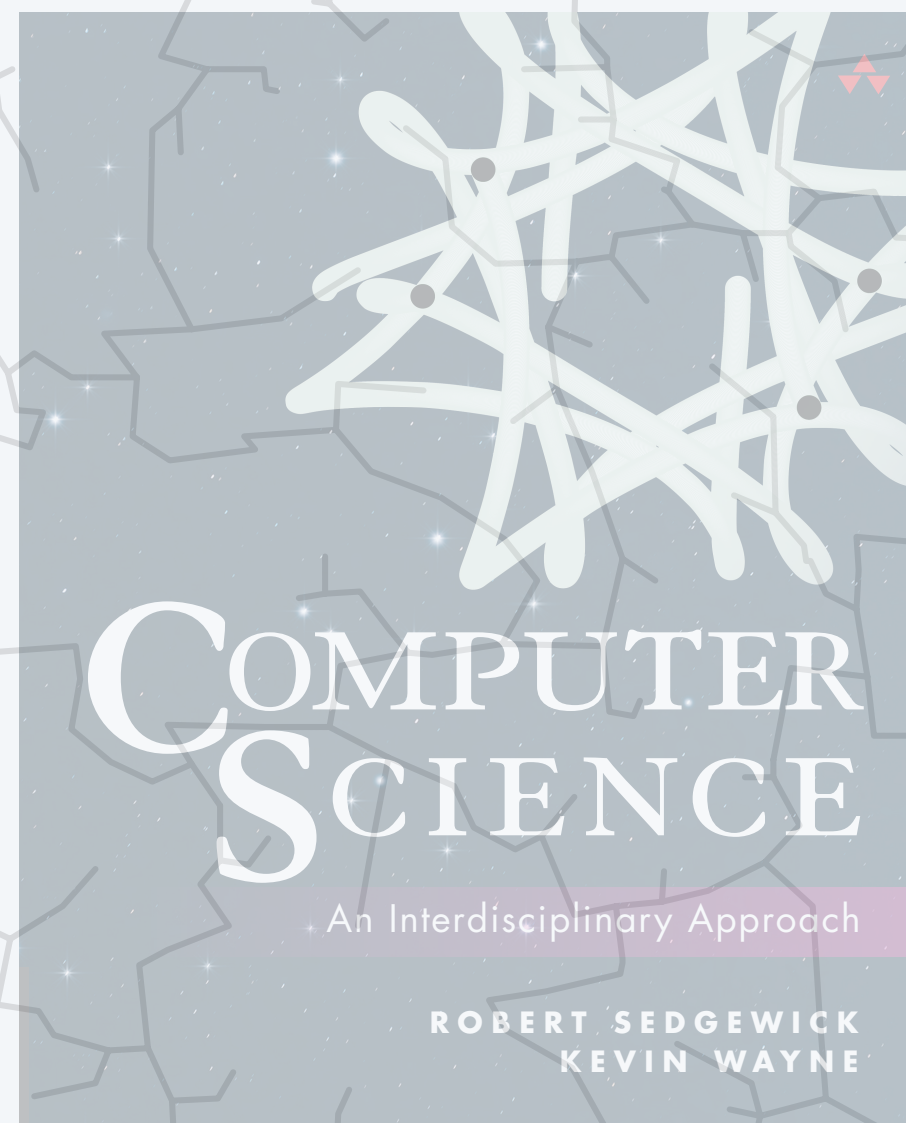
↑
string concatenation



```
~/cos126/datatypes> java Ruler  
0 1 0 2 0 1 0 3 0 1 0 2 0 1 0
```

| <i>ruler0</i> | <i>ruler1</i> | <i>ruler2</i> | <i>ruler3</i> |
|-------------------|-------------------|-------------------|---------------------------------|
| <i>undeclared</i> | <i>undeclared</i> | <i>undeclared</i> | <i>undeclared</i> |
| "0" | <i>undeclared</i> | <i>undeclared</i> | <i>undeclared</i> |
| "0" | "0 1 0" | <i>undeclared</i> | <i>undeclared</i> |
| "0" | "0 1 0" | "0 1 0 2 0 1 0" | <i>undeclared</i> |
| "0" | "0 1 0" | "0 1 0 2 0 1 0" | "0 1 0 2 0 1 0 3 0 1 0 2 0 1 0" |

trace of variables (after each statement)



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The *int* data type

Typical usage: math calculations involving integers; program control flow.

| | | | | | |
|------------------|--|-----------------|-----------------|---------------|------------------|
| values | <i>integers between -2^{31} and $2^{31} - 1$</i> | | | | |
| example literals | 1234 99 0 1000000 -3 | | | | |
| operations | <i>add</i> | <i>subtract</i> | <i>multiply</i> | <i>divide</i> | <i>remainder</i> |
| operators | + | - | * | / | % |

*only 2^{32} different int values
(not quite the same as integers)*

| expression | value | remark |
|-----------------------|-------------|-------------------------------|
| 20 + 3 | 23 | |
| 20 - 3 | 17 | |
| 20 * 3 | 60 | |
| 20 / 3 | 6 | <i>drop fractional part</i> |
| 20 % 3 | 2 | <i>remainder</i> |
| 20 / 0 | - | <i>division-by-zero error</i> |
| <u>2147483647</u> + 1 | -2147483648 | <i>integer overflow</i> |
| $2^{31} - 1$ | | |

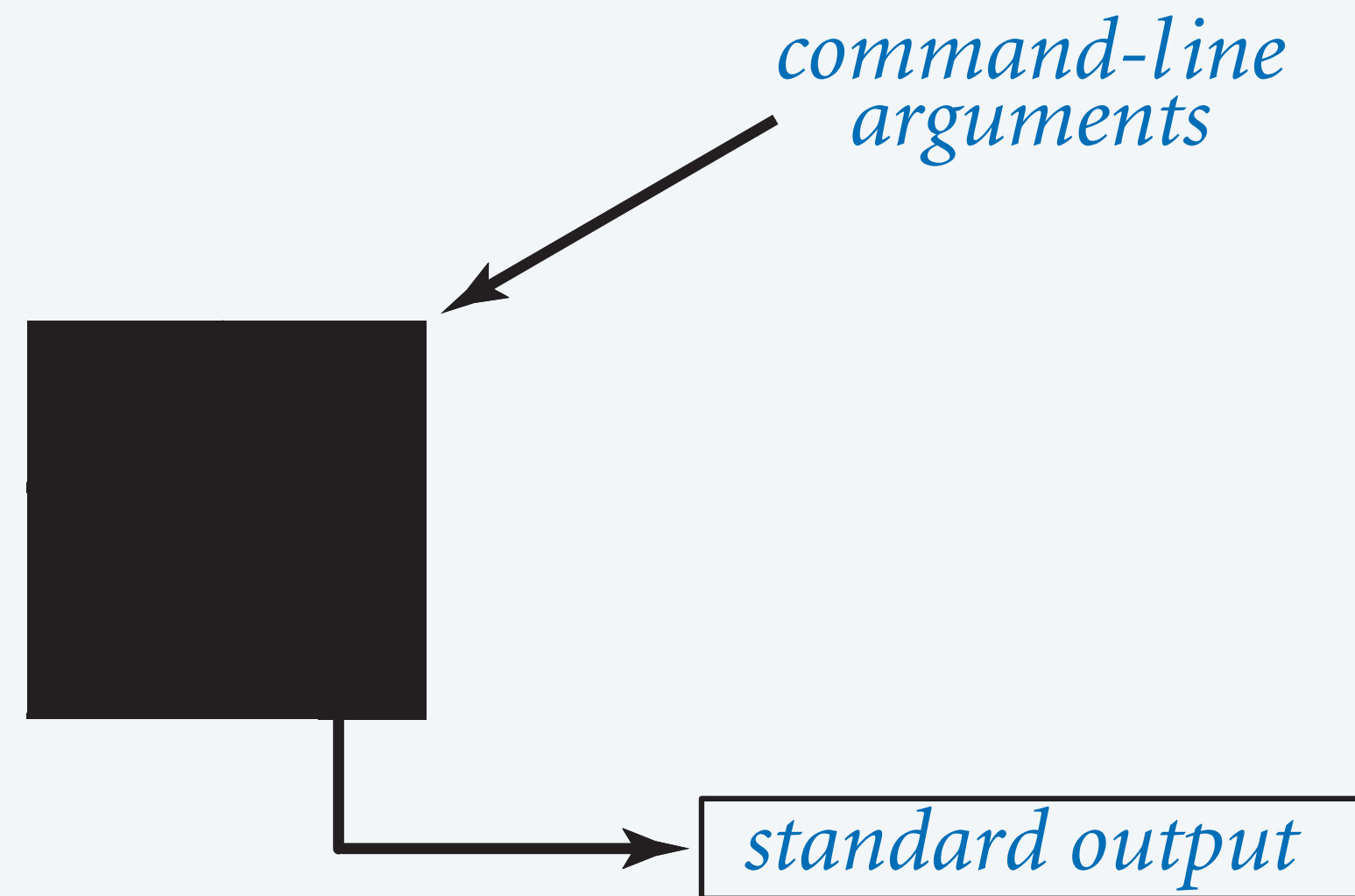
applying an int operator to two int operands always results in an int (or division-by-zero error)

don't use int with very large integers

Input and output

Java I/O model. [for now]

- Read strings from the command line.
- Print strings to standard output.



Q. How to read integers from the command line?

A. The system method `Integer.parseInt()` converts from a `String` to an `int`.

Q. How to print integers to standard output?

A. When a `String` is concatenated with an `int`, Java converts the `int` to a `String`.

Input and output with integers

```
public class IntOps {  
    public static void main(String[] args) {  
        int a = Integer.parseInt(args[0]);  
        int b = Integer.parseInt(args[1]);  
        int sum = a + b;  
        int prod = a * b;  
        int quot = a / b;  
        int rem = a % b;  
        System.out.println(a + " + " + b + " = " + sum);  
        System.out.println(a + " * " + b + " = " + prod);  
        System.out.println(a + " / " + b + " = " + quot);  
        System.out.println(a + " % " + b + " = " + rem);  
    }  
}
```

*converts from
String to int*

*converts from
int to String*

```
~/cos126/datatypes> java IntOps 20 3
```

```
20 + 3 = 23
```

```
20 * 3 = 60
```

```
20 / 3 = 6
```

```
20 % 3 = 2
```

← 20 = 6×3 + 2

```
~/cos126/datatypes> java IntOps 1234 10
```

```
1234 + 10 = 1244
```

```
1234 * 10 = 12340
```

```
1234 / 10 = 123
```

```
1234 % 10 = 4
```

← 1234 = 123×10 + 4

```
~/cos126/datatypes> java IntOps 1234 Hello
```

```
Exception in thread "main"
```

```
java.lang.NumberFormatException:
```

```
For input string: "Hello"
```

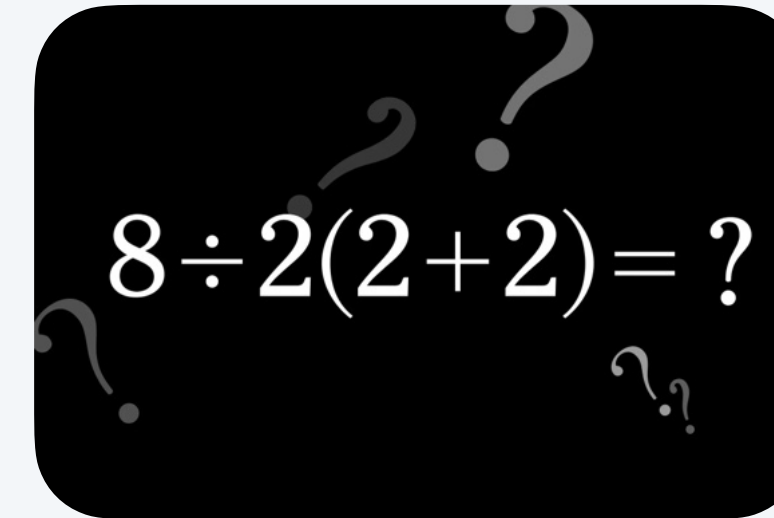
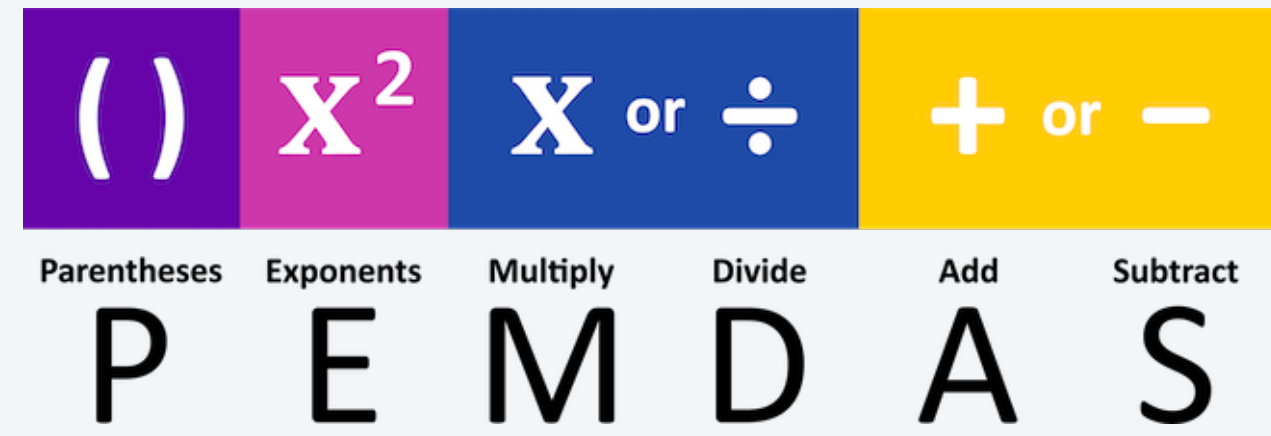
```
...
```

```
at IntOps.main(IntOps.java:4)
```

*← line number of
run-time error*

Order of operations

PEMDAS. Rules for evaluating an arithmetic expression.



internet meme

Operator precedence. Priority for grouping operands with operators in an expression.

Operator associativity. Rule for when two operators in an expression have same priority.

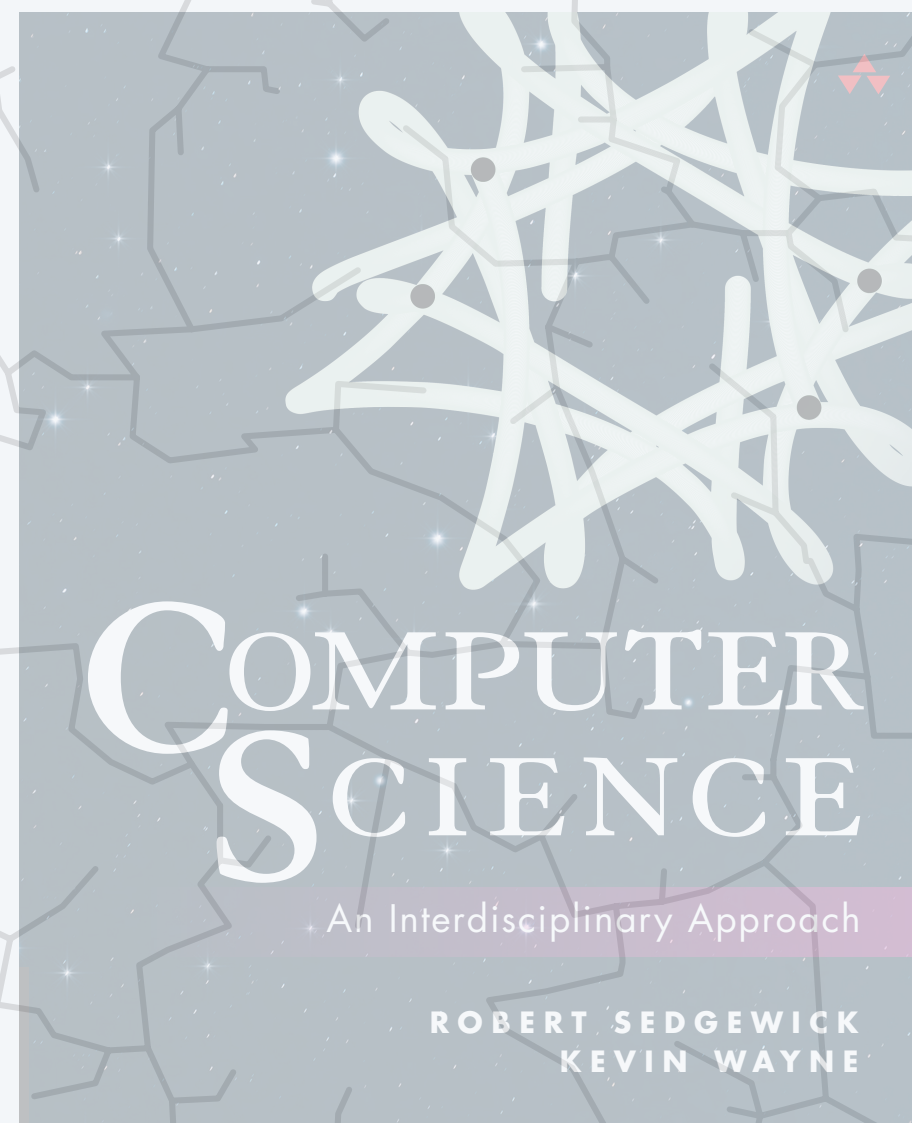
| expression | equivalent to | value | remark |
|-------------------|---------------------|-------|---|
| $3 * 5 - 2$ | $(3 * 5) - 2$ | 13 | <i>* has higher precedence than -</i> |
| $3 + 5 / 2$ | $3 + (5 / 2)$ | 5 | <i>/ has higher precedence than +</i> |
| $3 - 5 - 2$ | $(3 - 5) - 2$ | -4 | <i>left-to-right associative</i> |
| $(3 - 5) - 2$ | <i>itself</i> | -4 | <i>better style</i> |
| $8 / 2 * (2 + 2)$ | $(8 / 2) * (2 + 2)$ | 16 | <i>left-to-right associative</i> <i>(multiplication and division have same precedence)</i> |



What value does the following expression evaluate to?

```
1 + 2 + "ABC" + 3 + 4
```

- A. "12ABC34"
- B. "3ABC7"
- C. "3ABC34"
- D. "12ABC7"
- E. Compile-time error.



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The *double* data type

Typical usage: scientific calculations involving real numbers.

| | | | | | |
|------------------|------------------------------------|-----------------|--------------------|---------------|------------------|
| values | <i>IEEE floating-point numbers</i> | | | | |
| example literals | 18.25 | -2.0 | 1.4142135623730951 | 6.022E23 | |
| operations | <i>add</i> | <i>subtract</i> | <i>multiply</i> | <i>divide</i> | <i>remainder</i> |
| operators | + | - | * | / | % |

*only 2^{64} different double values
(not quite the same as real numbers)*

*6.022×10^{23}
(scientific notation)*

| expression | value | remark |
|--------------|--------------------|---------------------------|
| $1.5 + 0.25$ | 1.75 | |
| $1.5 - 0.25$ | 1.25 | |
| $1.5 * 2.0$ | 3.0 | |
| $5.0 / 3.0$ | 1.6666666666666667 | not exactly $\frac{5}{3}$ |
| $-1.0 / 0.0$ | -Infinity | not an error |
| $0.0 / 0.0$ | NaN | "not a number" |

*applying a double operator
to two double operands
always results in a double
(never results in an error)*

*only binary fractional values
can be represented exactly, such as
 $\frac{1}{4} + \frac{1}{16} + \frac{1}{128} = 0.3203125$
(but not $\frac{5}{3}$, $\frac{1}{10}$, or π)*

Excepts from Java's *Math* library

Math library function

description

static double abs(double a)
 static double max(double a, double b)
 static double min(double a, double b)

static double sin(double theta)
 static double cos(double theta)
 static double tan(double theta)

static double exp(double a)
 static double log(double a)
 static double sqrt(double a)
 static double pow(double a, double b)

static long round(double a)
 static double random()

static double E
 static double PI

absolute value of a
maximum of a and b
minimum of a and b

← *also defined for int*

sine (sin θ)
cosine (cos θ)
tangent (tan θ)

exponential (e^a)
natural logarithm ($\log_e a$)
positive square root (\sqrt{a})
power (a^b)

round to the nearest integer
pseudorandom number in [0, 1)

value of e (constant)
value of π (constant)

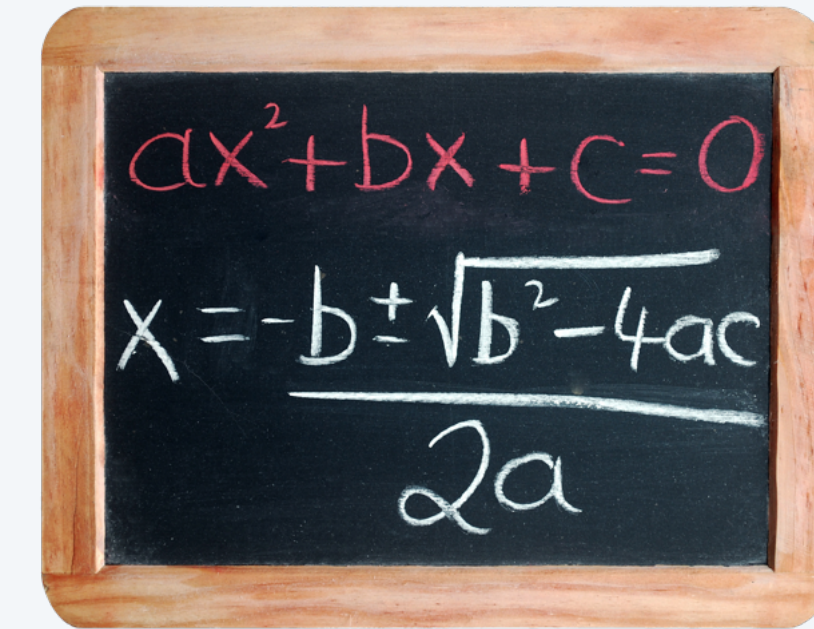


You can discard your calculator now (please).

| expression | value |
|--------------------|--------------------|
| Math.max(1.0, 2.5) | 2.5 |
| Math.cos(0.0) | 1.0 |
| Math.sqrt(2.0) | 1.4142135623730951 |
| Math.random() | 0.7707780210347349 |
| Math.PI | 3.141592653589793 |

Quadratic equation

Goal. Print the solutions to the equation $ax^2 + bx + c = 0$, assuming $a \neq 0$.



```
public class Quadratic {
    public static void main(String[] args) {

        // Parse coefficients from command-line.
        double a = Double.parseDouble(args[0]);
        double b = Double.parseDouble(args[1]);
        double c = Double.parseDouble(args[2]);

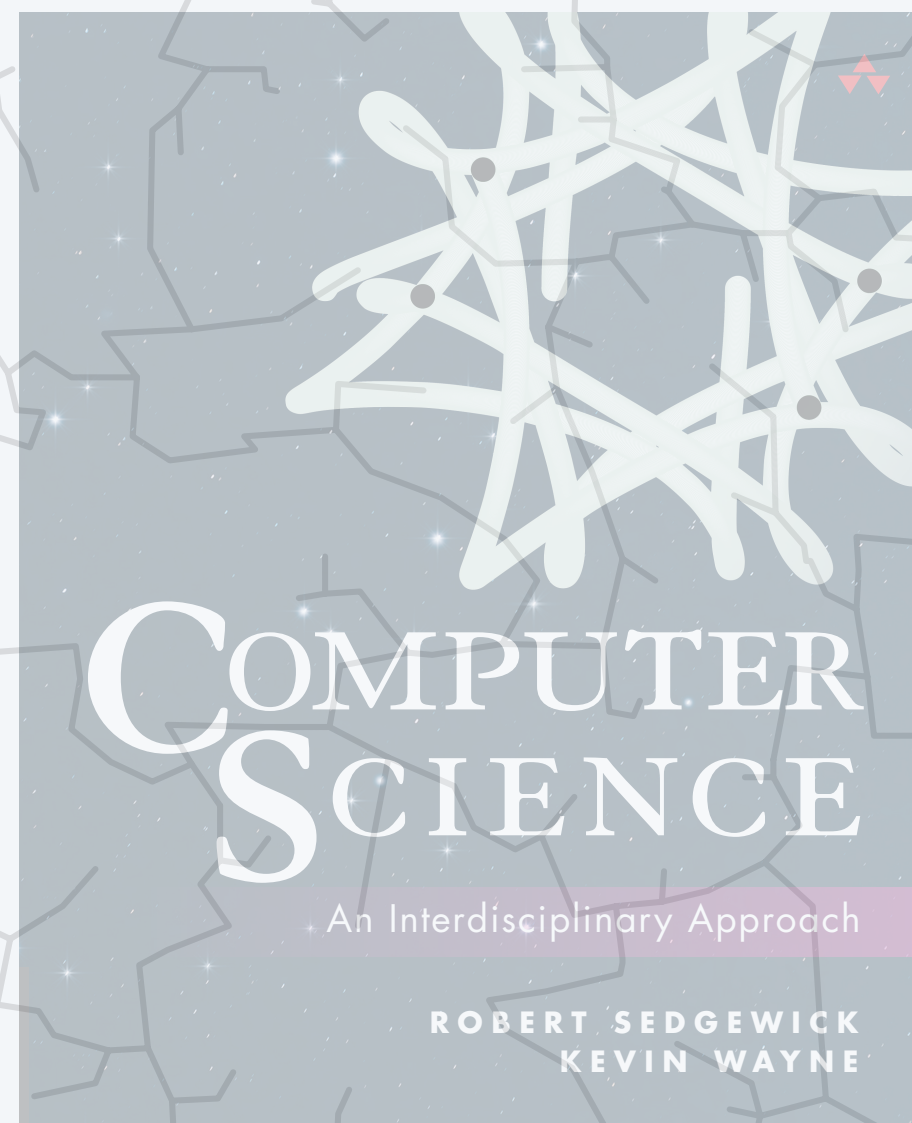
        // Calculate roots of ax^2 + bx + c = 0.
        double discriminant = b*b - 4.0*a*c;
        double d = Math.sqrt(discriminant);
        double root1 = (-b + d) / (2.0*a);
        double root2 = (-b - d) / (2.0*a);

        // Print the two roots.
        System.out.println(root1);
        System.out.println(root2);
    }
}
```

```
~/cos126/datatypes> java Quadratic 1.0 -3.0 2.0
2.0
1.0
x^2 - 3x + 2

~/cos126/datatypes> java Quadratic 1.0 -1.0 -1.0
1.618033988749895
-0.6180339887498949
x^2 - x - 1
← 1 ± √5
2

~/cos126/datatypes> java Quadratic 1.0 1.0 1.0
NaN
NaN
x^2 + x + 1
← -1 ± 3i
2
```



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The *boolean* data type

Typical usage: decision making in a program. *← stay tuned for conditionals and loops*

| | | | |
|------------|-----------------------|------------|----------------------------|
| values | <i>true and false</i> | | |
| literals | true false | | |
| operations | <i>not</i> | <i>and</i> | <i>or</i> |
| operators | ! | && | <i>← logical operators</i> |

| expression | value |
|---------------------|-------|
| <code>!false</code> | true |
| <code>!true</code> | false |

truth table for NOT

| expression | value |
|-------------------------------------|-------|
| <code>false && false</code> | false |
| <code>false && true</code> | false |
| <code>true && false</code> | false |
| <code>true && true</code> | true |

truth table for AND


| expression | value |
|-----------------------------|-------|
| <code>false false</code> | false |
| <code>false true</code> | true |
| <code>true false</code> | true |
| <code>true true</code> | true |

truth table for OR



Equality and comparison operators

Equality and comparison operators. To **compare** numeric values.

- Operands: two numeric expressions.  *can be literals, variable, or arbitrary expressions*
- Evaluates to: a value of type *boolean*.

| operator | meaning | true | false |
|--------------------|------------------------------|------------------------|------------------------|
| <code>==</code> | <i>equal</i> | <code>2 == 2</code> | <code>2 == 3</code> |
| <code>!=</code> | <i>not equal</i> | <code>3 != 2</code> | <code>2 != 2</code> |
| <code><</code> | <i>less than</i> | <code>2 < 13</code> | <code>13 < 2</code> |
| <code><=</code> | <i>less than or equal</i> | <code>2 <= 2</code> | <code>3 <= 2</code> |
| <code>></code> | <i>greater than</i> | <code>13 > 2</code> | <code>2 > 13</code> |
| <code>>=</code> | <i>greater than or equal</i> | <code>2 >= 2</code> | <code>2 >= 3</code> |

equality and comparison operators in Java

Equality and comparison operators: examples

| | | |
|-------------------------------|--|--|
| zero denominator? | <code>denominator == 0</code> | |
| non-negative discriminant? | <code>(b*b - 4.0*a*c) >= 0.0</code> | ← parentheses for clarity: arithmetic operators have higher precedence than equality/comparison operators |
| divisible by 60? | <code>(minutes % 60) == 0</code> | |
| RGB color is not black? | <code>(red > 0) (green > 0) (blue > 0)</code> | ← compound boolean expressions |
| valid month? | <code>(month >= 1) && (month <= 12)</code> | |
| invalid month? | <code>!((month >= 1) && (month <= 12))</code> | |
| floating-point roundoff error | <code>(0.1 * 3.0) == 0.3</code> | ← don't do this! (evaluates to false) |
| string equality | <code>args[0] == "Hello"</code> | ← or this! (always evaluates to false) |

Example of computing with booleans: leap year test

Q. Is a given year a leap year? ← *Gregorian calendar*

A. Yes if **either**: (Case A:) divisible by 400 or (Case B:) divisible by 4 but not 100.

```
public class LeapYear {
    public static void main(String[] args) {

        int year = Integer.parseInt(args[0]);
        boolean isLeapYear;

        // Case B: divisible by 4 but not 100
        isLeapYear = (year % 4 == 0) && (year % 100 != 0);

        // ...or Case A: divisible by 400
        isLeapYear = isLeapYear || (year % 400 == 0);

        System.out.println(isLeapYear);
    }
}
```

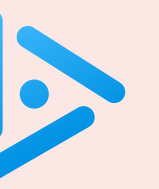
```
~/cos126/datatypes> java LeapYear 2024
true

~/cos126/datatypes> java LeapYear 2023
false

~/cos126/datatypes> java LeapYear 1900
false

~/cos126/datatypes> java LeapYear 2000
true
```

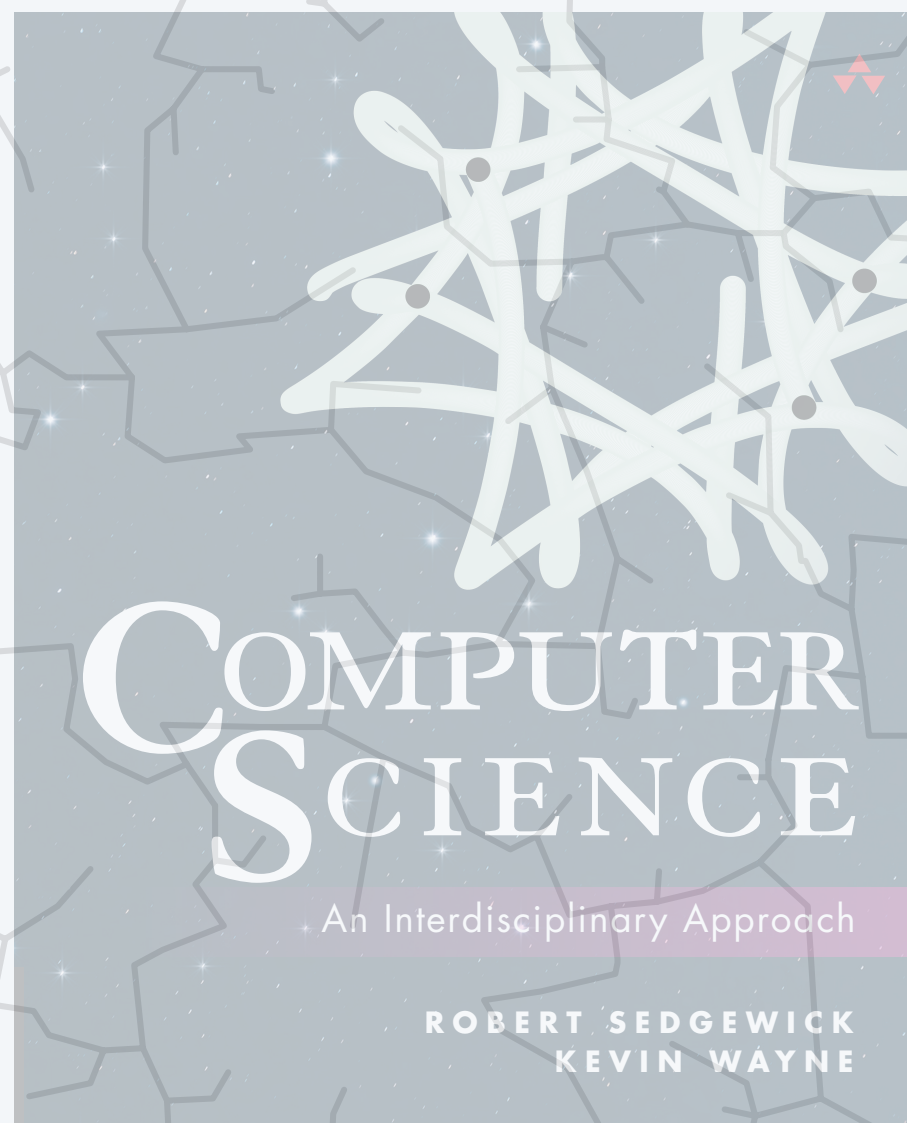
*if argument to System.out.println() is of type boolean,
it prints either true or false*



What does the following expression evaluate to?

```
1 <= month <= 12
```

- A. Works: equivalent to `(month >= 1) && (month <= 12)`.
- B. Compile-time error: equivalent to `(1 <= month) <= 12`.



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Data types

Types limit the allowable operations on values and determine the meaning of those operations.

```
public class StringMultiply {  
    public static void main(String[] args) {  
        String s = "123" * "456";  
    }  
}
```

```
~/cos126/datatypes> javac StringMultiply.java  
StringMultiply.java:3: error: bad operand types  
for binary operator '*'  
        String s = "123" * "456";  
                        ^  
    first type: String  
    second type: String  
1 error
```

Java compiler. The compiler checks for type mismatch errors in your code.

Data types

Types limit the allowable operations on values and determine the meaning of those operations.

| operator | int | double | boolean | String |
|----------|-------------------------|-----------------------|--------------------|----------------------|
| + | <i>addition</i> | <i>addition</i> | <i>no</i> | <i>concatenation</i> |
| - | <i>subtraction</i> | <i>subtraction</i> | <i>no</i> | <i>no</i> |
| * | <i>multiplication</i> | <i>multiplication</i> | <i>no</i> | <i>no</i> |
| / | <i>integer division</i> | <i>division</i> | <i>no</i> | <i>no</i> |
| && | <i>no</i> | <i>no</i> | <i>logical AND</i> | <i>no</i> |
| | <i>no</i> | <i>no</i> | <i>logical OR</i> | <i>no</i> |
| ! | <i>no</i> | <i>no</i> | <i>logical NOT</i> | <i>no</i> |
| < | <i>less than</i> | <i>less than</i> | <i>no</i> | <i>no</i> |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

← *can't subtract, multiply, or divide two String or boolean values (compile-time errors)*

Static typing. Every Java variable and expression has a type that is known at compile time.

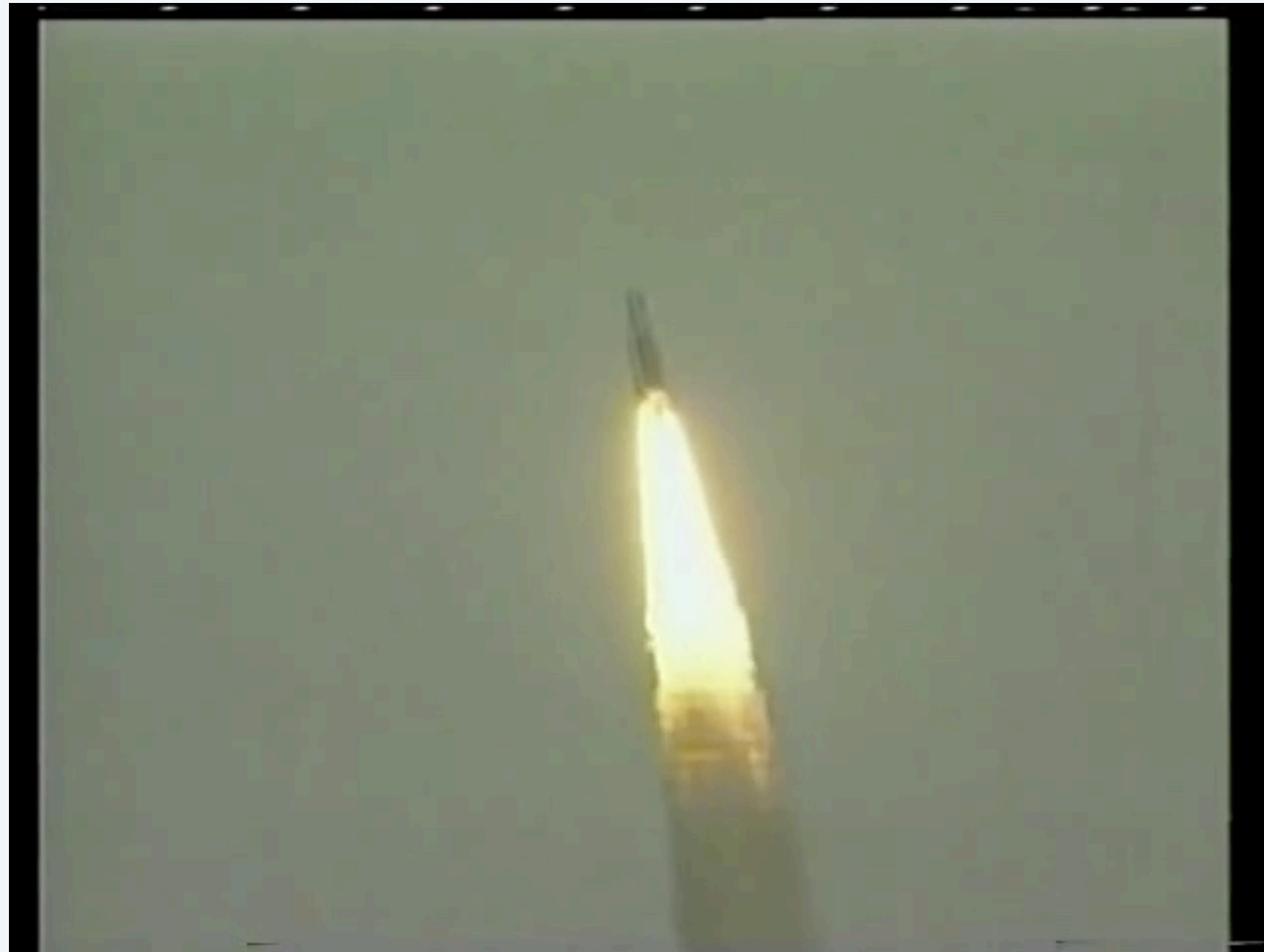
- Benefit: compiler catches entire class of programming errors automatically.
- Drawback: extra boilerplate code.

Type-conversion catastrophe

Ariane 5 rocket.

- European Space Agency spent a decade and \$7 billion in research and development.
- Rocket self-destructed 39 seconds after first launch.
- Source of bug: unsafe type conversion of 64-bit floating-point number to 16-bit integer.

*code worked fine in Ariane 4
(but Ariane 5 velocity was much higher)*



https://www.youtube.com/watch?v=PK_yguLapgA

Type conversions with built-in types

Type conversion is an essential aspect of programming.

Automatic type conversions.

- String conversion: from any type to *String* (via string concatenation).
- Numeric promotion: from *int* to *double* (when a *double* is expected).

every int can be exactly represented as a double

| expression | type | value |
|-------------|--------|----------|
| "x = " + 99 | String | "x = 99" |
| 11 * 0.25 | double | 2.75 |

System methods.

- *Integer.parseInt()* from *String* to *int*.
- *Double.parseDouble()* from *String* to *double*.

| expression | type | value |
|-----------------------------------|--------|-------|
| <i>Integer.parseInt</i> ("126") | int | 126 |
| <i>Double.parseDouble</i> ("2.5") | double | 2.5 |

Explicit casts from one type to another.

- Cast from *double* to *int*. *discards fractional part*
- Cast from *int* to *double*.

| | expression | type | value |
|----------------------------|-------------------|--------|---------|
| | (int) 2.71828; | int | 2 |
| cast operator | (double) sum / n; | double | average |
| cast has higher precedence | | | |
| | ↑ ↑ | | |
| | two int variables | | |

Example of type conversion

Q. What is type and value of each expression on the left?

| expression | type | value | remark |
|----------------------------|---------------------------|-------|--|
| <code>(7 / 2) * 2.0</code> | double | 6.0 | <i>integer division; then promotion to double</i> |
| <code>(7 / 2.0) * 2</code> | double | 7.0 | <i>promotion to double; then floating-point division</i> |
| <code>"12" + 6</code> | String | "126" | <i>conversion to String</i> |
| <code>0 == false</code> | <i>compile-time error</i> | | <i>can't compare int to boolean</i> |

Simulate the rolling of a fair die

Goal. Given an integer $n > 0$, generate a uniformly random integer between 1 and n . ← *each possible integer is equally likely*



$n = 6$



$n = 10$



$n = 100$

Generate pseudo-random integers

Problem. Given an integer $n > 0$, generate a uniformly random integer between 0 and $n - 1$.

Useful system method. `Math.random()` returns a pseudorandom `double` value in $[0, 1)$. ← *can return 0.0*
can't return 1.0

Approach. Scale to desired range, round down to nearest integer. *not truly random, but close enough for most applications*

```
public class RandomInt {
    public static void main(String[] args) {
        int n = Integer.parseInt(args[0]);
        double r = Math.random();
        int result = (int) (r * n);
        System.out.println(result);
    }
}
```

String to int
(system method)

double to int
(cast)

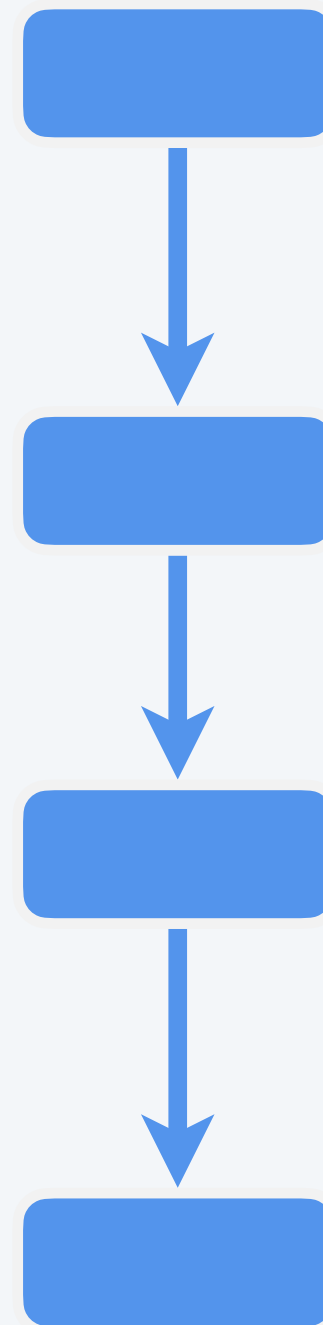
int to double
(automatic)

```
~/cos126/datatypes> java RandomInt 6
3
~/cos126/datatypes> java RandomInt 6
0
~/cos126/datatypes> java RandomInt 6
5
~/cos126/datatypes> java RandomInt 10000
3184
```

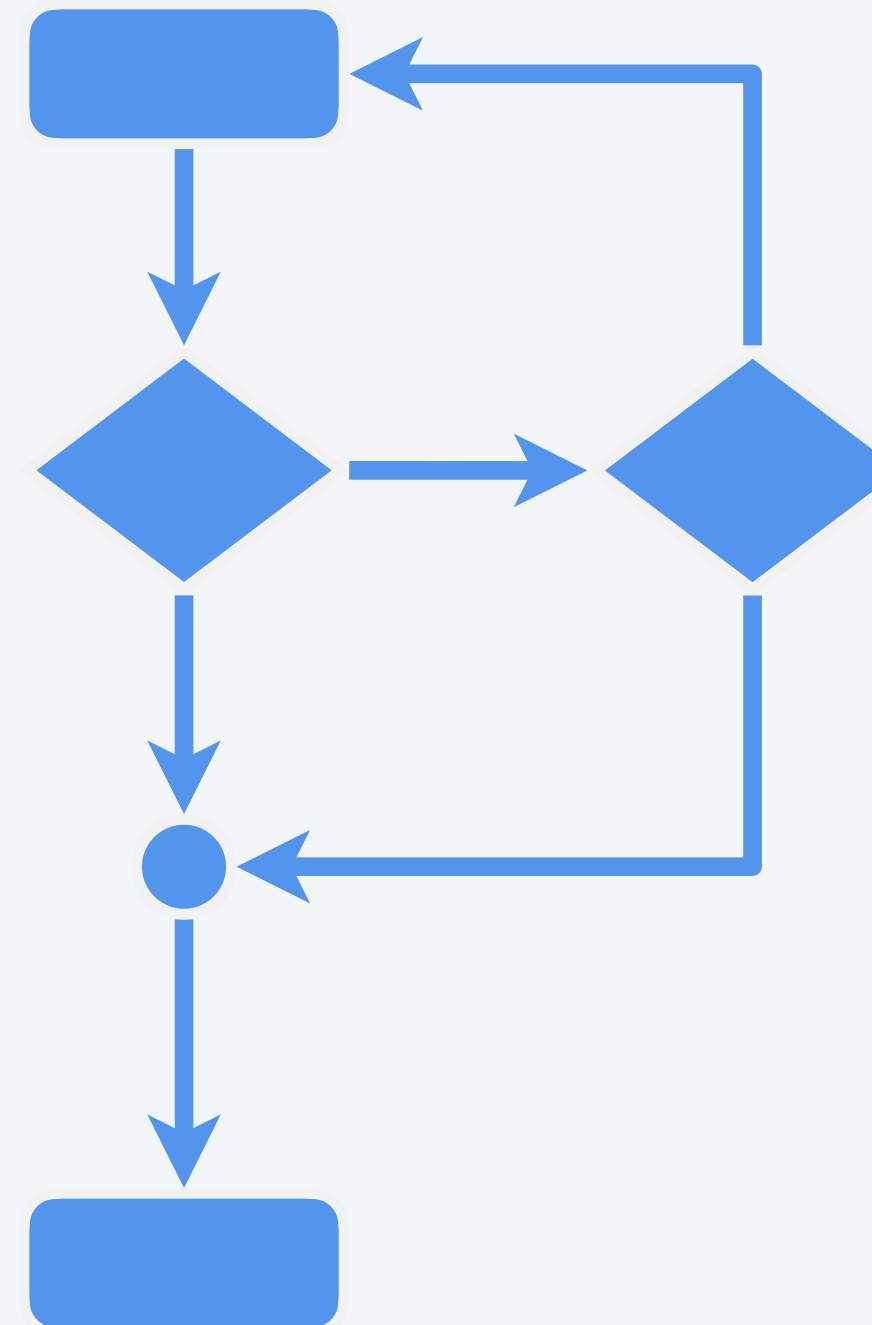
Overview

This lecture. Write programs with **declaration**, **assignment**, and **print** statements.

Next week. Write programs with **conditionals** and **loops**.



straight-line control flow



control flow with conditionals and loops

Credits

| media | source | license |
|--|------------------------------|--------------------------|
| <i>PEMDAS</i> | <u>Mometrix</u> | |
| <i>PEMDAS meme</i> | <u>New York Times</u> | |
| <i>Scientific Calculator</i> | <u>Wikimedia</u> | <u>CC BY-SA 3.0</u> |
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| <i>Scud Missile Hits a U.S. Barracks</i> | <u>New York Times</u> | |
| <i>!FALSE</i> | <u>redbubble.com</u> | |
| <i>Ariane 5 Rocket Launch</i> | <u>European Space Agency</u> | |
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