Passive Dynamics and Particle Systems

COS 426, Spring 2016
Princeton University

Syllabus

I. Image processing

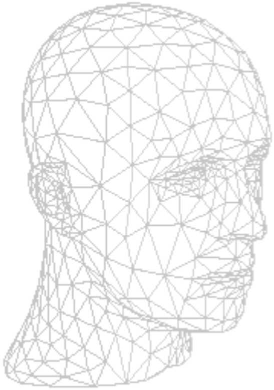
II. Modeling

III. Rendering

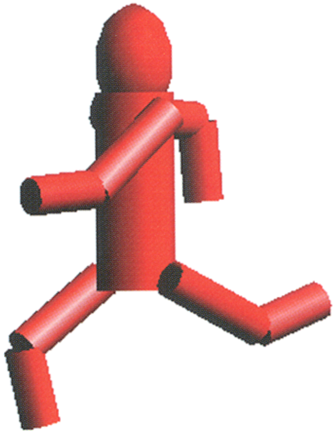
IV. Animation

Image Processing
(Rusty Coleman, CS426, Fall99)

Rendering
(Michael Bostock, CS426, Fall99)



Modeling
(Dennis Zorin, CalTech)



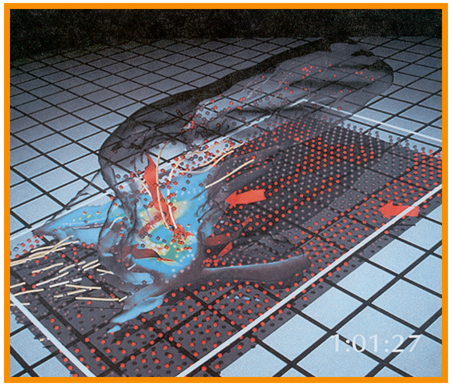
Animation
(Angel, Plate 1)

Animation & Simulation

- Animation
  - Make objects change over time according to scripted actions

Pixar

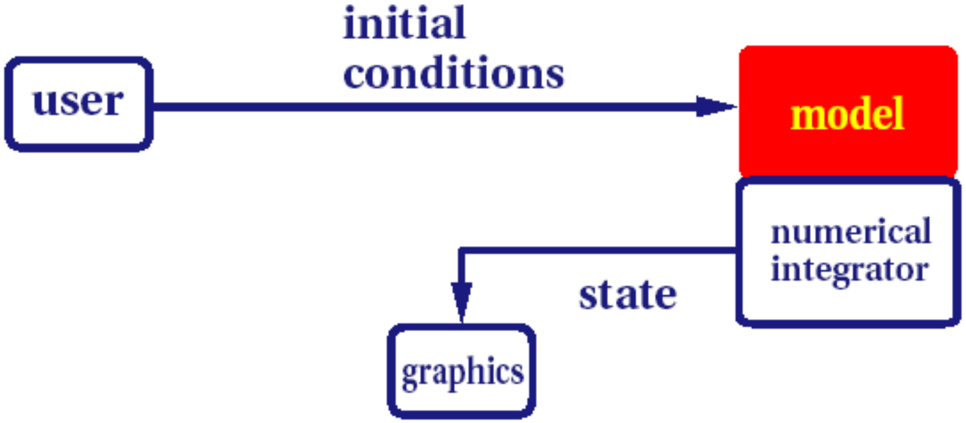
- Simulation / dynamics
  - Predict how objects change over time according to physical laws



University of Illinois

Dynamics

Passive--no muscles or motors


user

initial conditions

model

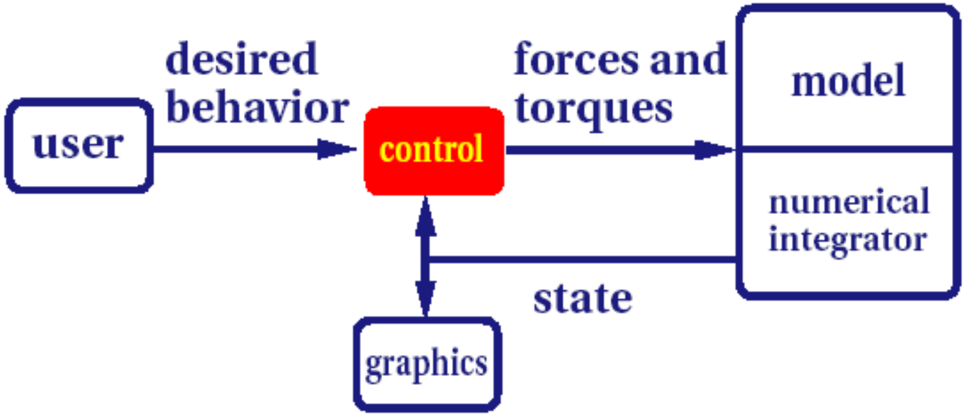
numerical integrator

state

graphics

particle systems
leaves
water spray
clothing

Active--internal source of energy


user

desired behavior

control

forces and torques

model

numerical integrator

state

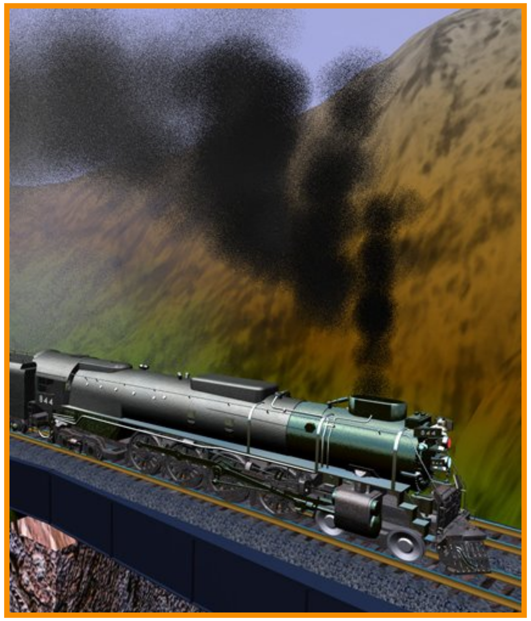
graphics

running human
trotting dog
swimming fish

Hodgins

Passive Dynamics

- No muscles or motors
  - Smoke
  - Water
  - Cloth
  - Fire
  - Fireworks
  - Dice

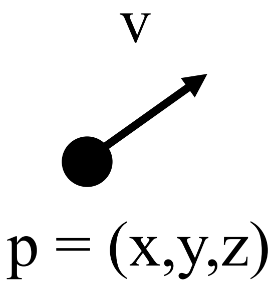


Passive Dynamics

- Physical laws
  - Newton’s laws
  - Hooke’s law
  - Etc.
- Physical phenomena
  - Gravity
  - Momentum
  - Friction
  - Collisions
  - Elasticity
  - Fracture

Particle Systems

- A particle is a point mass
  - Position
  - Velocity
  - Mass
  - Drag
  - Elasticity
  - Lifetime
  - Color



- Use many particles to model complex phenomena
  - Keep array of particles
  - Newton’s laws

Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

Creating Particles

- Where to create particles?
  - Predefined source
  - Where particle density is low
  - Surface of shape
  - etc.

Reeves

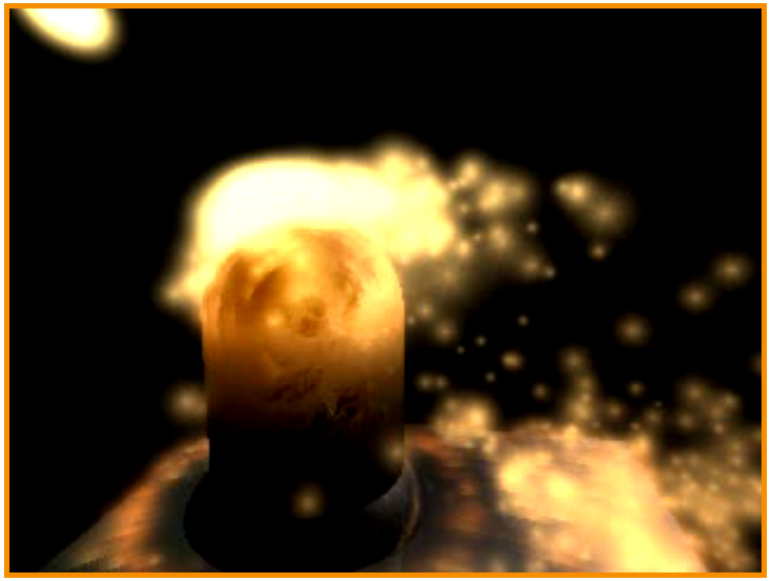
Creating Particles

- Where to create particles?
  - Predefined source
  - Where particle density is low
  - Surface of shape
  - etc.

Reeves

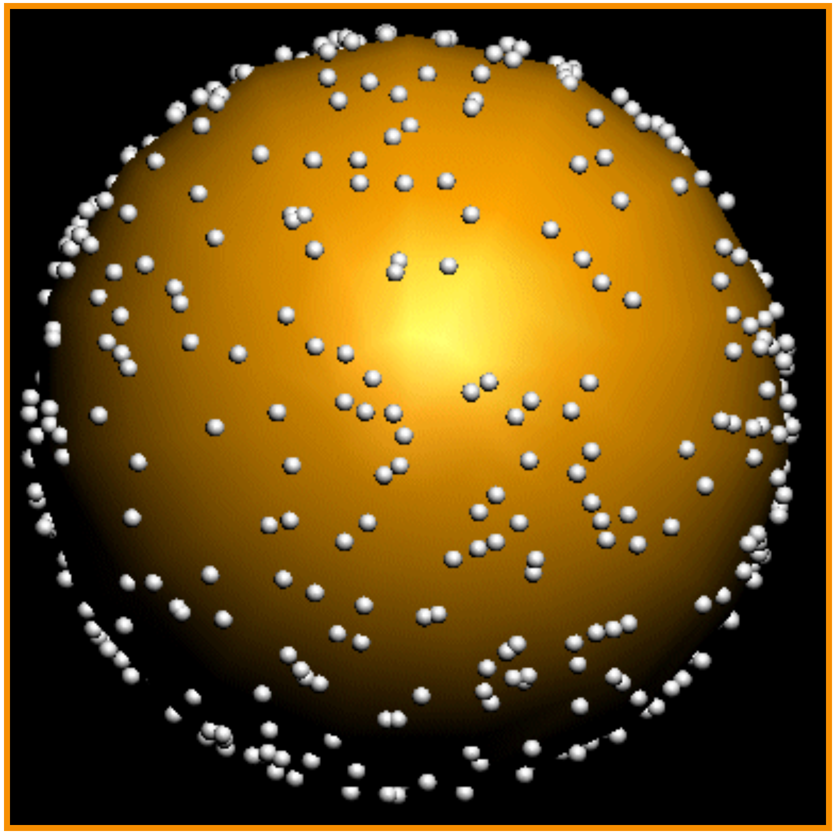
Creating Particles

- Example: particles emanating from shape
  - Line
  - Box
  - **Circle**
  - **Sphere**
  - Cylinder
  - Cone
  - Mesh



Creating Particles

- Example: particles emanating from sphere



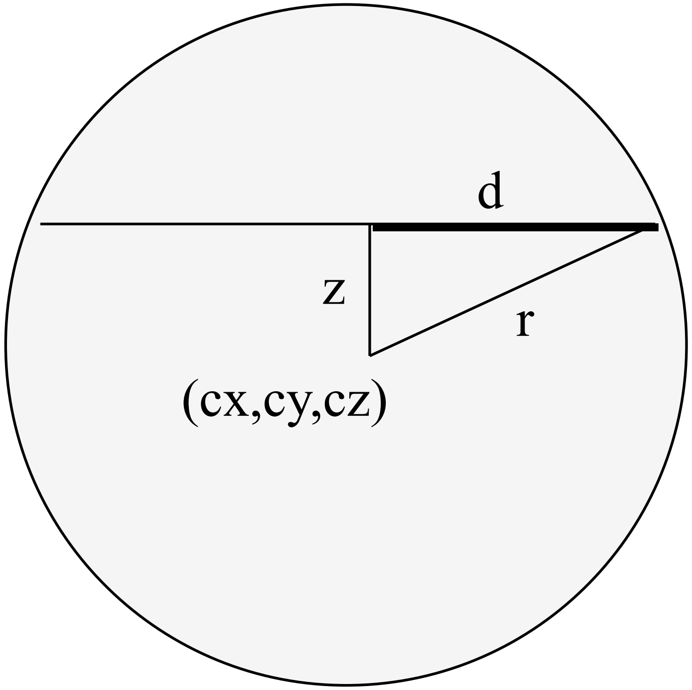
nigels.com

Creating Particles

- Example: particles emanating from sphere

Selecting random position on surface of sphere

1. z = random $[-r, r]$
2. phi = random $[0, 2\pi)$
3. $d = \mathrm{sqrt}(r^2 - z^2)$
4. px = cx + d * cos(phi)
5. py = cy + d * sin(phi)
6. pz = cz + z

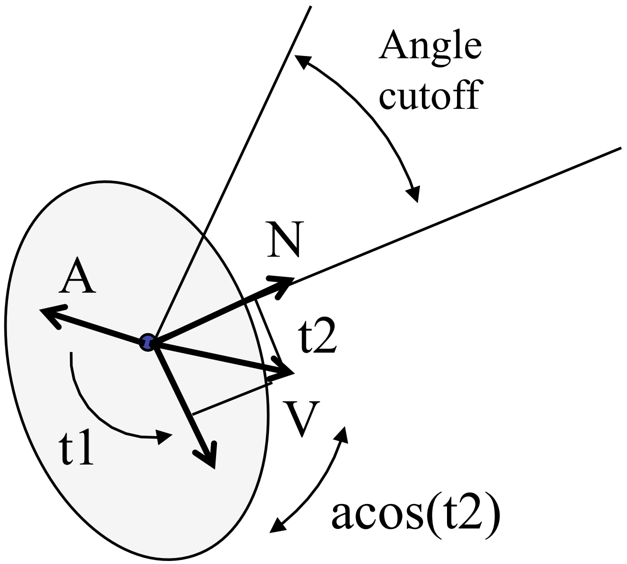


Creating Particles

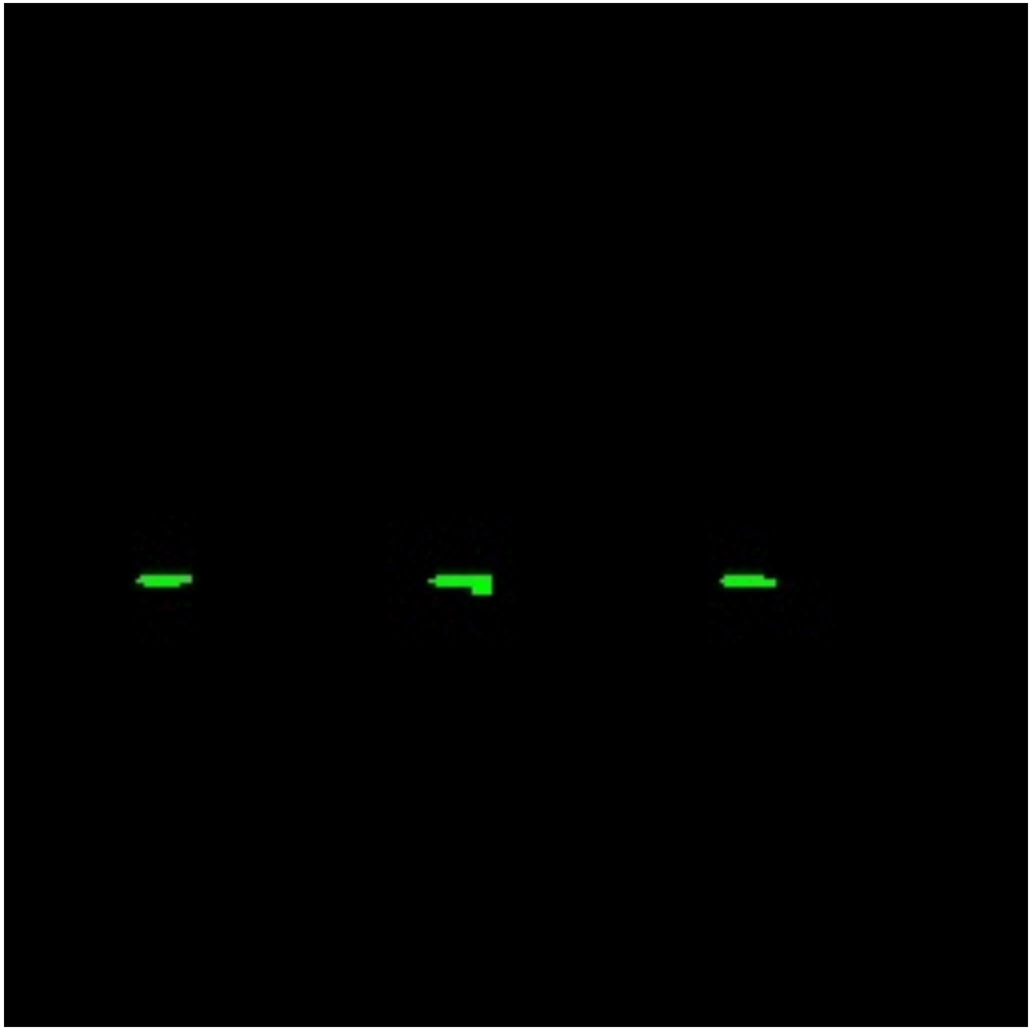
- Example: particles emanating from sphere

Selecting random direction within angle cutoff of normal

1. N = surface normal
2. A = any vector on tangent plane
3. t1 = random [0, 2π)
3. t2 = random [0, sin(angle cutoff))
4. V = rotate A around N by t1
5. V = rotate V around VxN by acos(t2)



Example: Fountains



Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

Equations of Motion

- Newton's Law for a point mass
  - f = ma

- Computing particle motion requires solving second-order differential equation

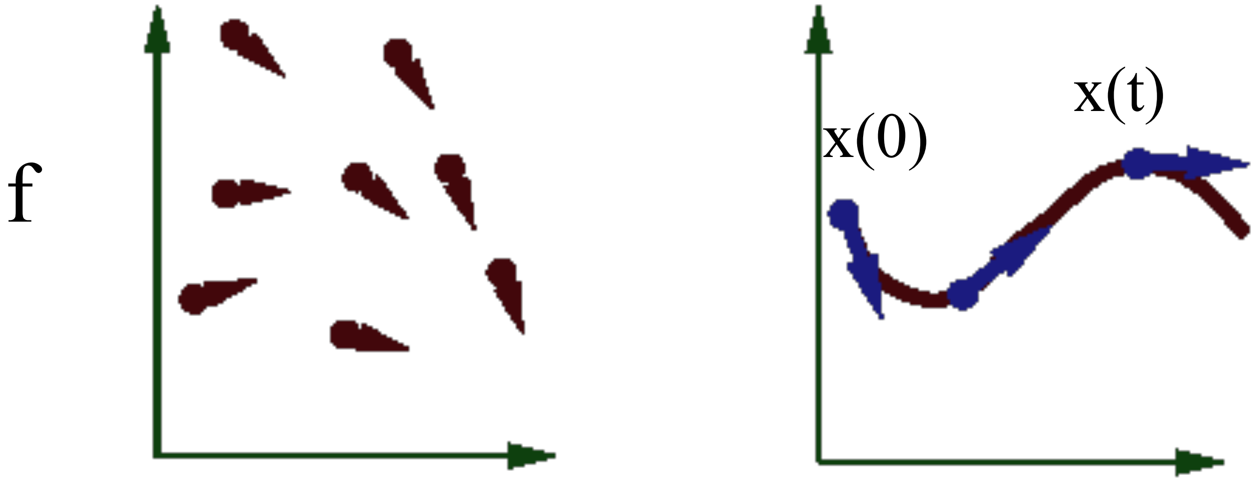
$$\ddot{x} = \frac{f(x, \dot{x}, t)}{m}$$

- Add variable v to form coupled first-order differential equations: "state-space form"

$$\begin{cases} \dot{x} = v \\ \dot{v} = \dfrac{f}{m} \end{cases}$$

Solving the Equations of Motion

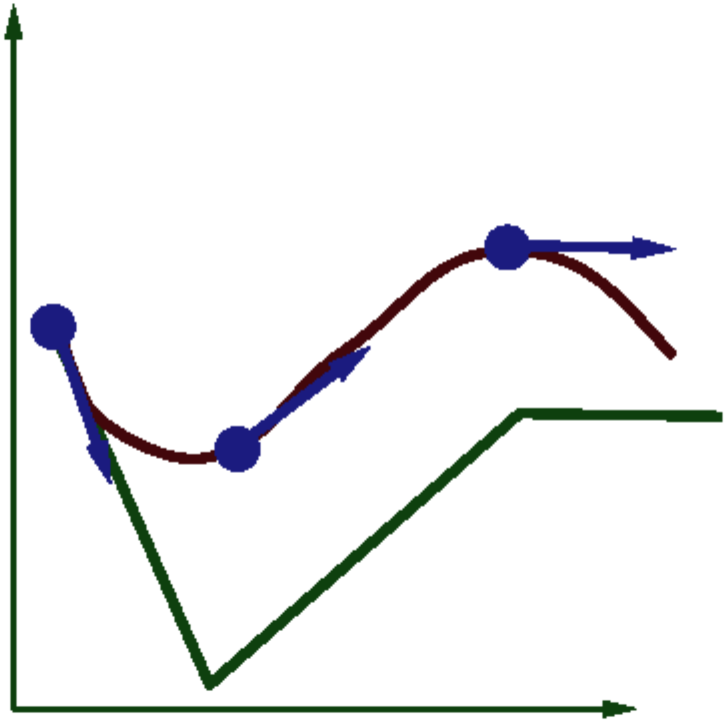
- Initial value problem
    - Know x(0), v(0)
    - Can compute force (and therefore acceleration) for any position / velocity / time
    - Compute x(t) by forward integration



Hodgins

Solving the Equations of Motion

- Forward (explicit) Euler integration
    - $x(t+\Delta t) \leftarrow x(t) + \Delta t\, v(t)$
    - $v(t+\Delta t) \leftarrow v(t) + \Delta t\, f\big(x(t), v(t), t\big) / m$



Hodgins

Solving the Equations of Motion

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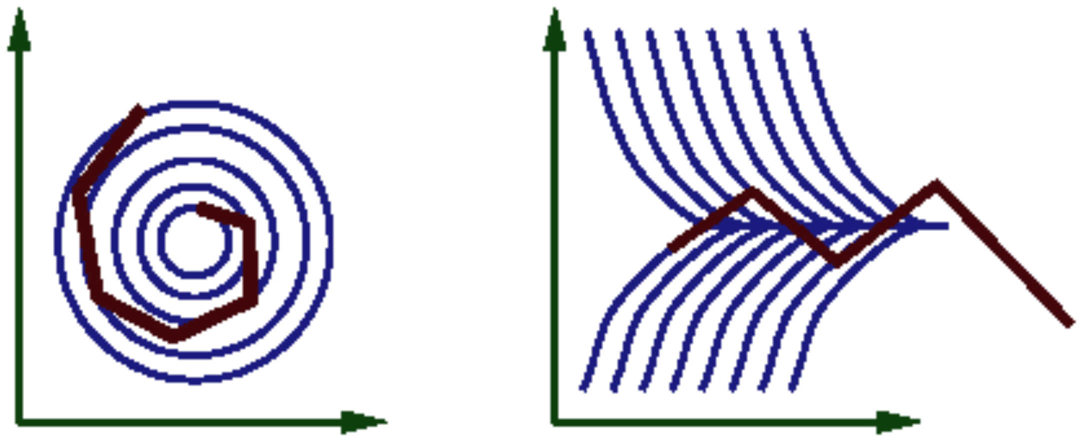
    - $x(t+\Delta t) \leftarrow x(t) + \Delta t\, v(t)$
    - $v(t+\Delta t) \leftarrow v(t) + \Delta t\, f\big(x(t), v(t), t\big) / m$
- Problem:
    - Accuracy decreases as Δt gets bigger

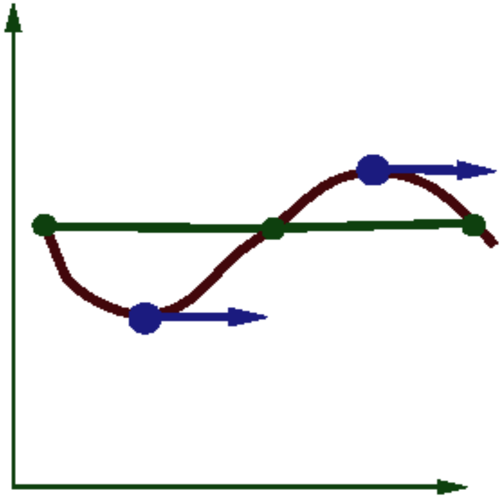


Hodgins

Solving the Equations of Motion

- Midpoint method (2nd-order Runge-Kutta)
  1. Compute an Euler step
  2. Evaluate f at the midpoint of Euler step
  3. Compute new position / velocity using midpoint velocity / acceleration

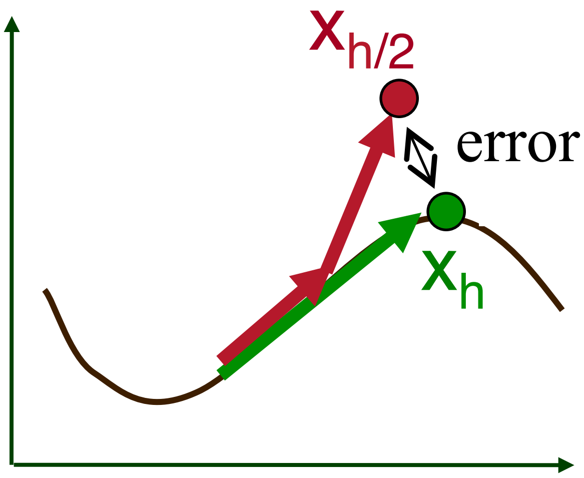
    - $x_{mid} \leftarrow x(t) + \Delta t / 2 * v(t)$
    - $v_{mid} \leftarrow v(t) + \Delta t / 2 * f(x(t), v(t), t) / m$
    - $x(t+\Delta t) \leftarrow x(t) + \Delta t\, v_{mid}$
    - $v(t+\Delta t) \leftarrow v(t) + \Delta t\, f(x_{mid}, v_{mid}, t) / m$

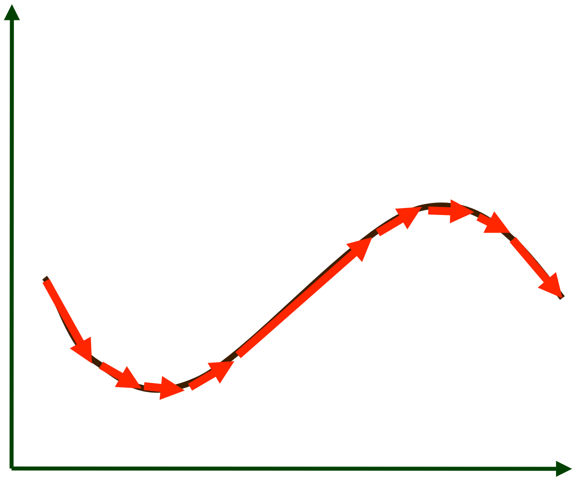


Hodgins

Solving the Equations of Motion

- Adaptive step size
  - Repeat until error is below threshold
    1. Compute x_h by taking one step of size h
    2. Compute $x_{h/2}$ by taking 2 steps of size h / 2
    3. Compute error = | $x_h - x_{h/2}$ |
    4. If (error < threshold) break
    5. Else, reduce step size and try again





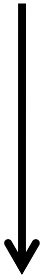
Particle System Forces

- Force fields
  - Gravity, wind, pressure
- Viscosity/damping
  - Drag, friction
- Collisions
  - Static objects in scene
  - Other particles
- Attraction and repulsion
  - Springs between neighboring particles (mesh)
  - Gravitational pull, charge

Particle System Forces

- Gravity
  - Force due to gravitational pull (of earth)
  - g = acceleration due to gravity (m/s^2)

$$f_g = mg$$

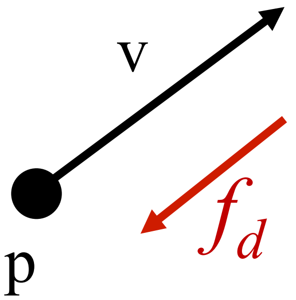


$$g = (0, -9.80665, 0)$$

Particle System Forces

- Drag
  - Force due to resistance of medium
  - k_{drag} = drag coefficient (kg/s)

$$f_d = -k_{drag} v$$

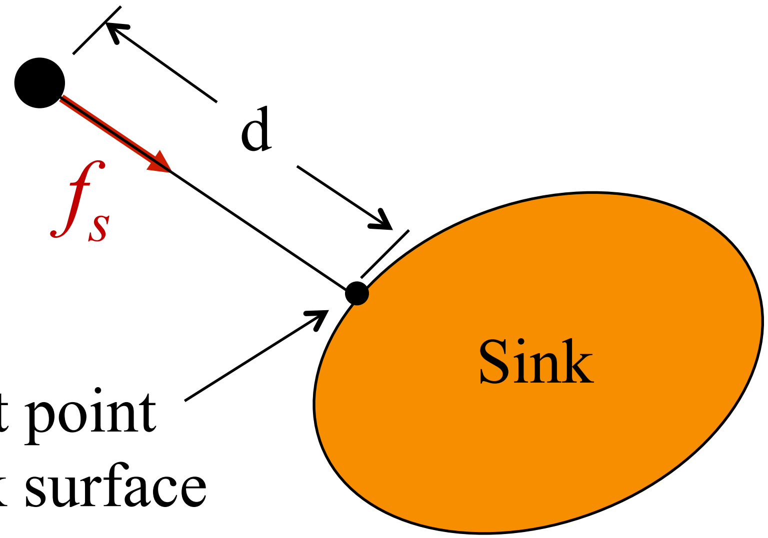


  - Air resistance sometimes taken as proportional to v^2

Particle System Forces

- Sinks
  - Force due to attractor in scene

$$f_s = \frac{\text{intensity}}{ca + la \cdot d + qa \cdot d^2}$$

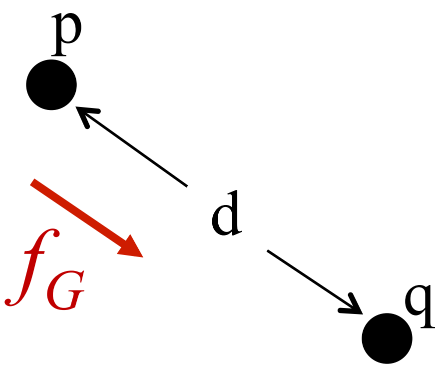


Particle System Forces

- Gravitational pull of other particles
  - Newton’s universal law of gravitation

$$f_G = G\frac{m_1 \cdot m_2}{d^2}$$

$$G = 6.67428 \times 10^{-11} \ \mathrm{N\,m^2\,kg^{-2}}$$



Particle System Forces

- Springs
  - Hooke's law

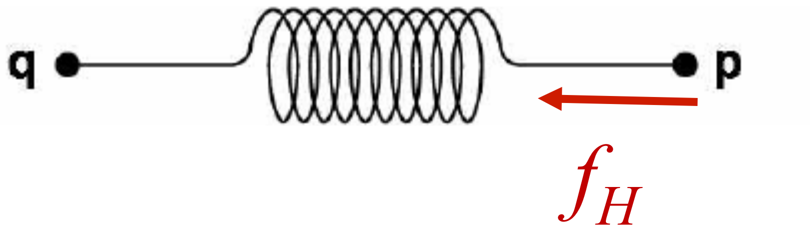
$$f_H(p) = k_s\left(d(p,q) - s\right)\ D$$

$$D = (q - p)/\|q - p\|$$

$$d(p,q) = \|q - p\|$$

s = resting length

k_s = spring coefficient



Particle System Forces

- Springs
  - Hooke's law with damping

$$f_H(p) = \left[k_s\left(d(p,q) - s\right) + k_d\left(v(q) - v(p)\right)\cdot D\right] D$$

$D = (q-p)/\|q-p\|$

$d(p,q) = \|q-p\|$

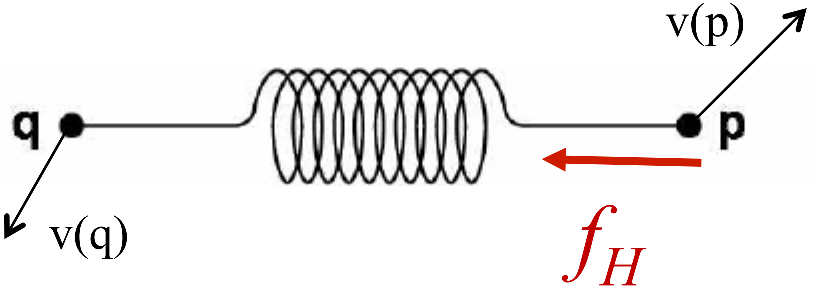
s = resting length

k_s = spring coefficient

k_d = damping coefficient

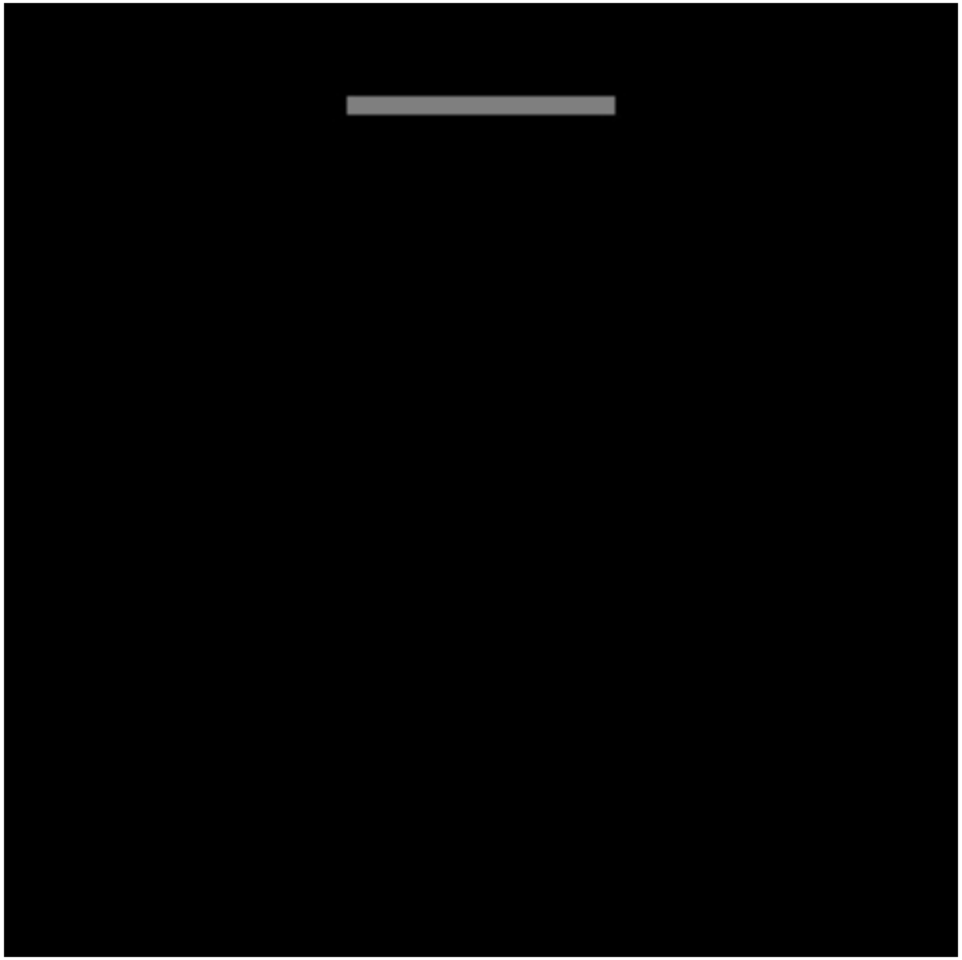
$v(p)$ = velocity of p

$v(q)$ = velocity of q



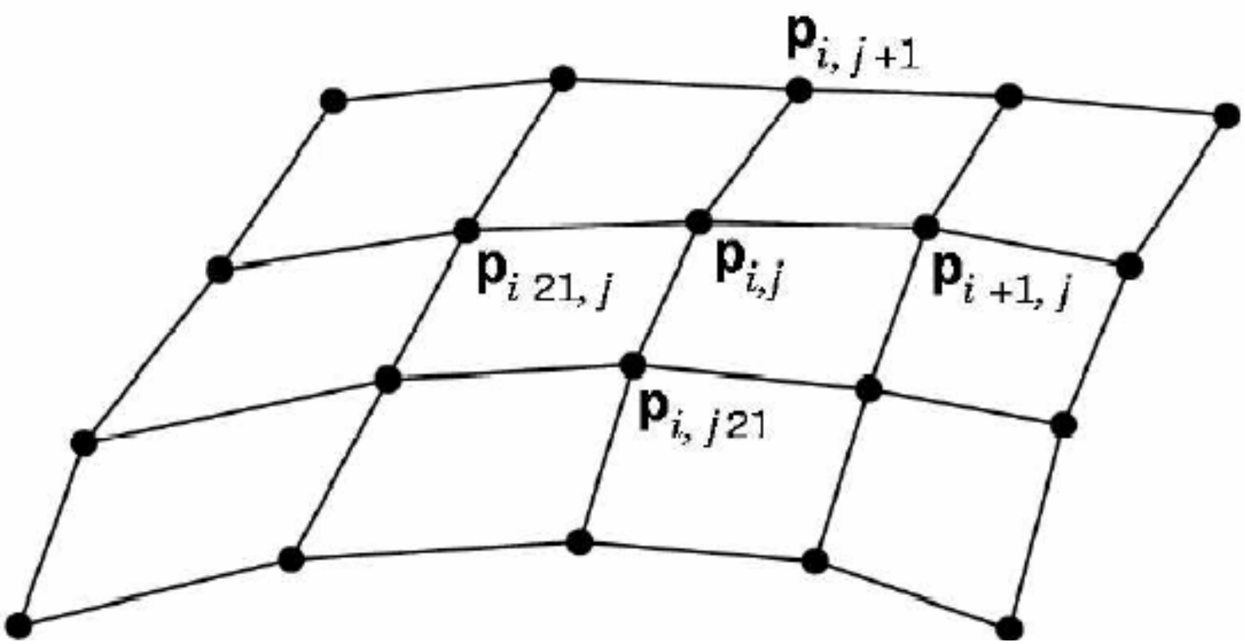
$$k_d \sim 2\sqrt{mk_s}$$

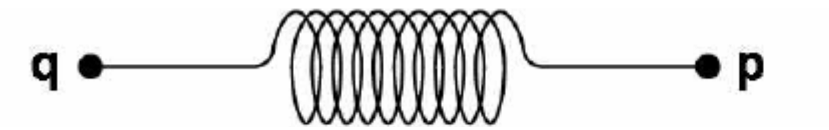
Example: Rope



Particle System Forces

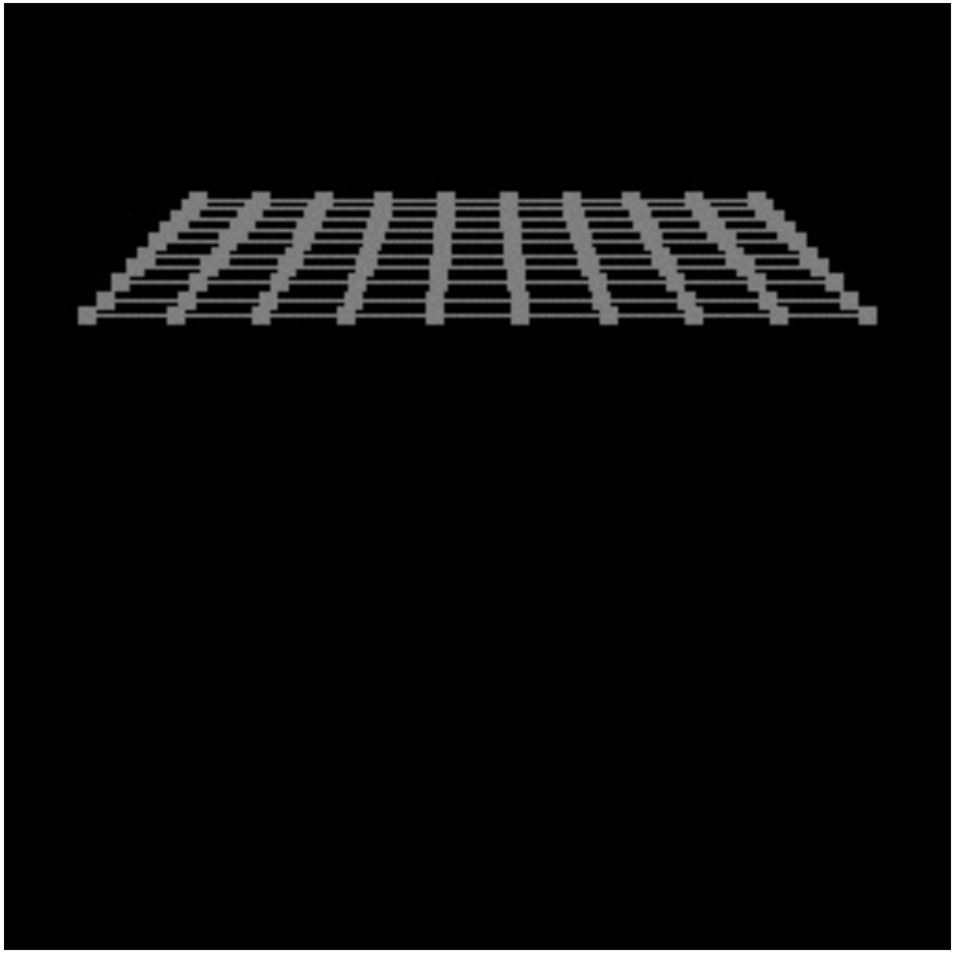
- Spring-mass mesh





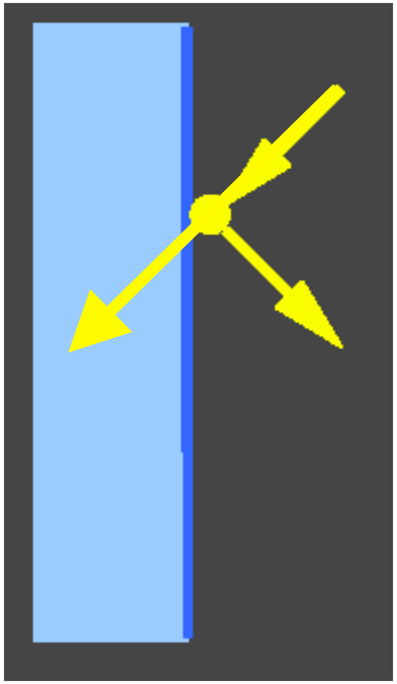
Hodgins

Example: Cloth



Particle System Forces

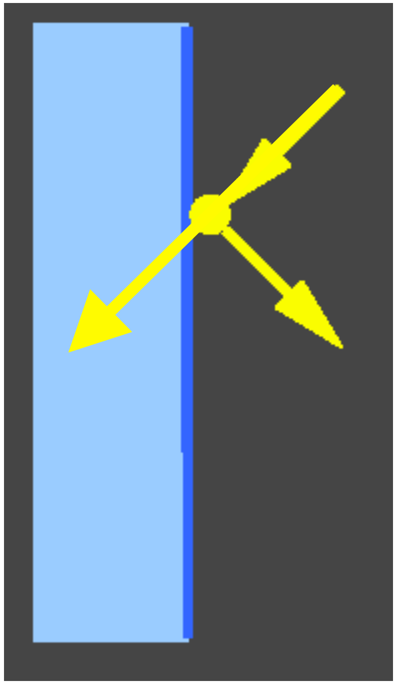
- Collisions
  - Collision detection
  - Collision response



Witkin

Particle System Forces

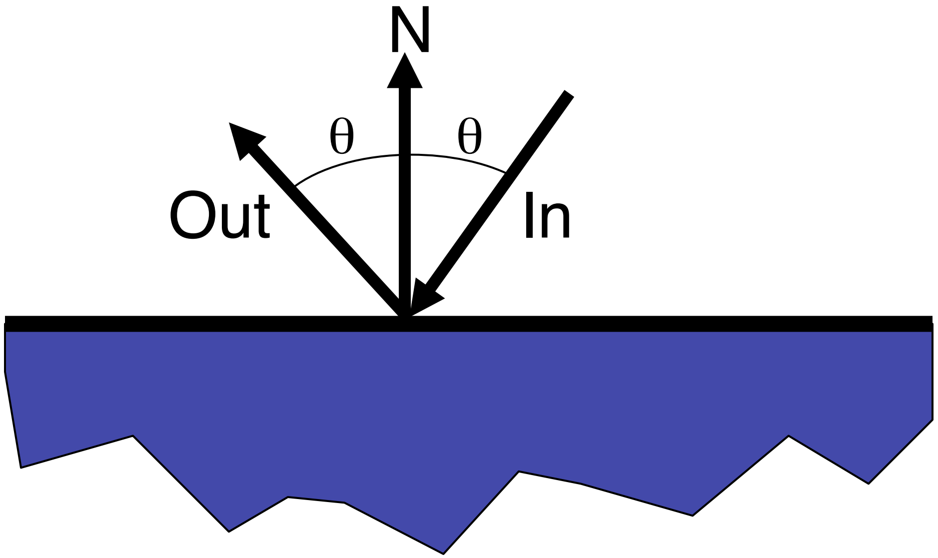
- Collision detection
    - Intersect ray with scene
    - Compute up to Δt at time of first collision, and then continue from there



Witkin

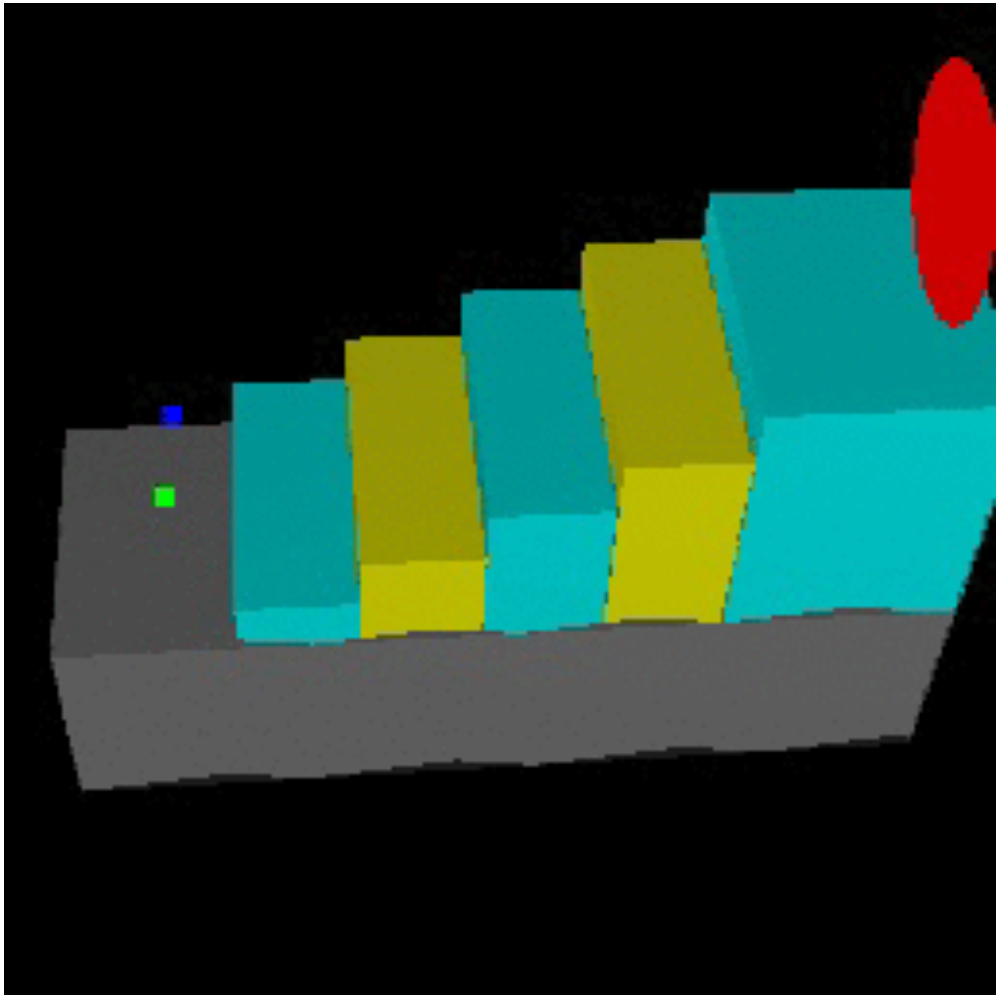
Particle System Forces

- Collision response
  - No friction: elastic collision (for $m_{target} >> m_{particle}$: specular reflection)



  - Otherwise, total momentum conserved, energy dissipated if inelastic

Example: Bouncing



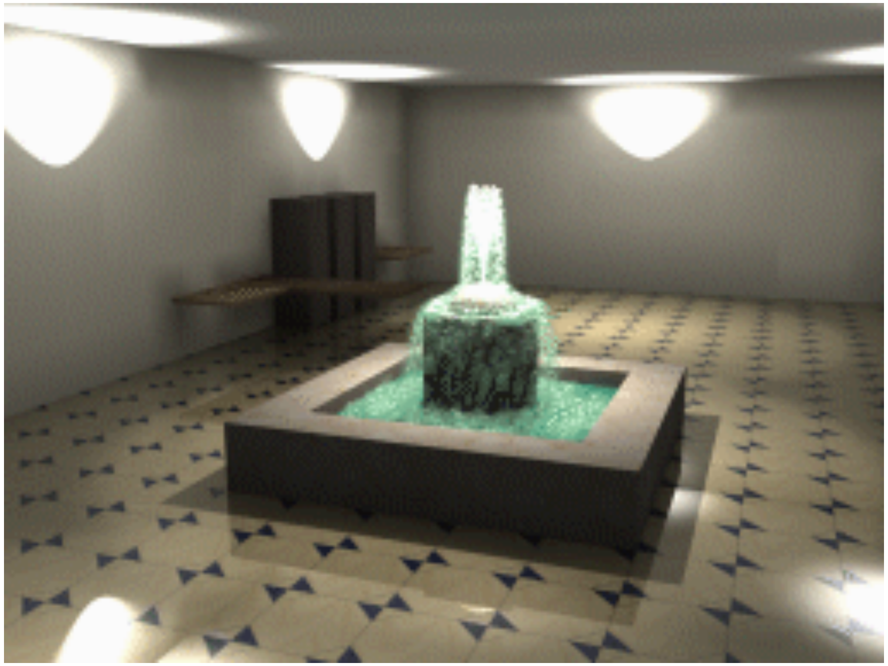
Ning Jin
COS 426, 2013

Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

Deleting Particles

- When to delete particles?
  - When life span expires
  - When intersect predefined sink surface
  - Where density is high
  - Random



McAllister

Particle Systems

- For each frame:
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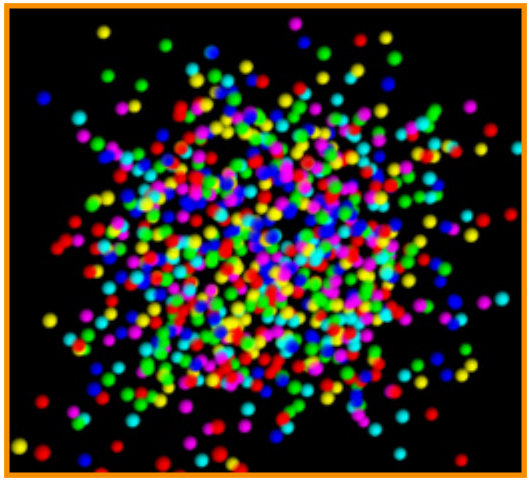
  - Render particles

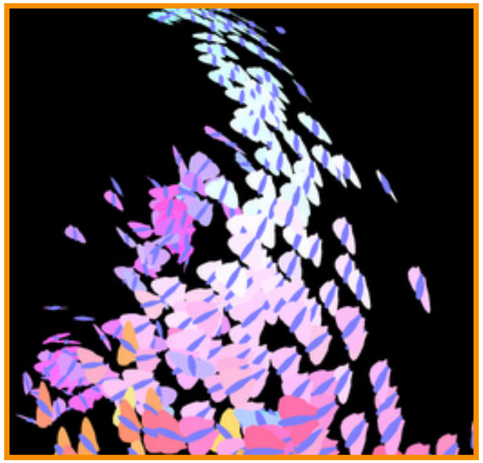
Rendering Particles

- Rendering styles
  - ➢ Points
  - Polygons
  - Shapes
  - Trails
  - etc.



Rendering Particles

- Rendering styles
    - Points
    - ➢ Textured polygons: sprites
    - Shapes
    - Trails
    - etc.



Rendering Particles

- Rendering styles
  - Points
  - Polygons
  - ➢ Shapes
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  - etc.

Rendering Particles

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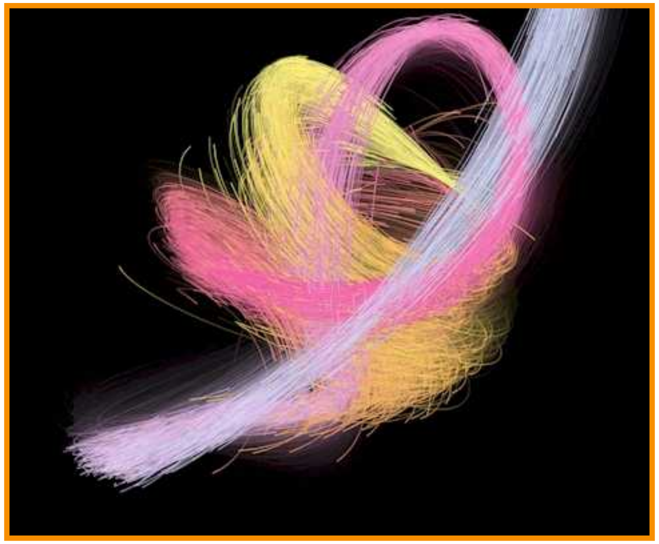
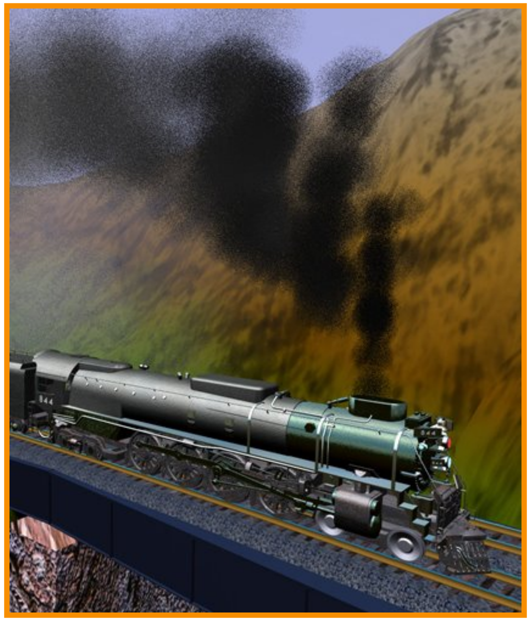
McAllister

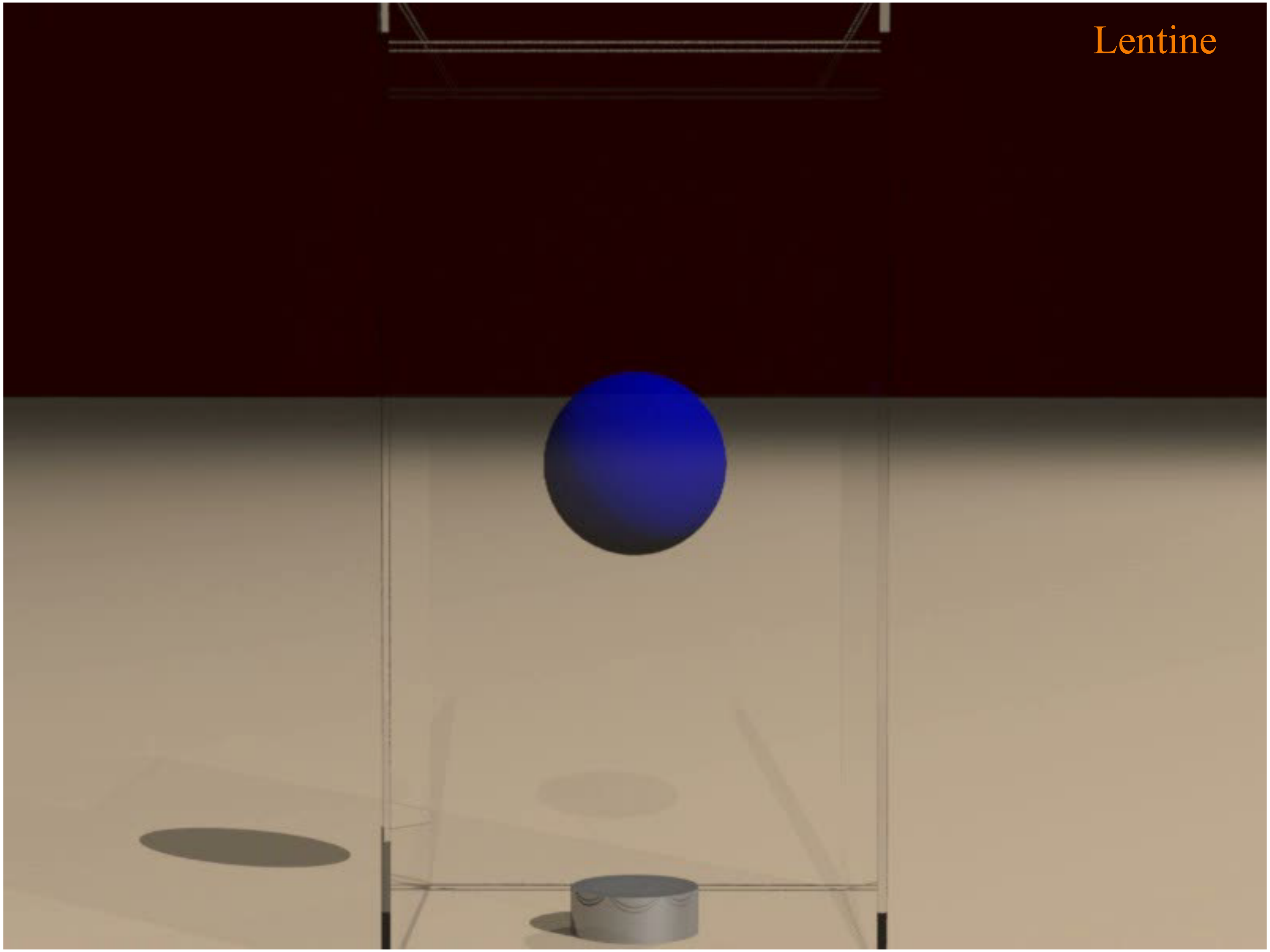


Putting it All Together

- Examples
    - Smoke
    - Water
    - Cloth
    - Fire
    - Fireworks
    - Dice



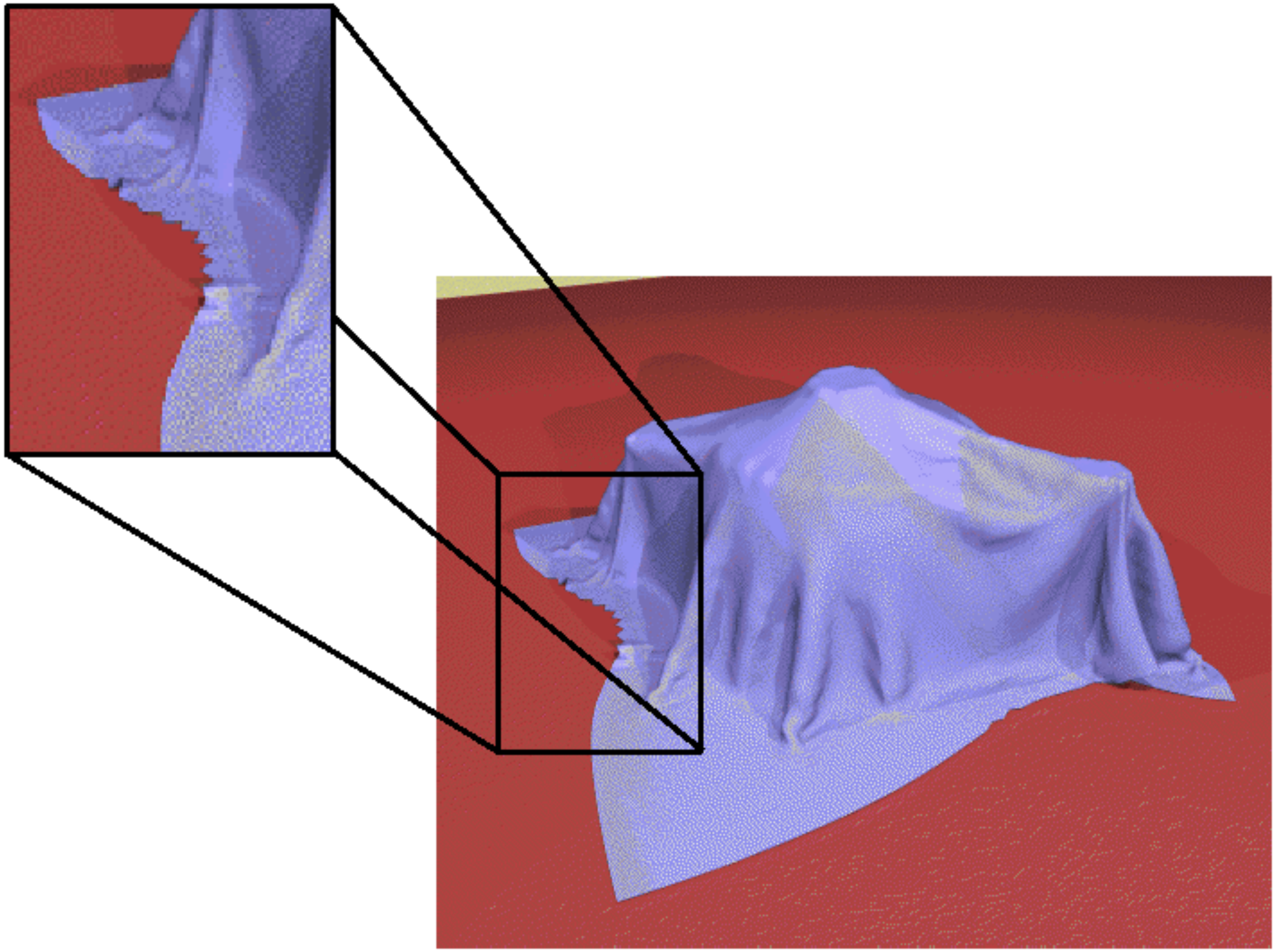
Example: “Smoke”



Lentine

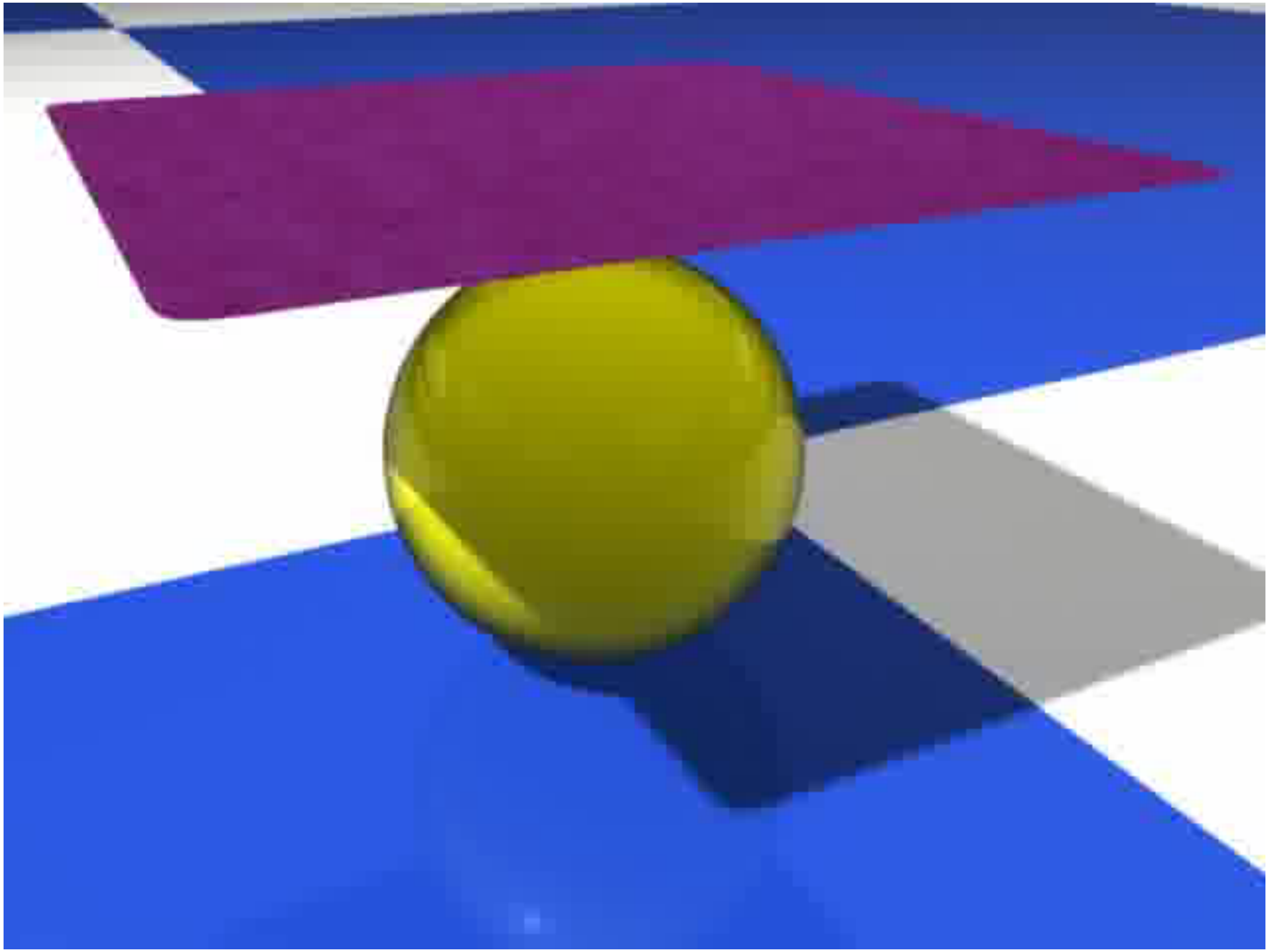
Example: Fire

Example: Cloth



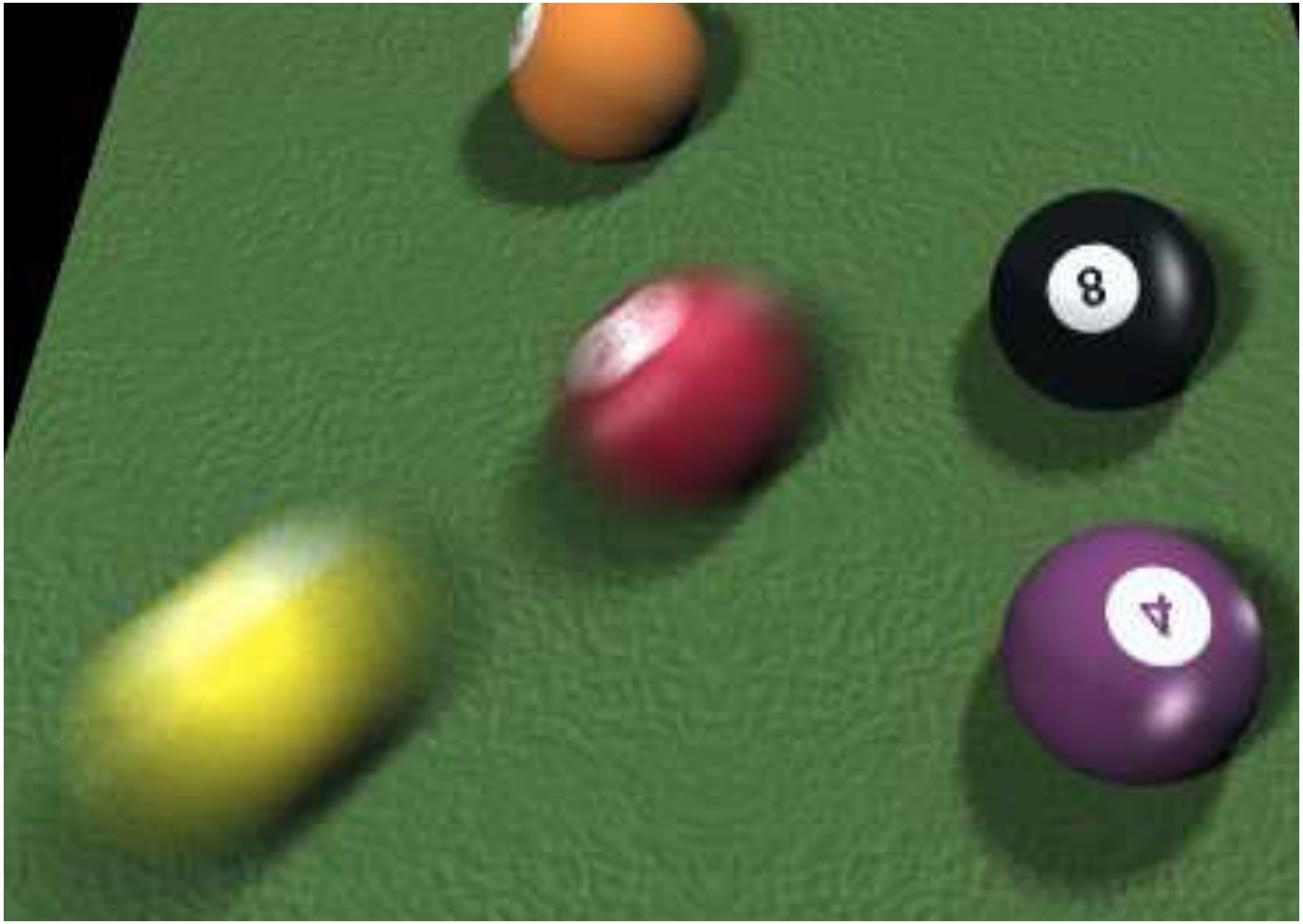
Breen

Example: Cloth

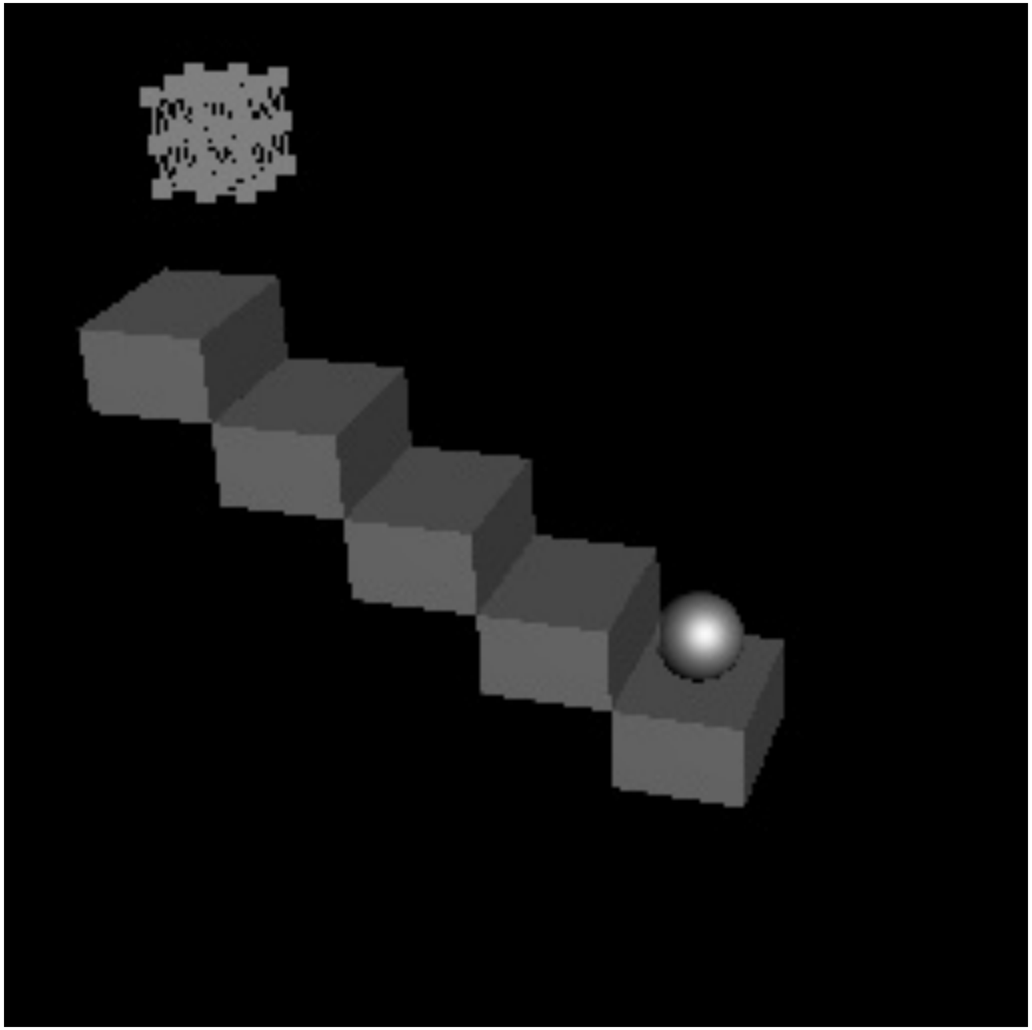


Bender

Example: Bouncing Particles

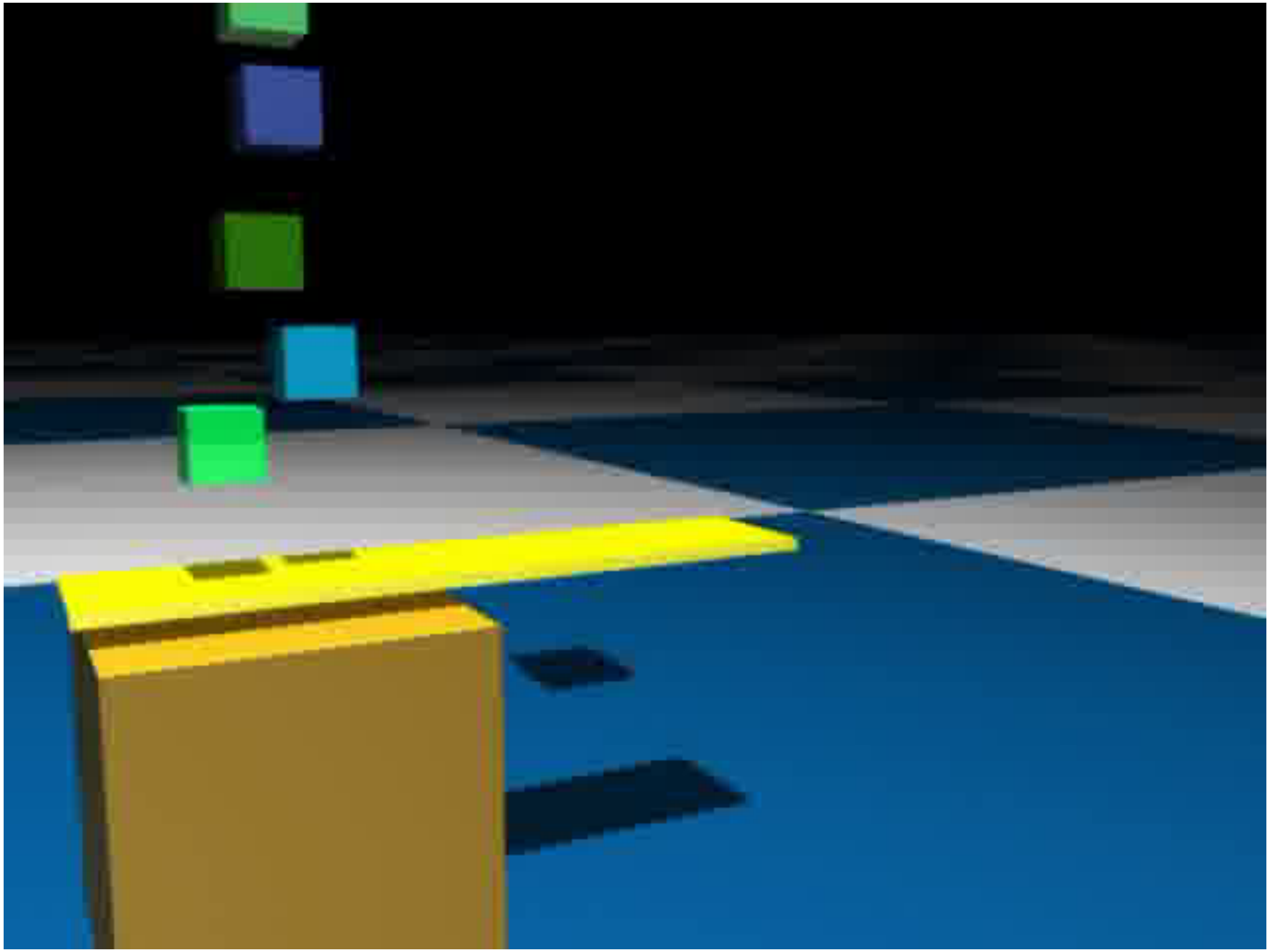


Example: Bouncing Particles



Zhaoyang Xu
COS 426, 2007

Example: More Bouncing



Bender

Example: Flocks & Herds

Reynolds

Summary

- Particle systems
  - Lots of particles
  - Simple physics
- Interesting behaviors
  - Waterfalls
  - Smoke
  - Cloth
  - Flocks
- Solving motion equations
  - For each step, first sum forces, then update position and velocity