
# Polygonal Meshes

COS 426, Spring 2016

Princeton University

# 3D Object Representations


Points
- Range image
- Point cloud

Surfaces
- Polygonal mesh
- Parametric
- Subdivision
- Implicit

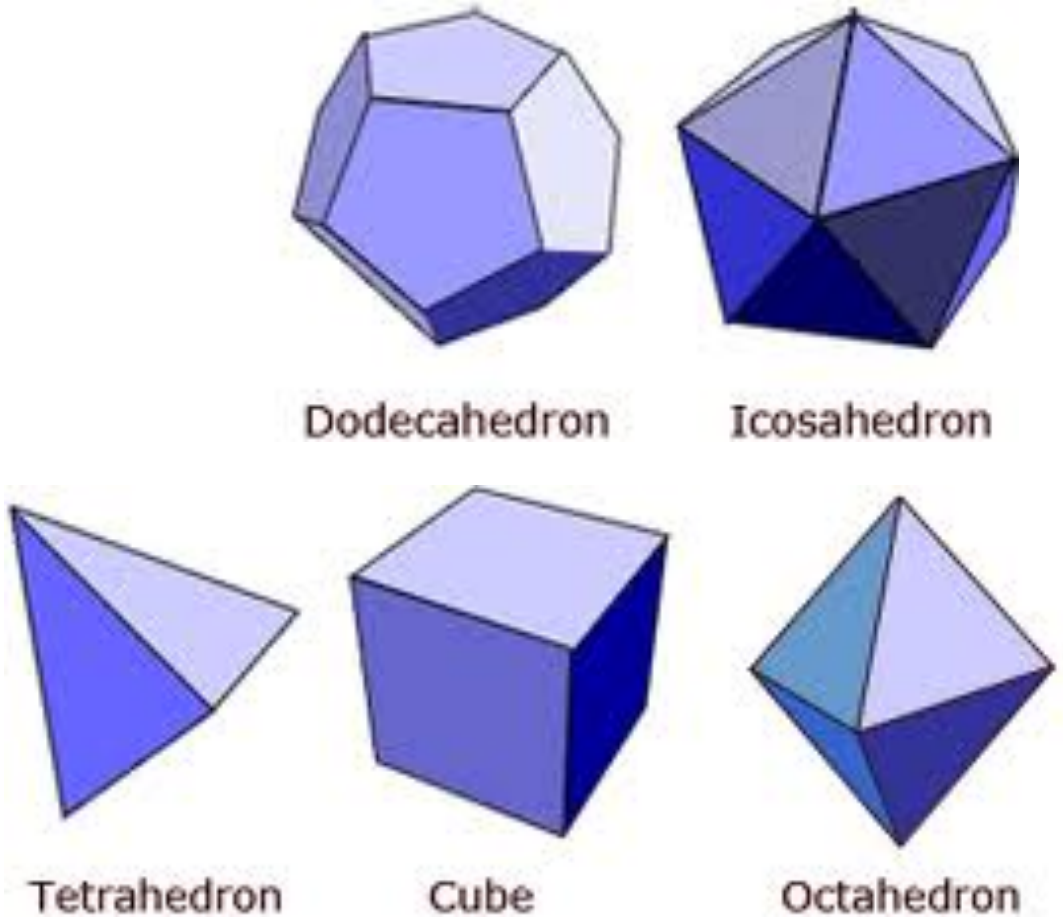
Solids
- Voxels
- BSP tree
- CSG
- Sweep

High-level structures
- Scene graph
- Application specific

# 3D Polygonal Mesh

Set of polygons representing a 2D surface embedded in 3D

Platonic Solids



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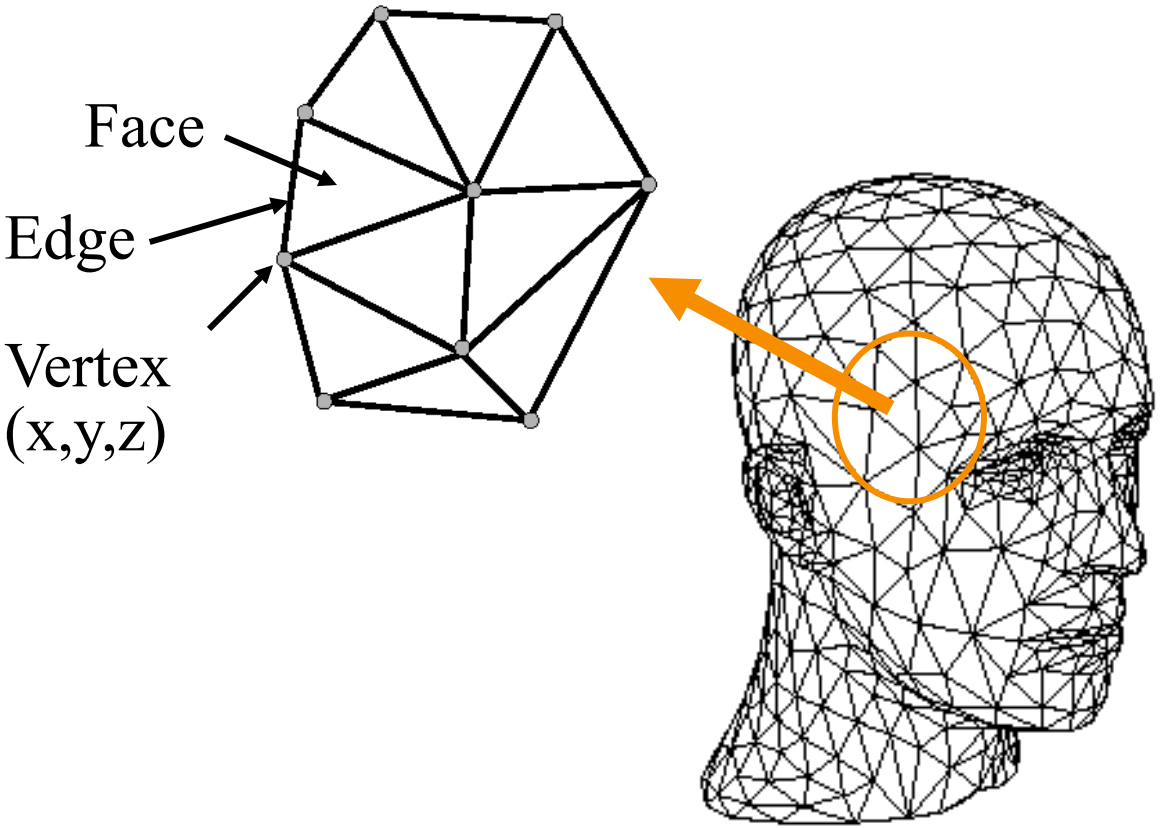

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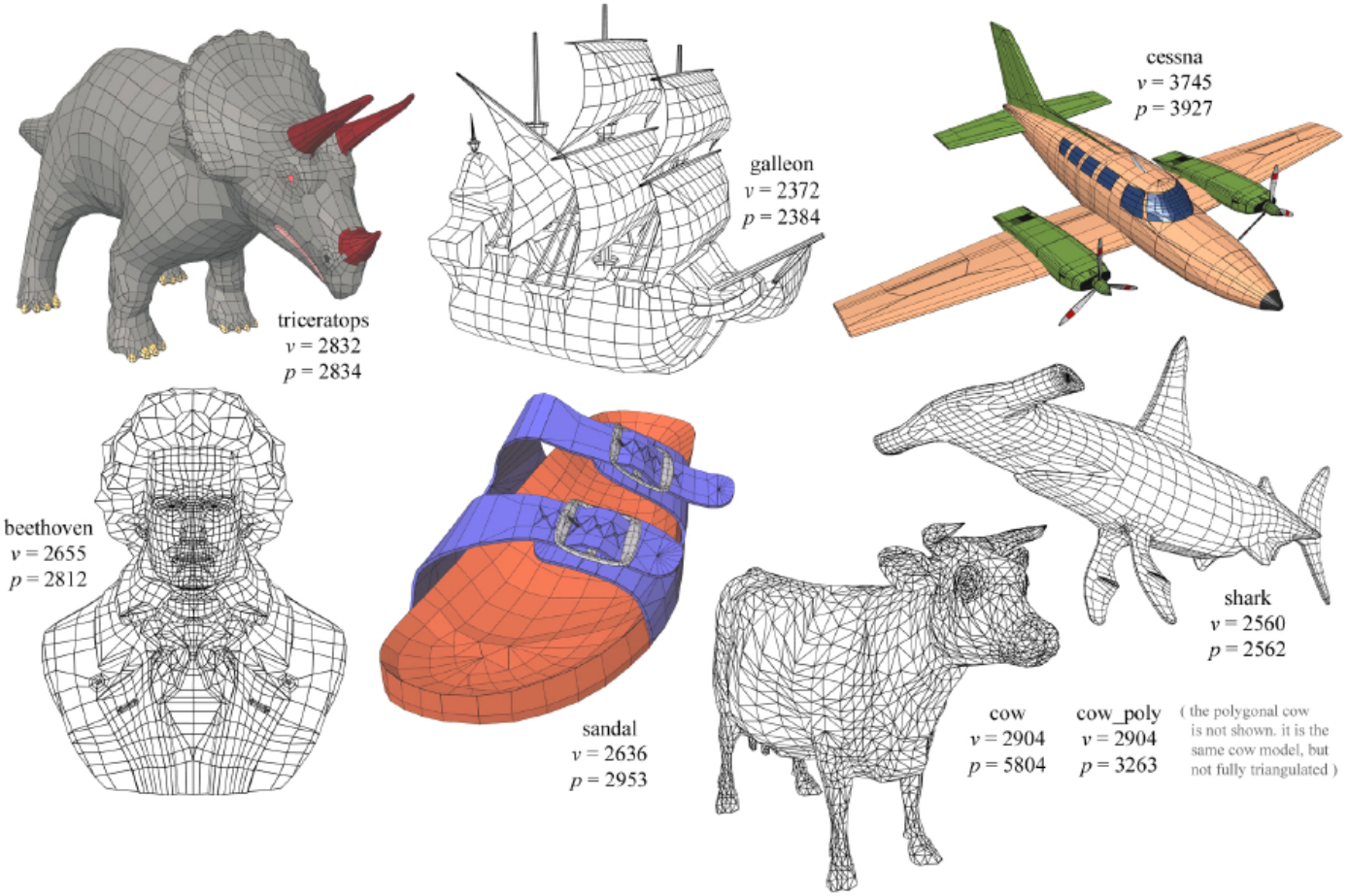
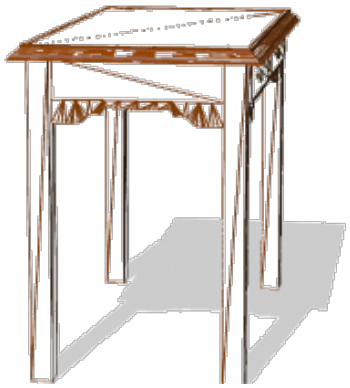
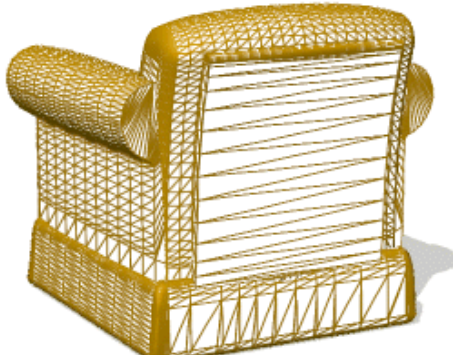


# 3D Polygonal Meshes

Why are they of interest?

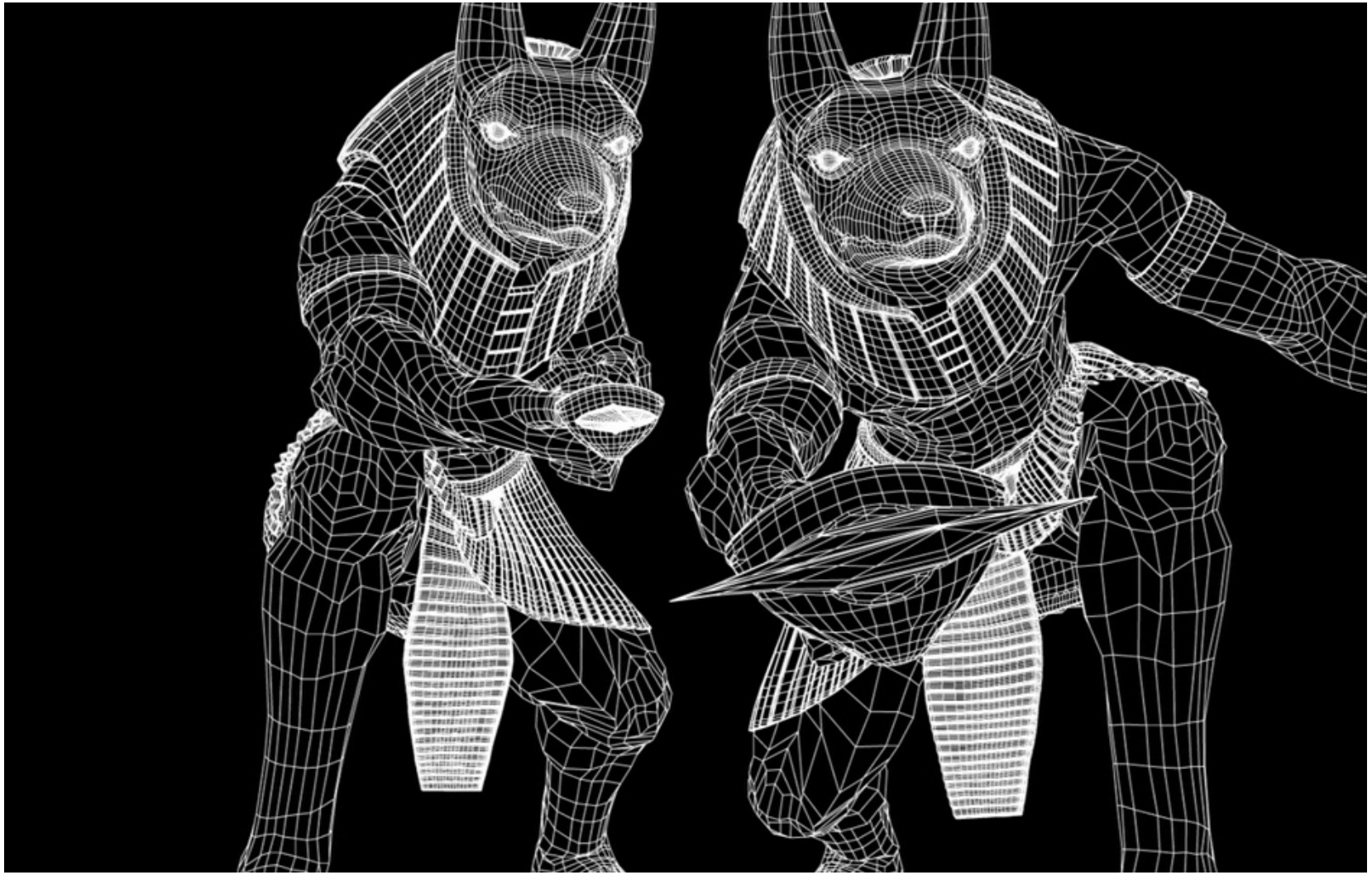
- Simple, common representation
- Rendering with hardware support
- Output of many acquisition tools
- Input to many simulation/analysis tools





Viewpoint

# 3D Polygonal Meshes

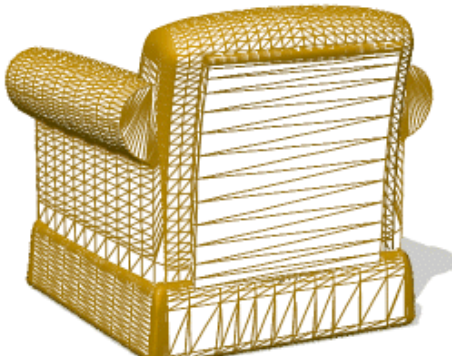


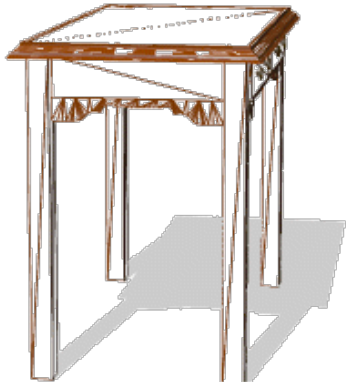
http://www.fxguide.com/featured/Comic_Horrors_Rocks_Statues_and_VanDyke/

# 3D Polygonal Meshes

Properties

- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.





Viewpoint

# Outline


Acquisition ←

Processing

Representation

# Polygonal Mesh Acquisition


Interactive modeling
- Polygon editors
- Interchange formats

Scanners
- Laser range scanners
- Geological survey

Procedural generation
- Surface of revolution
- Sweep

Simulations
- Physical processes

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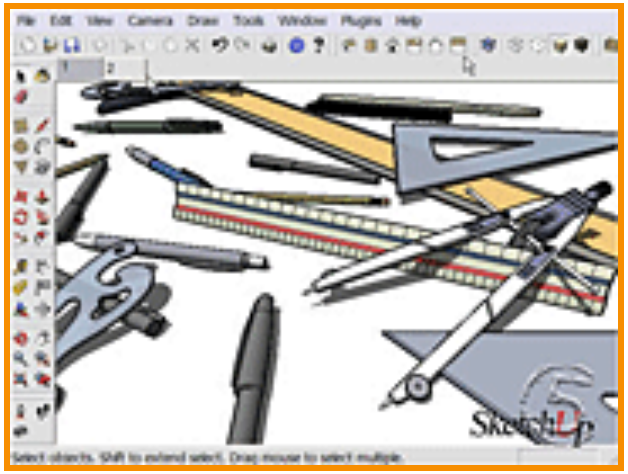
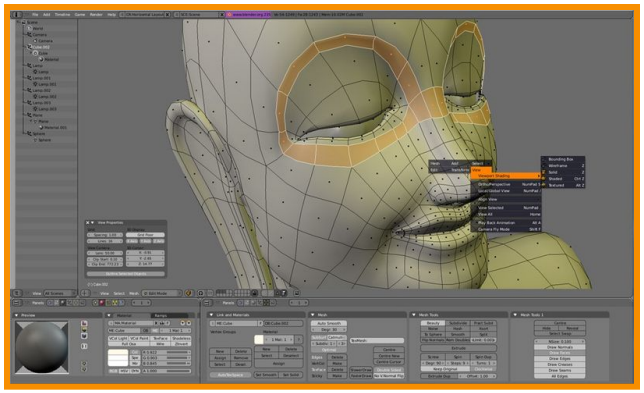


Sketchup



Blender

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Jose Maria De Espona

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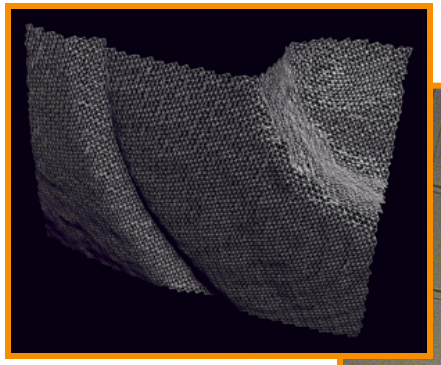

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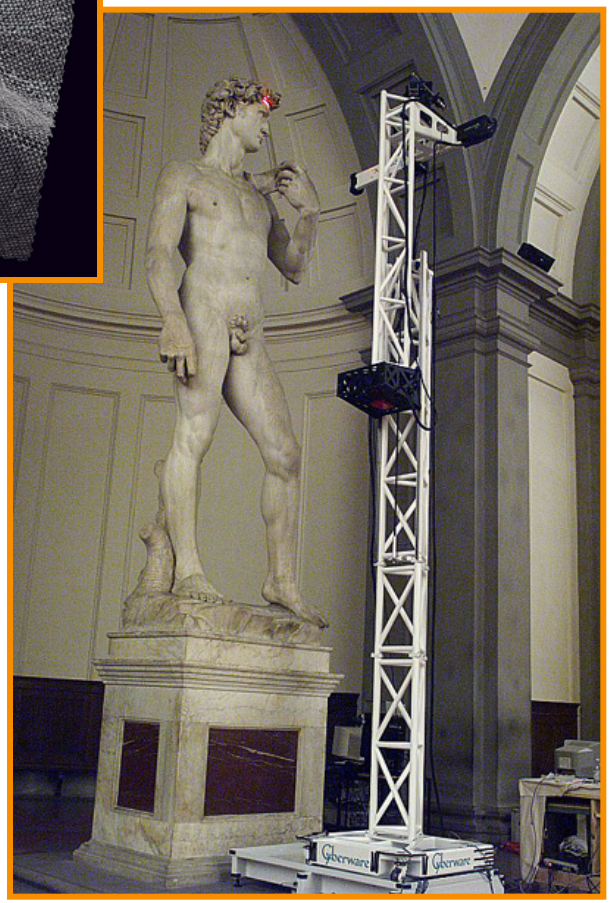

Digital Michelangelo Project
Stanford

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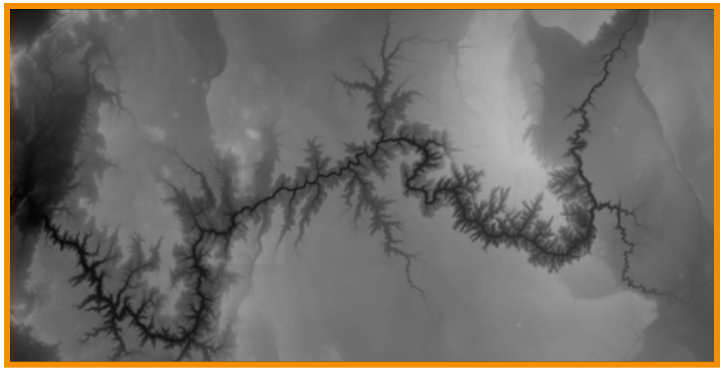

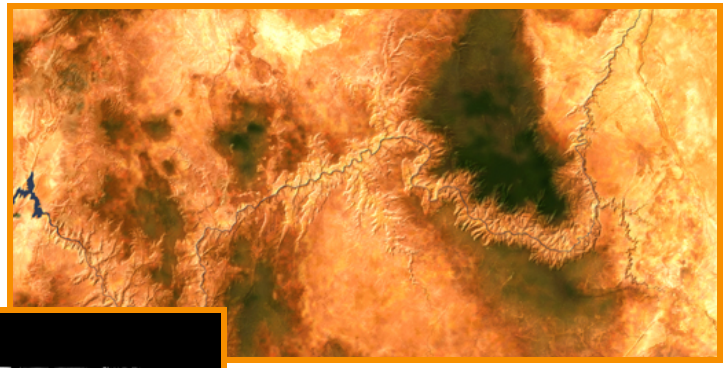

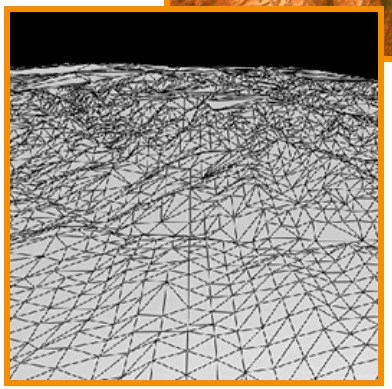

Large Geometric Model Repository Georgia Tech

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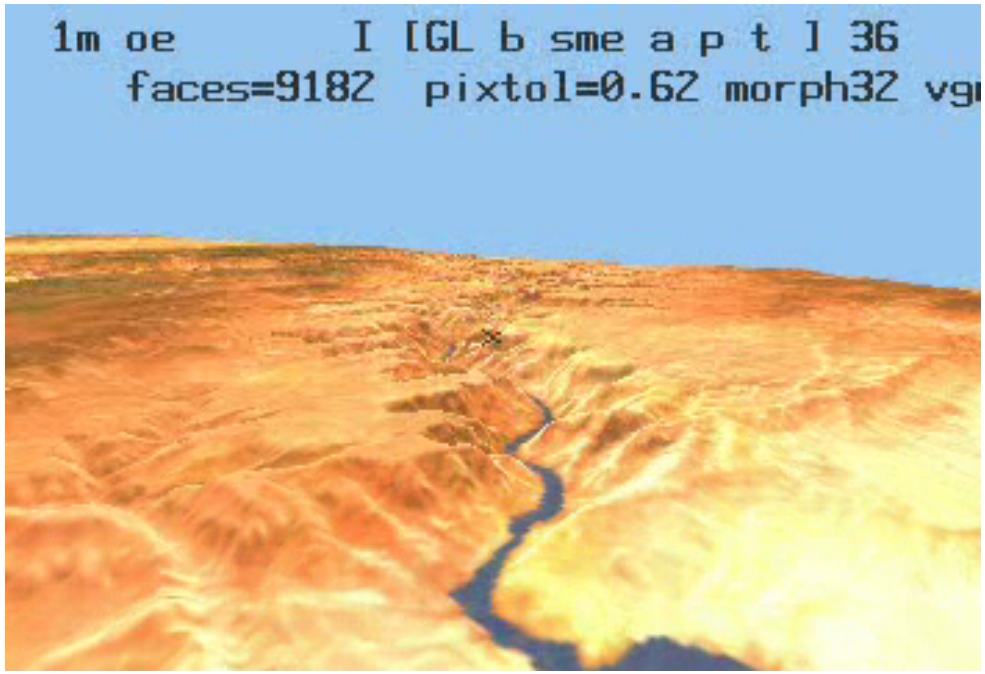

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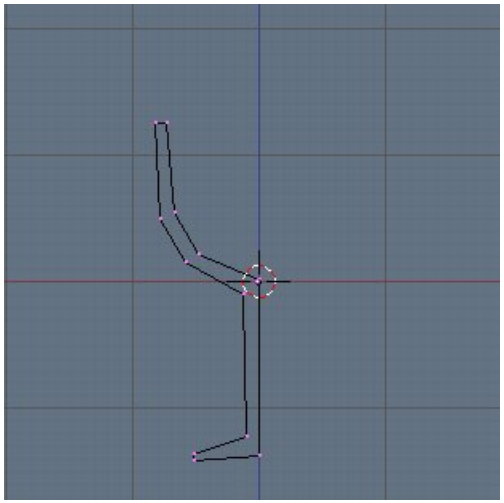

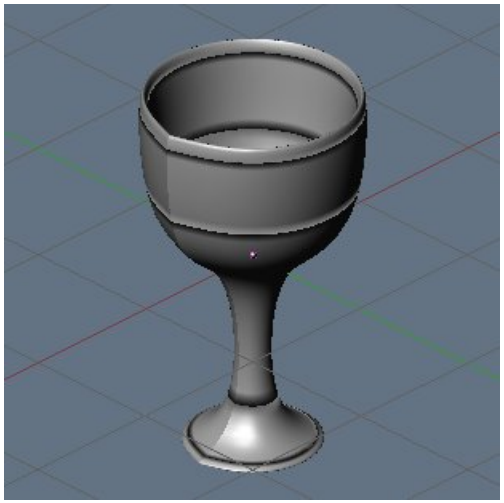

sphynx.co.uk

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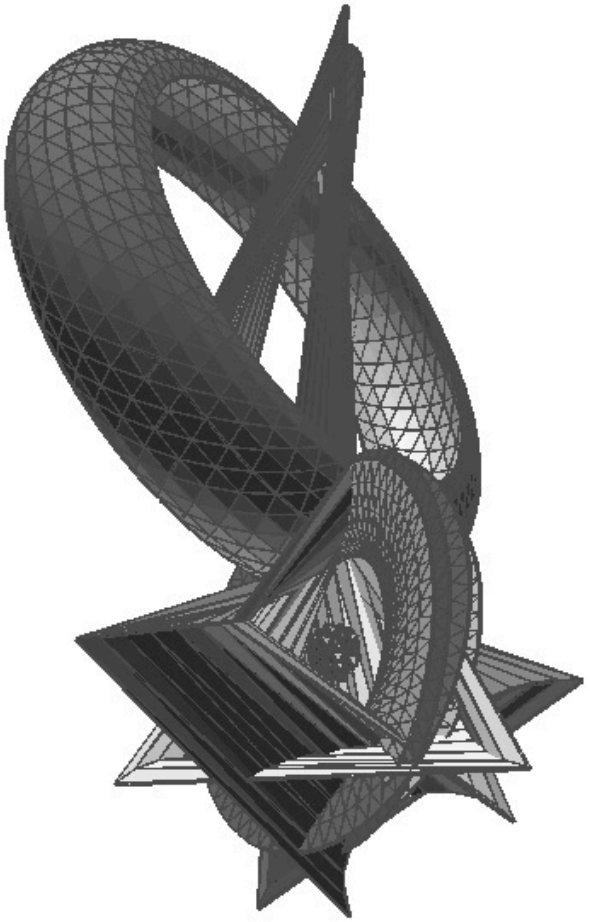

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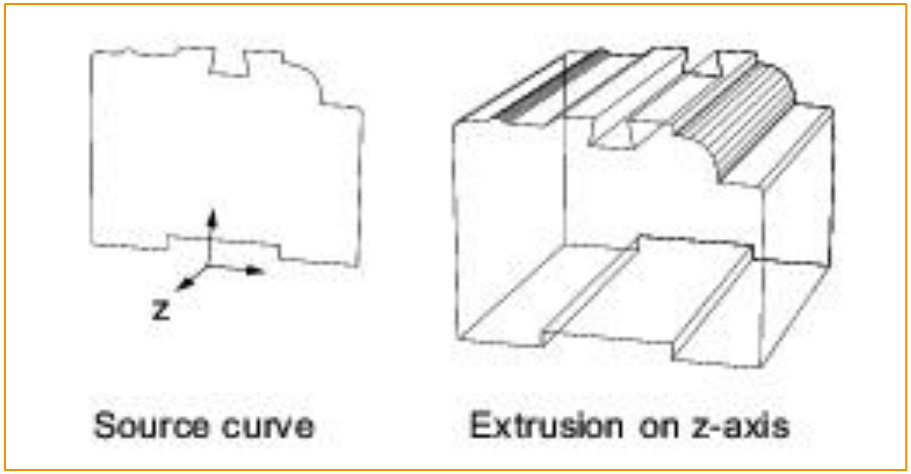


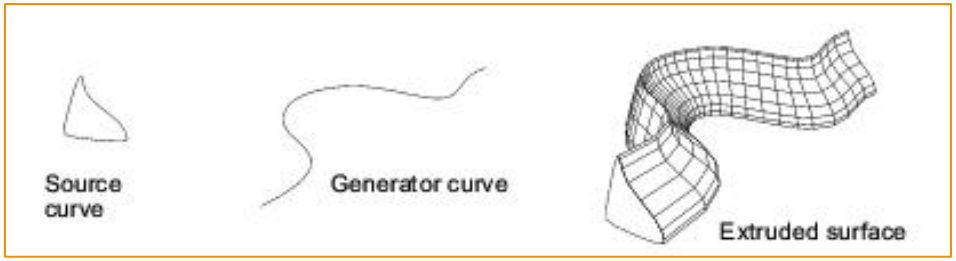

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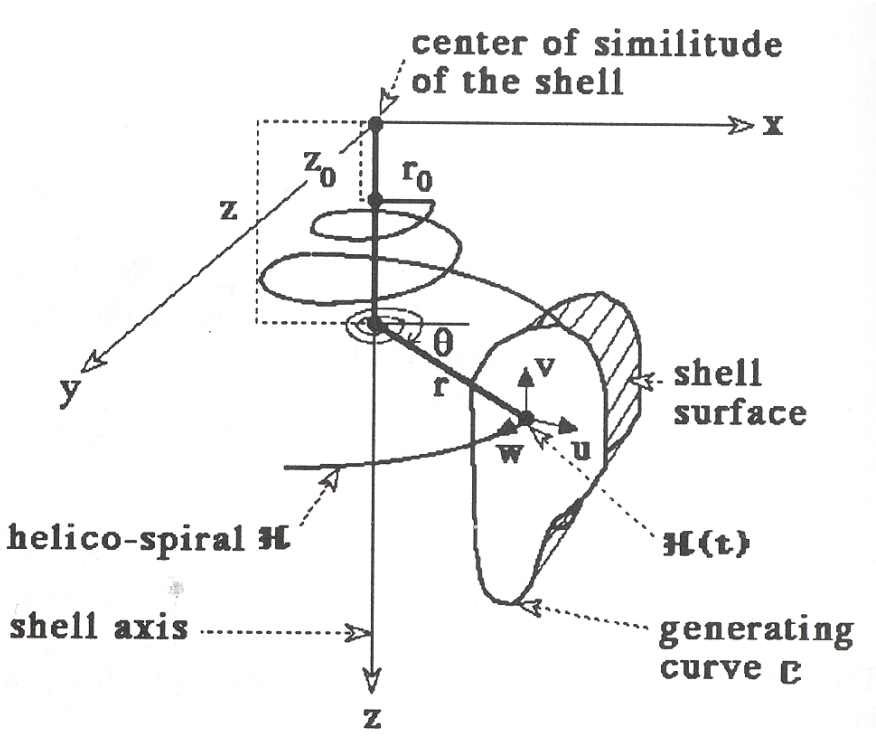


Fowler et al., 1992

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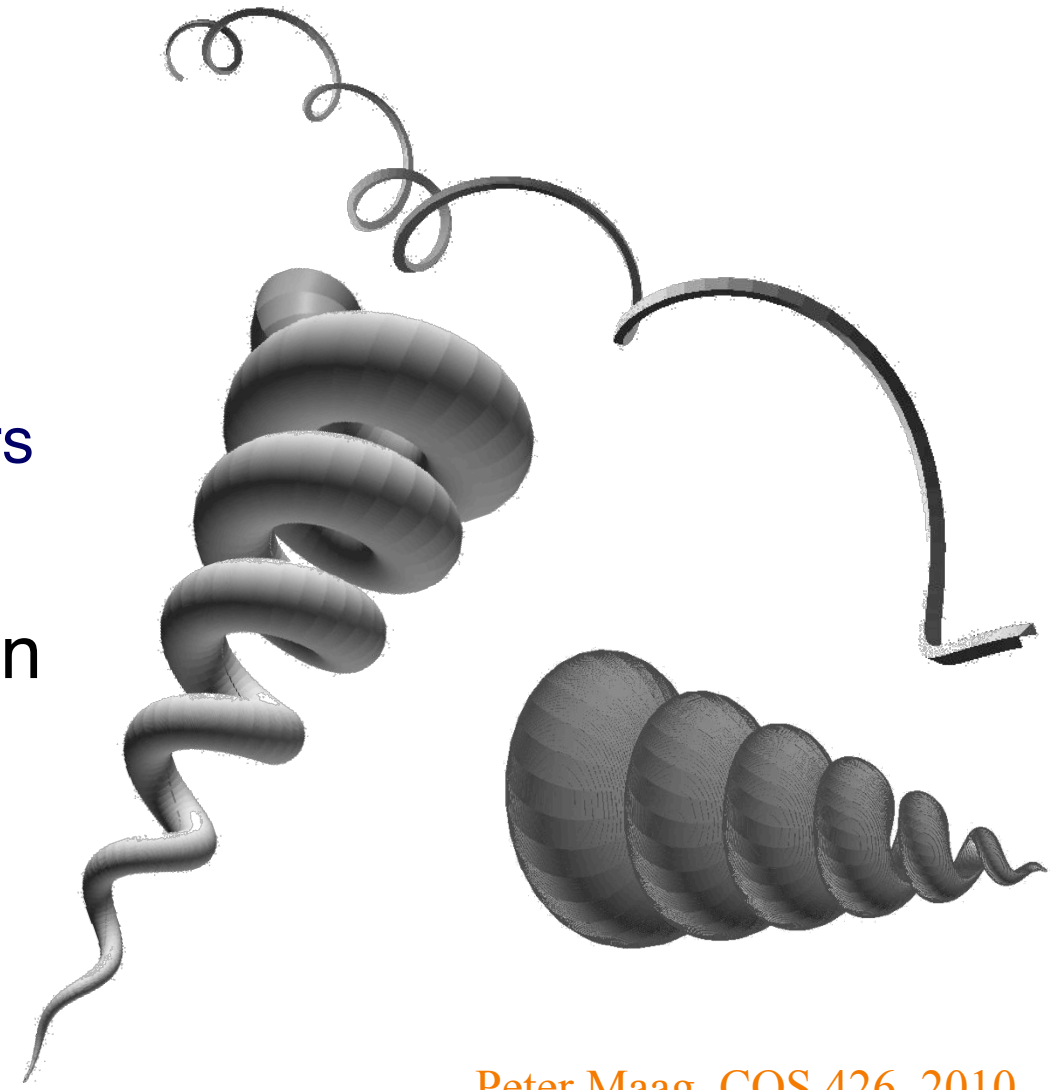

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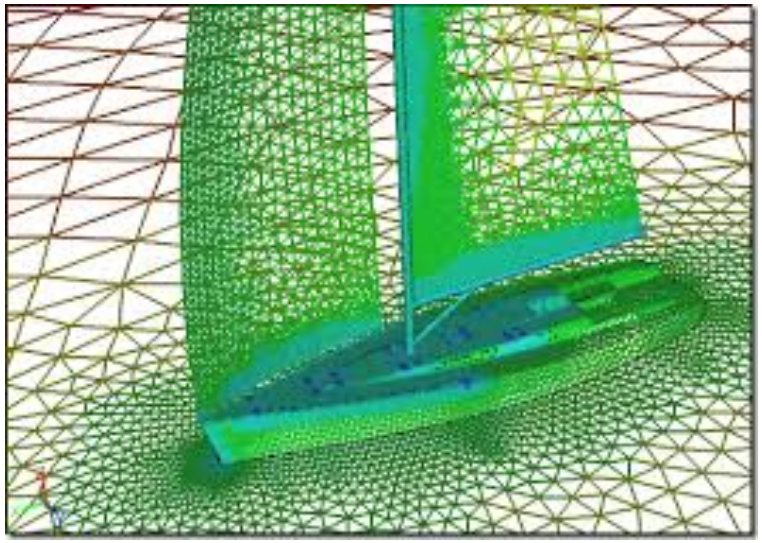
SGI



symscape

# Outline


Acquisition

Processing ←

Representation

# Polygonal Mesh Processing


Analysis
- Normals
- Curvature

Warps
- Rotate
- Deform

Filters
- Smooth
- Sharpen
- Truncate
- Bevel

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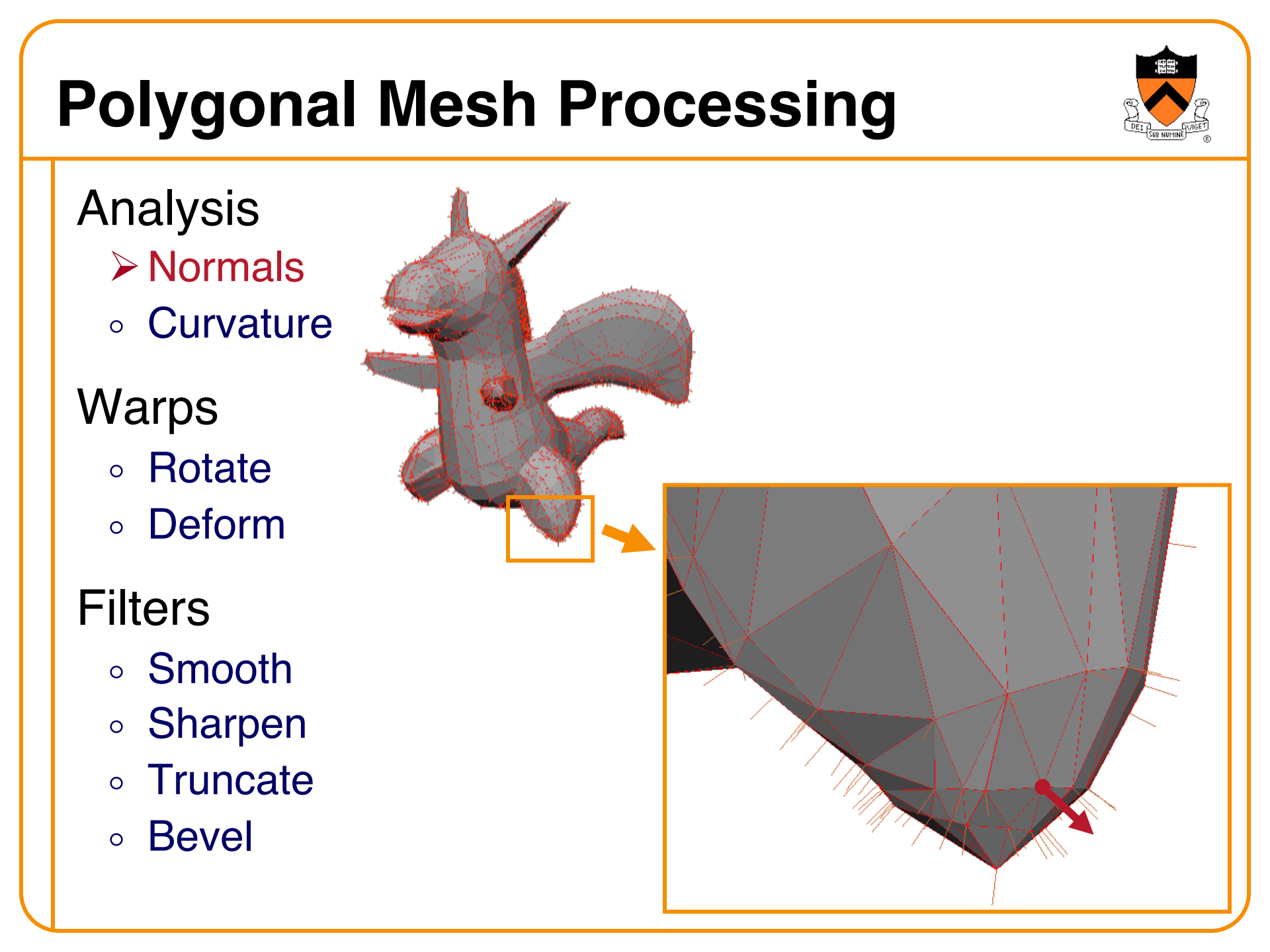


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"The bunny with normal vertices shown.
Reminded me of an album cover so I made it into one."

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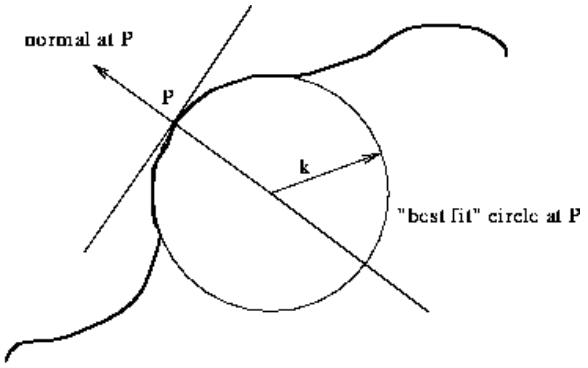


Figure 32: curvature of curve at $P$ is $1/k$

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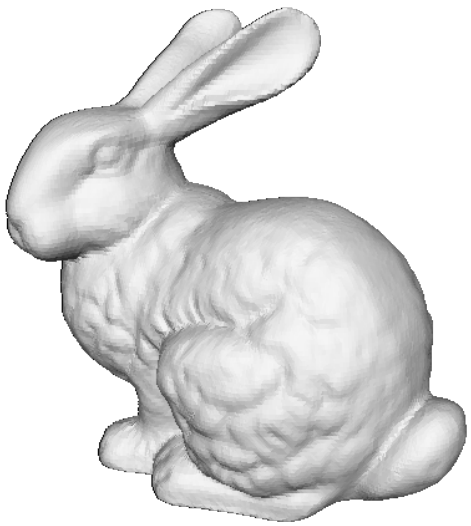

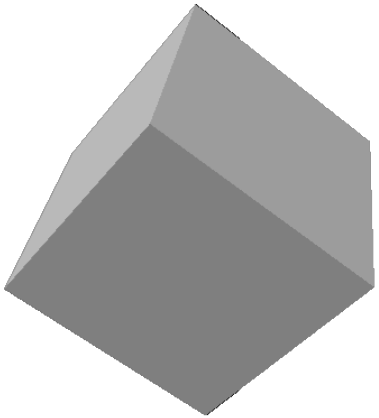

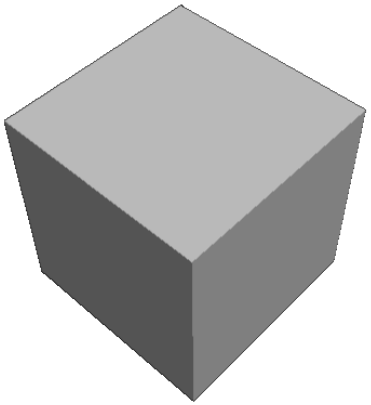

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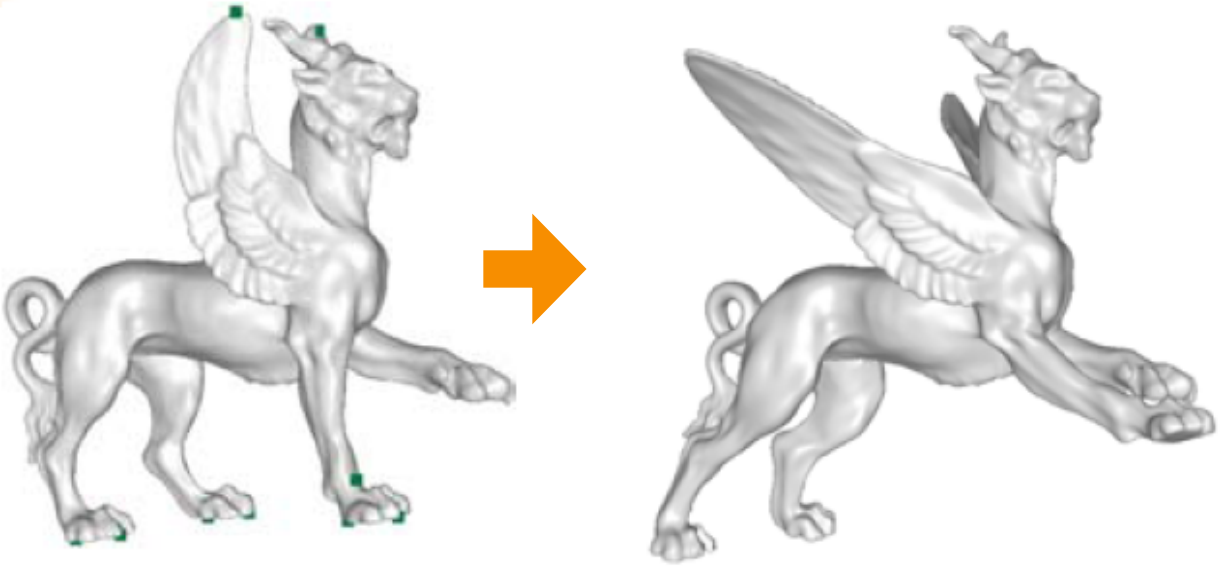

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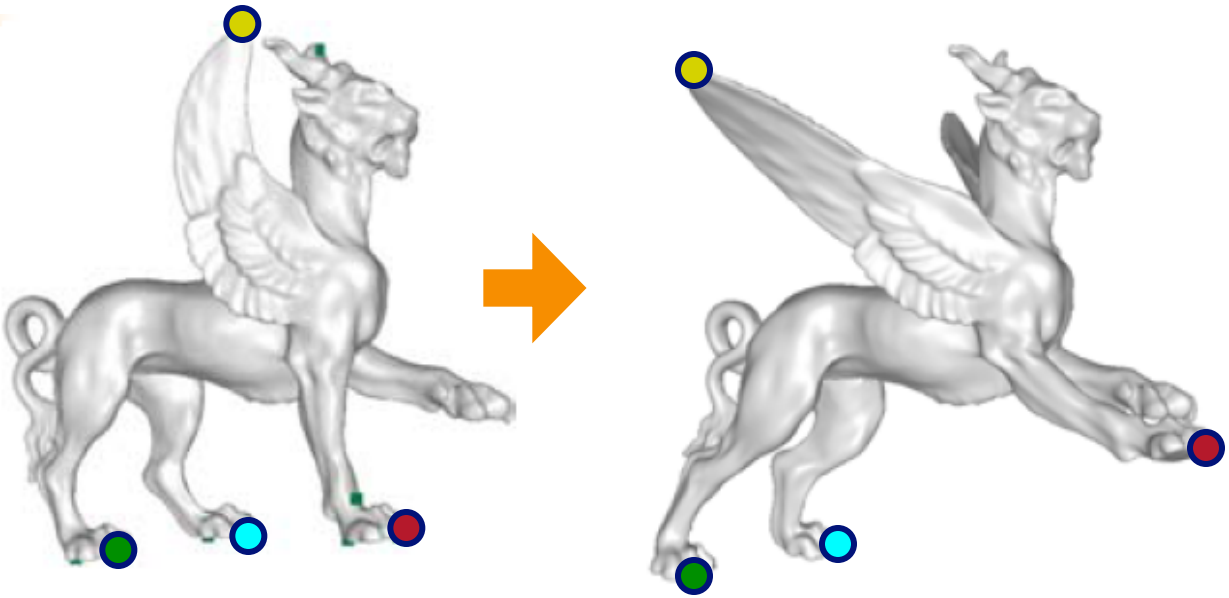

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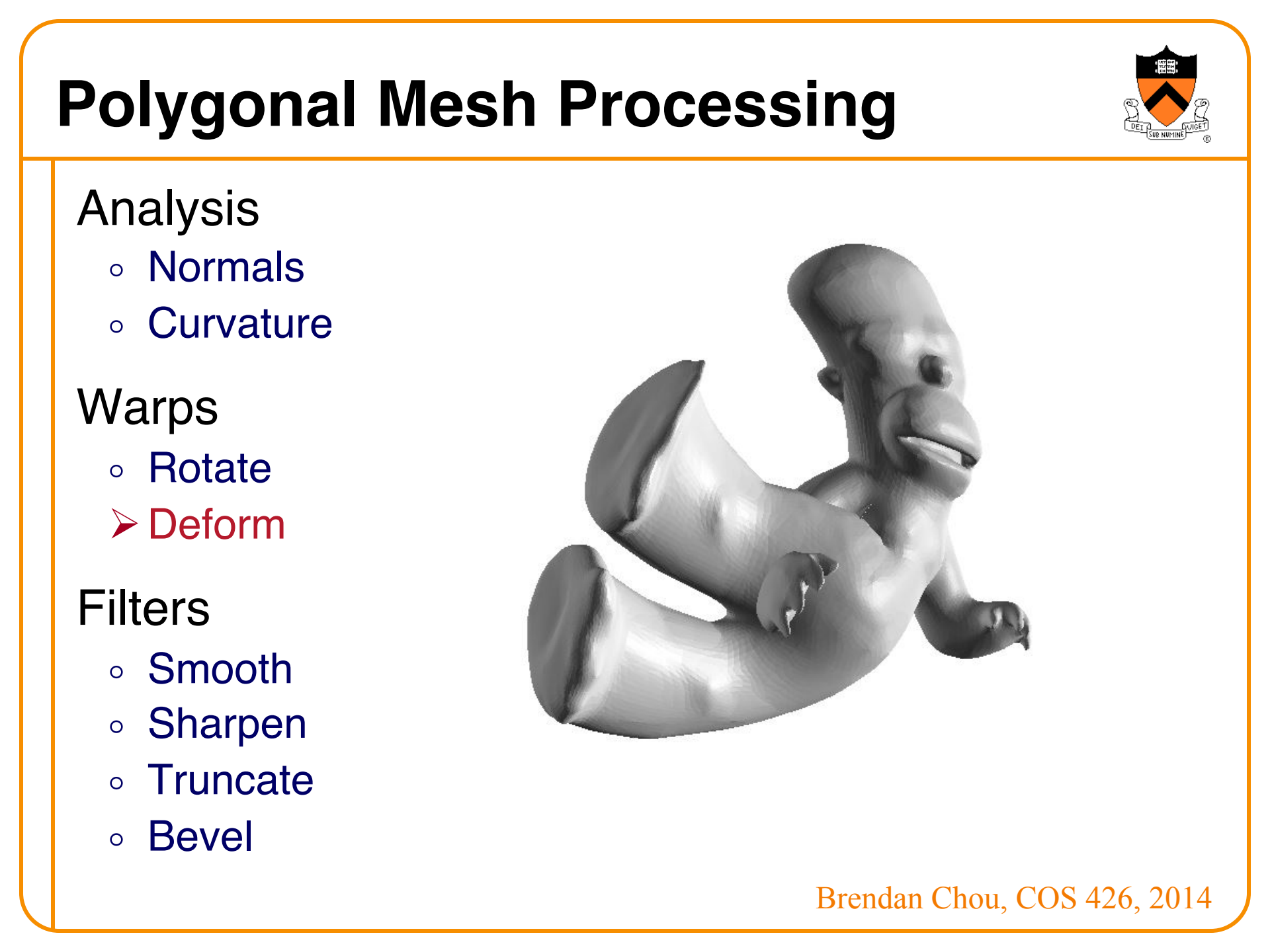


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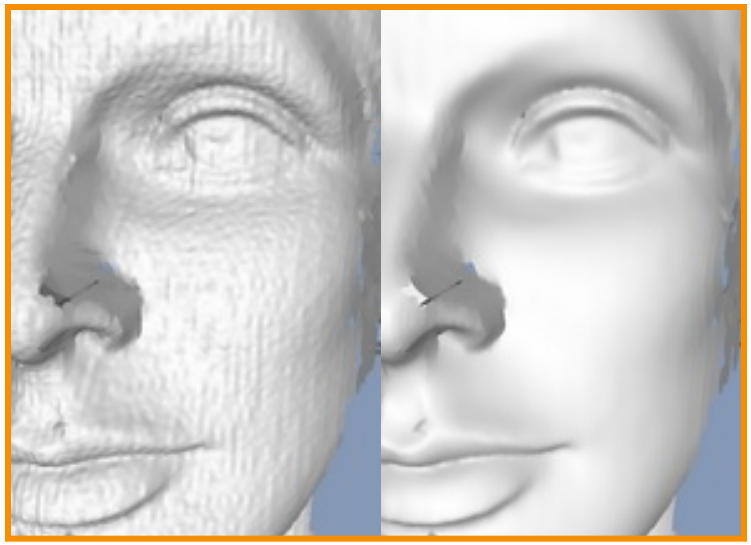

Thouis "Ray" Jones

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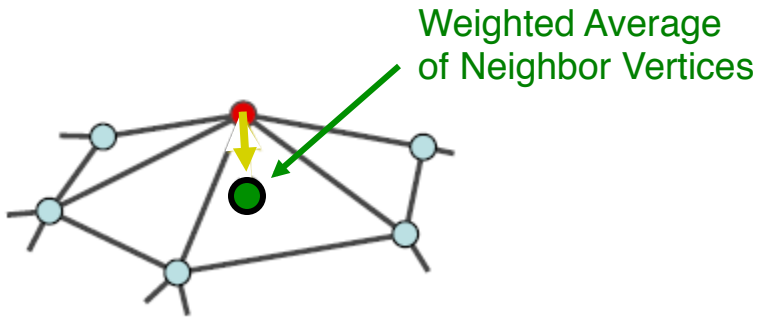


Olga Sorkine

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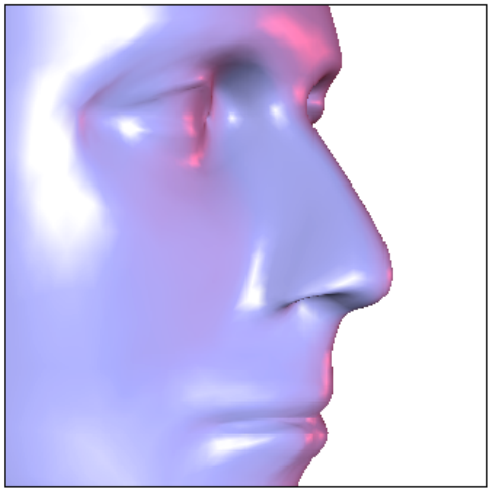

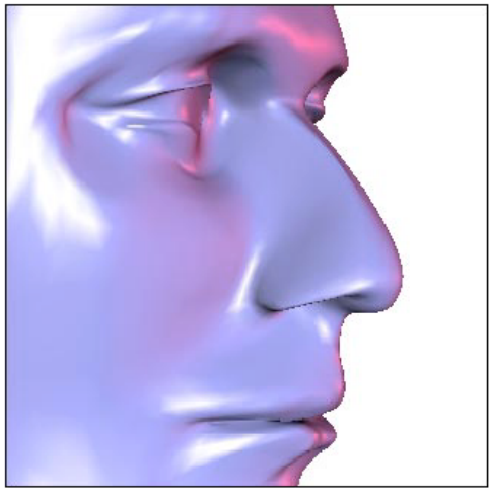

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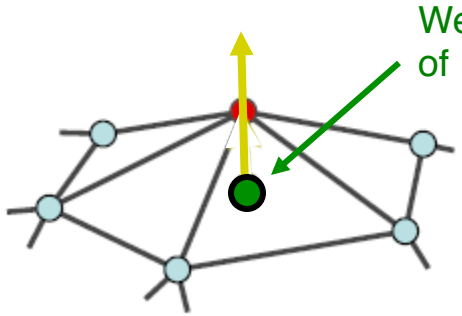


Olga Sorkine

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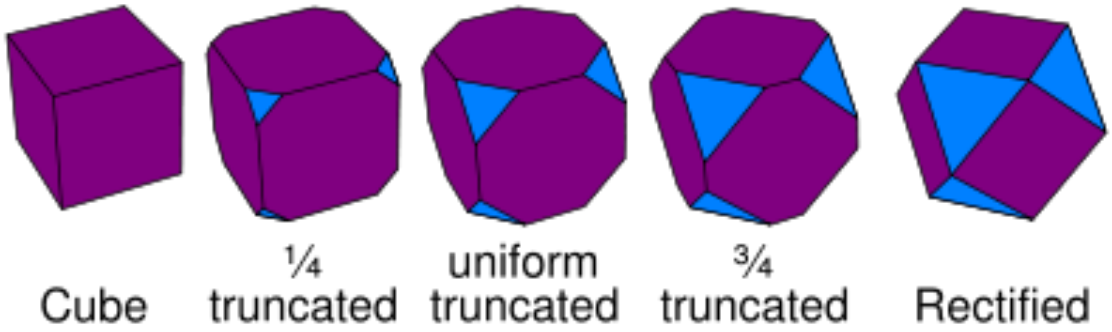


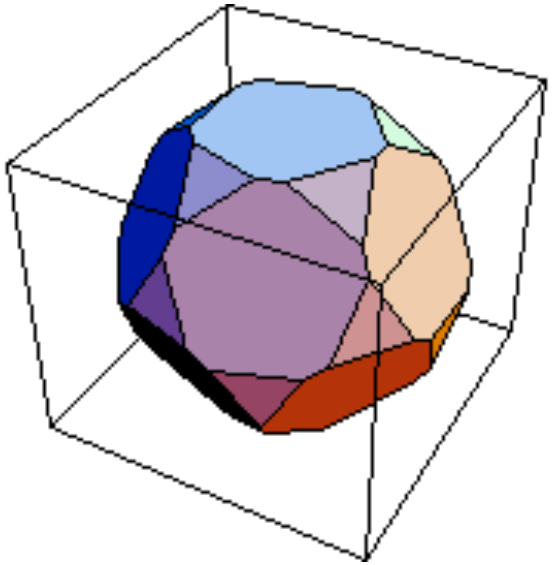

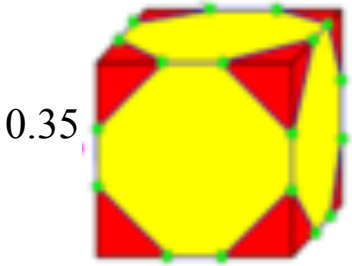


Conway

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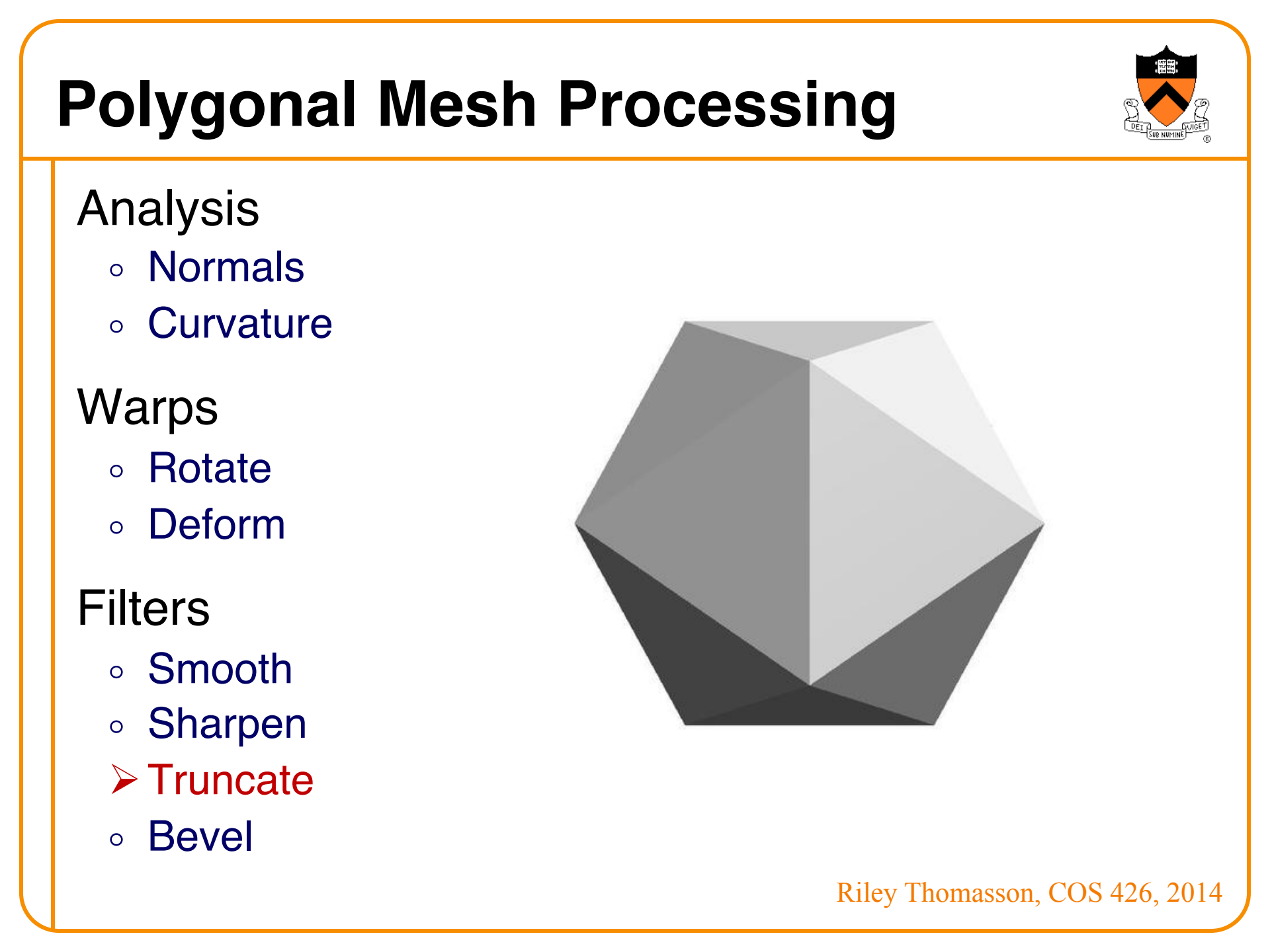


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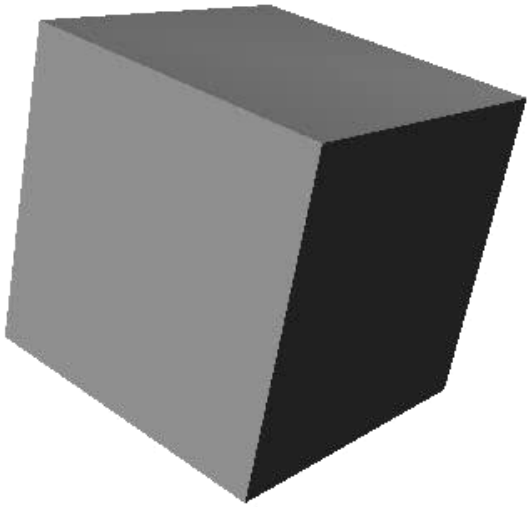

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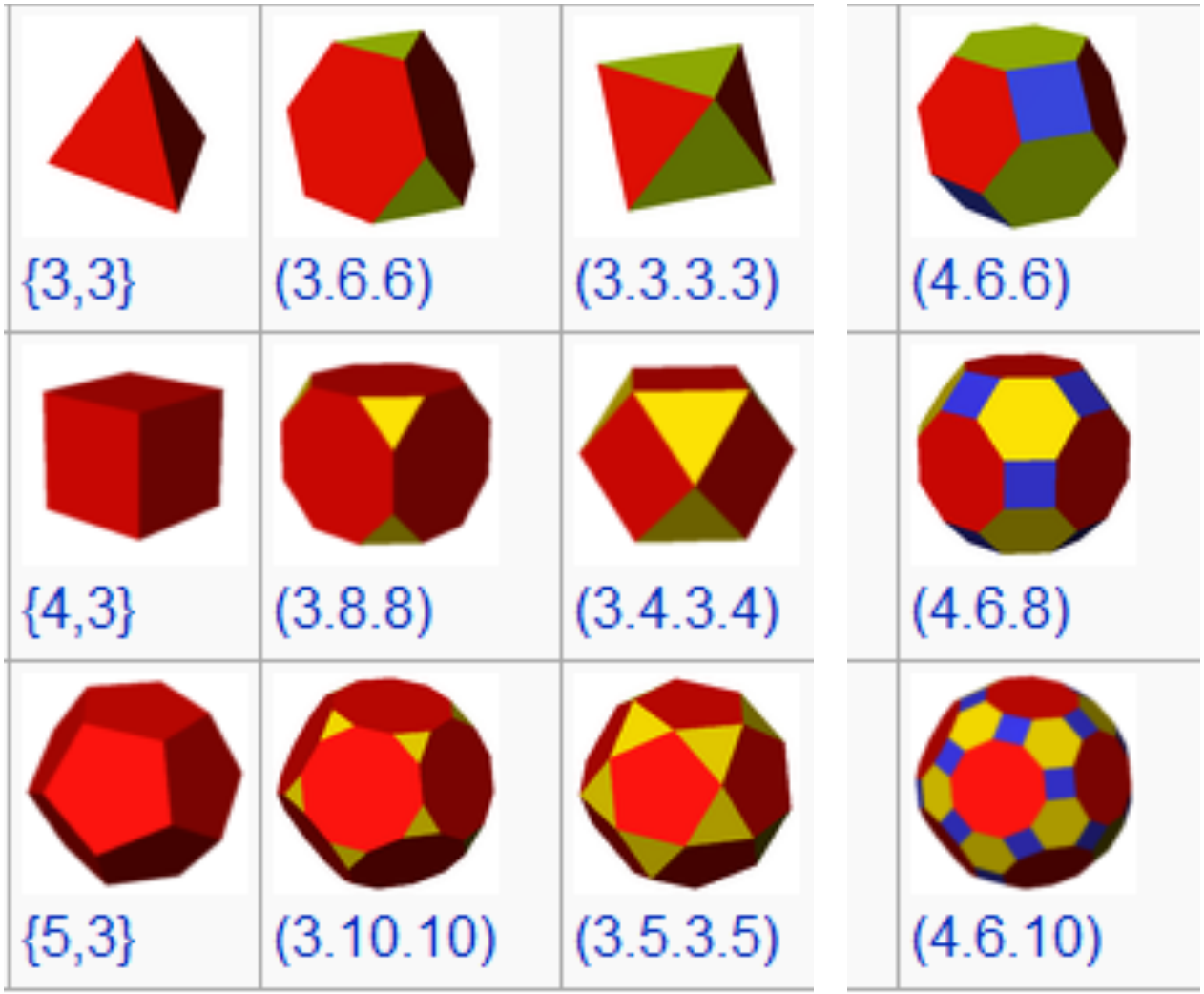
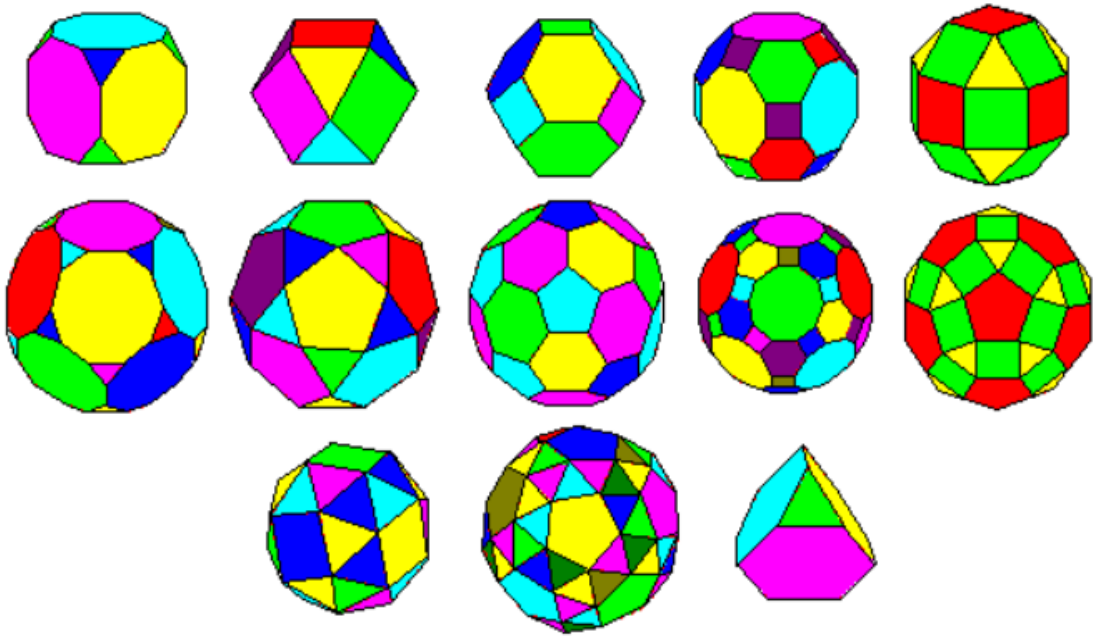



Wikipedia

# Polygonal Mesh Processing


- Analysis
  - Normals
  - Curvature
- Warps
  - Rotate
  - Deform
- Filters
  - Smooth
  - Sharpen
  - ➢ Truncate
  - Bevel



Archimedean Polyhedra
http://www.uwgb.edu/dutchs/symmetry/archpol.htm

# Polygonal Mesh Processing

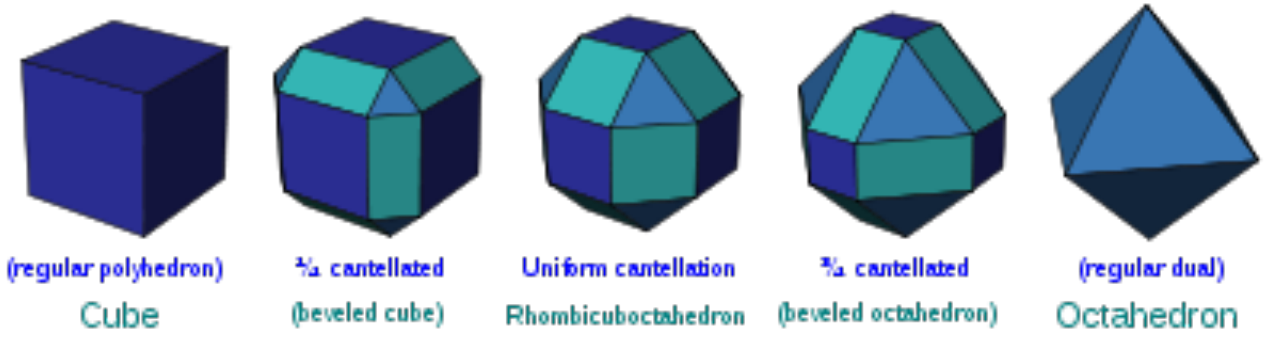
- Analysis
  - No
  - Cu
- Warp
  - Re
  - De
- Filter
  - Sm
  - Sh
  - ➢ Tr
  - Bevel

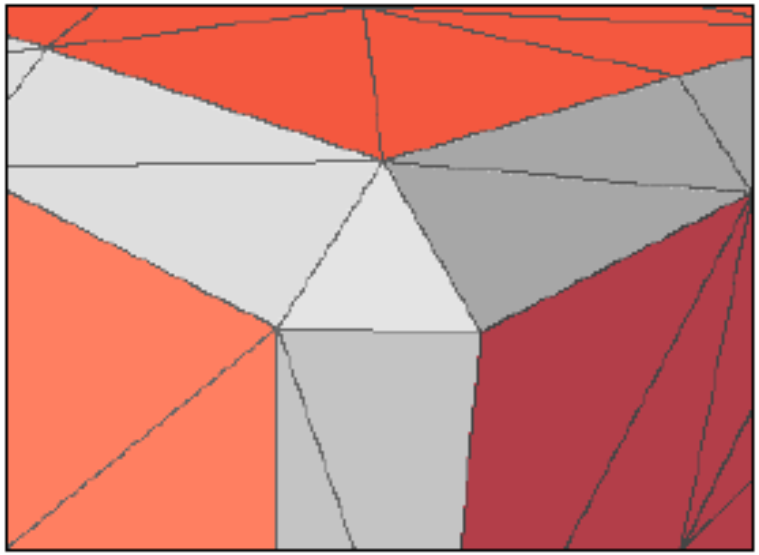
Carlo Sèquin

# Polygonal Mesh Processing


- Analysis
  - Normals
  - Curvature
- Warps
  - Rotate
  - Deform
- Filters
  - Smooth
  - Sharpen
  - Truncate
  - ➢ Bevel

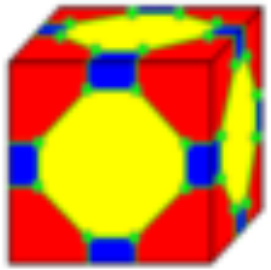
Wikipedia



Jarek Rossignac

0.40



Conway

# Polygonal Mesh Processing


Analysis
- Normals
- Curvature

Warps
- Rotate
- Deform

Filters
- Smooth
- Sharpen
- Truncate
- ➢ Bevel

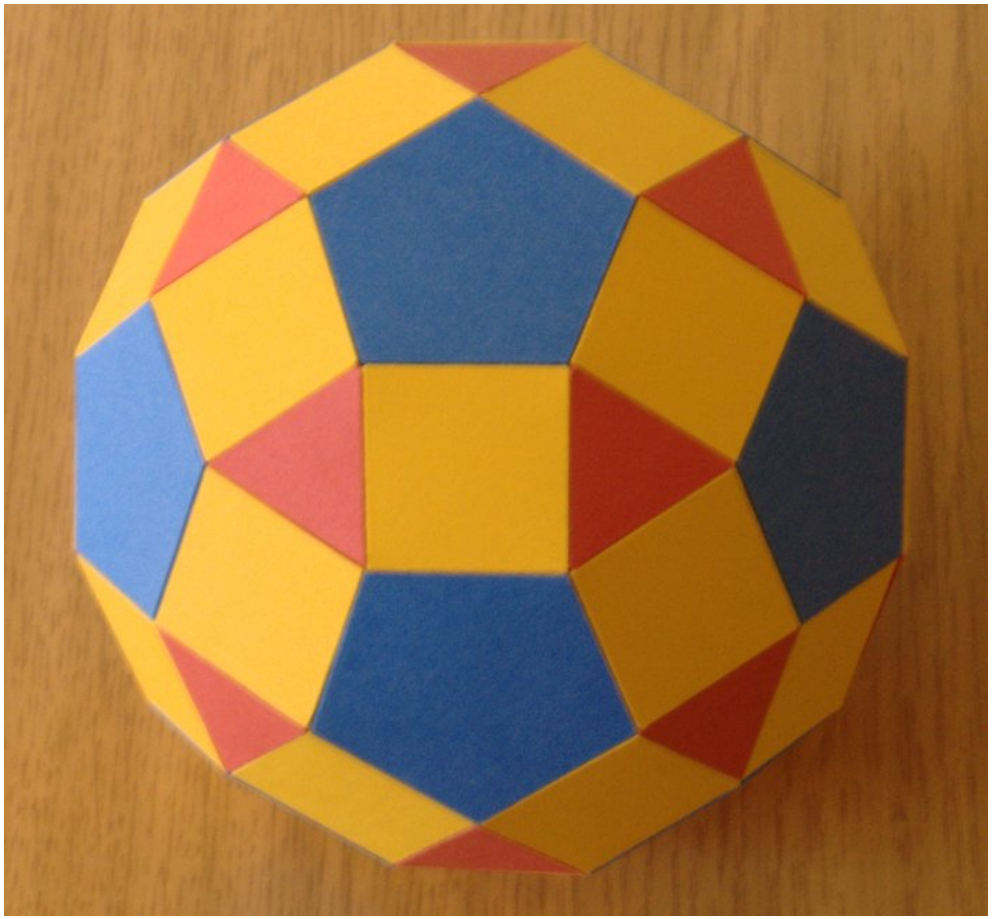
# Polygonal Mesh Processing




Analysis
- Normals
- Curvature

Warps
- Rotate
- Deform

Filters
- Smooth
- Sharpen
- Truncate
- ➢ Bevel



# Polygonal Mesh Processing

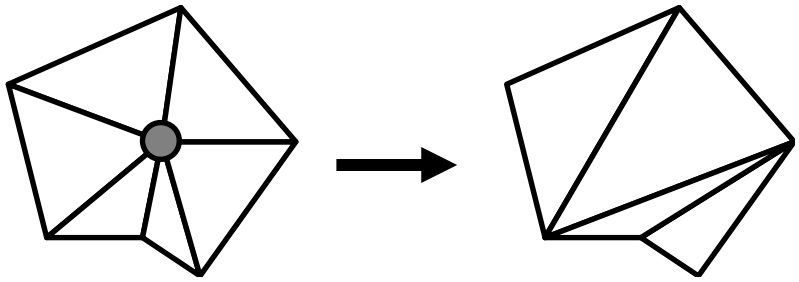

Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- Fill holes
- Fix self-intersections

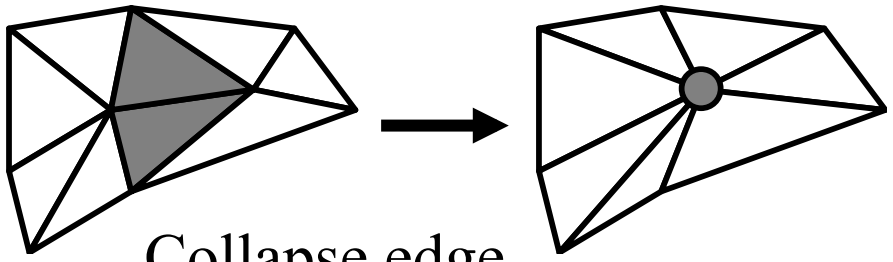
Boolean operations
- Crop
- Subtract

# Polygonal Mesh Processing

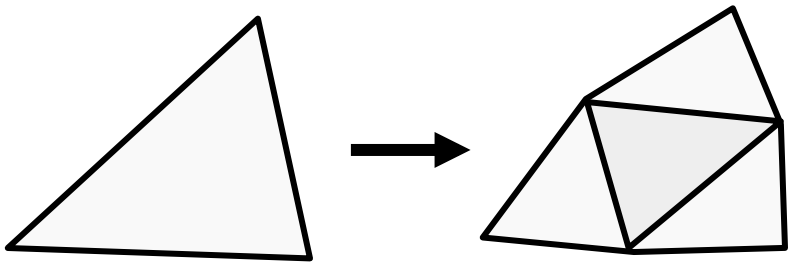

- Remeshing
  - Subdivide
  - Resample
  - Simplify
- Topological fixup
  - Fill holes
  - Fix self-intersections
- Boolean operations
  - Crop
  - Subtract



Remove Vertex



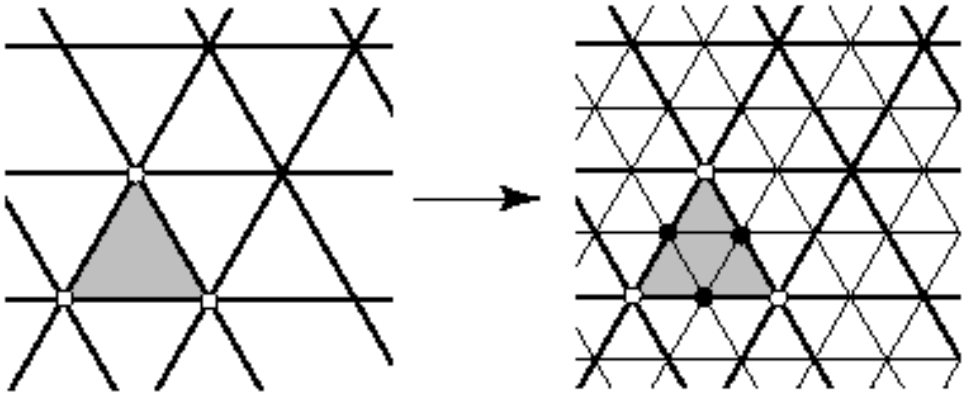
Collapse edge

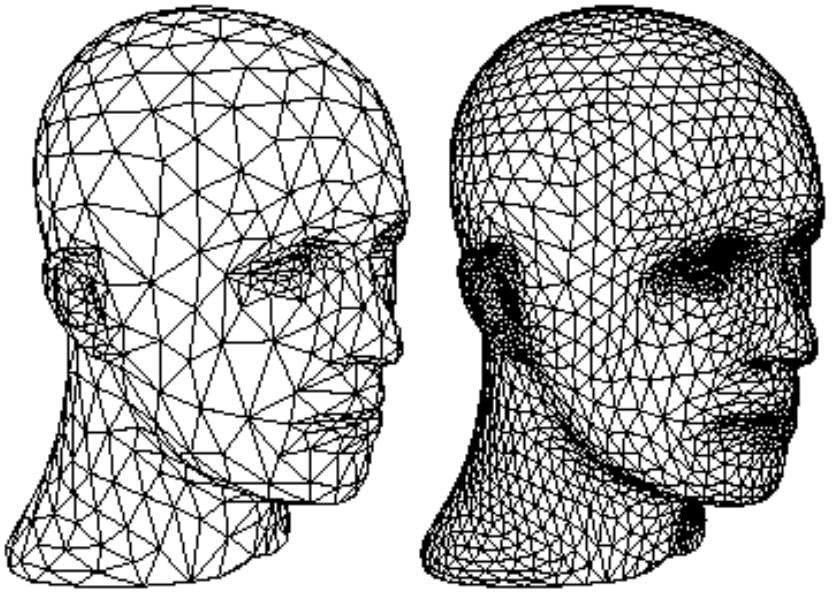


Subdivide face

# Polygonal Mesh Processing


- Remeshing
  - Subdivide
  - Resample
  - Simplify
- Topological fixup
  - Fill holes
  - Fix self-intersections
- Boolean operations
  - Crop
  - Subtract





Zorin & Schroeder

# Polygonal Mesh Processing

Remeshing
- Subdivide
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Topological fixup
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Boolean operations
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- Subtract

# Polygonal Mesh Processing


Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
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Boolean operations
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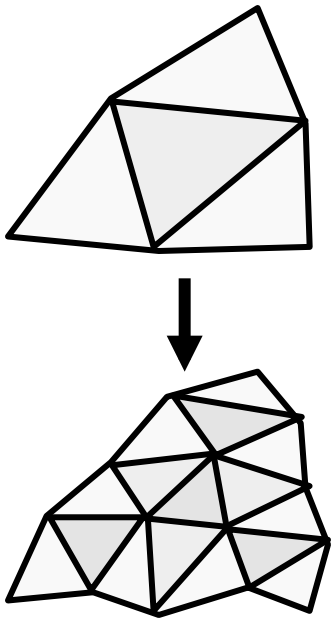
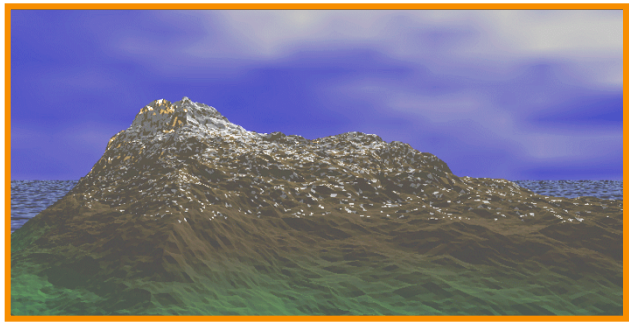


Fractal Landscape



*Dirk Balfanz, Igor Guskov, Sanjeev Kumar, & Rudro Samanta,*

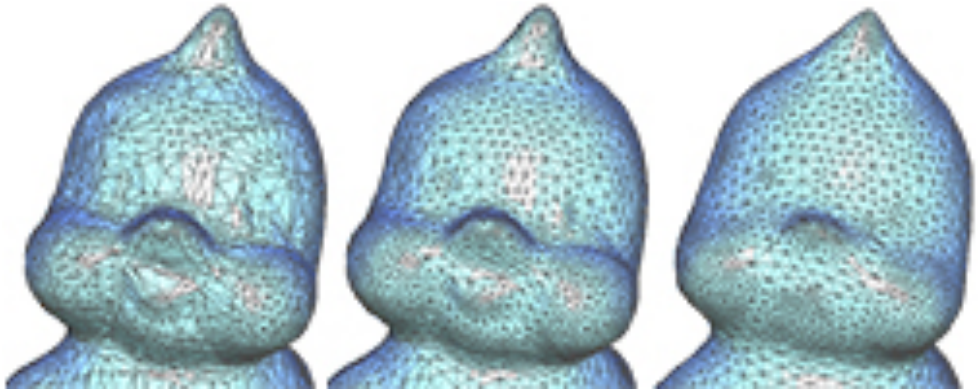
# Polygonal Mesh Processing




Remeshing
- Subdivide
- ➢ Resample
- Simplify

Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations
- Crop
- Subtract



Original

Resampled

- more uniform distribution
- triangles with nicer aspect

# Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
- ➢ Simplify

Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations
- Crop
- Subtract

Garland

# Polygonal Mesh Processing


Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup ←
- Fill holes
- Fix self-intersections

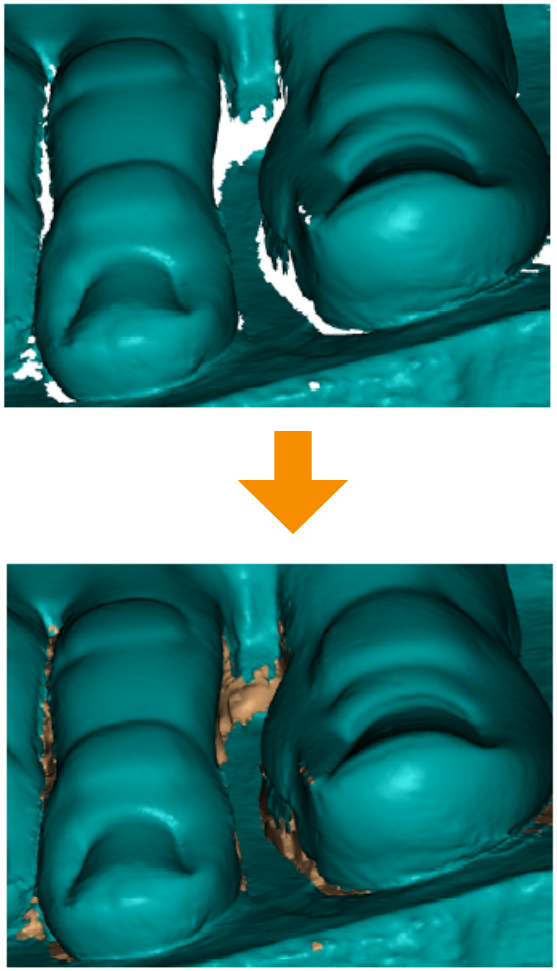
Boolean operations
- Crop
- Subtract

# Polygonal Mesh Processing


Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- ➢ Fill holes
- Fix self-intersections

Boolean operations
- Crop
- Subtract



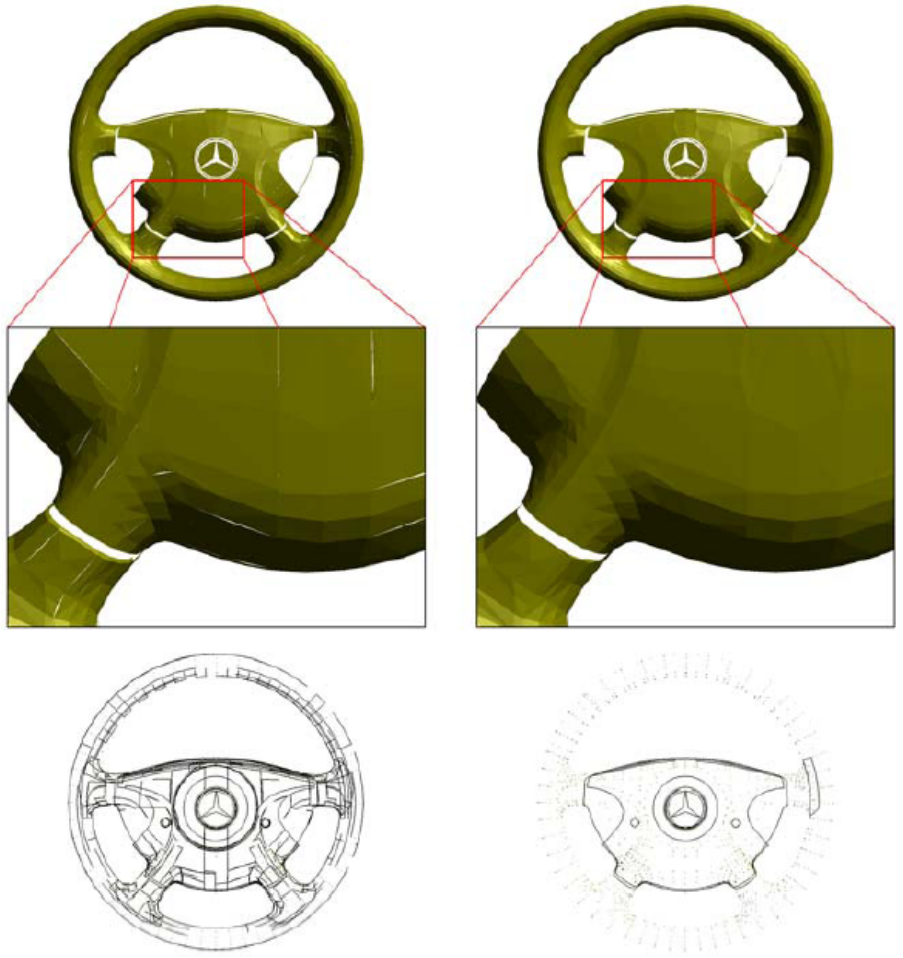
Podolak

# Polygonal Mesh Processing

Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- Fill holes
- ➢ Fix self-intersections

Boolean operations
- Crop
- Subtract



Borodin

# Polygonal Mesh Processing


Remeshing
- Subdivide
- Resample
- Simplify

Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations ←
- Crop
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# Polygonal Mesh Processing


Remeshing
- Subdivide
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Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations
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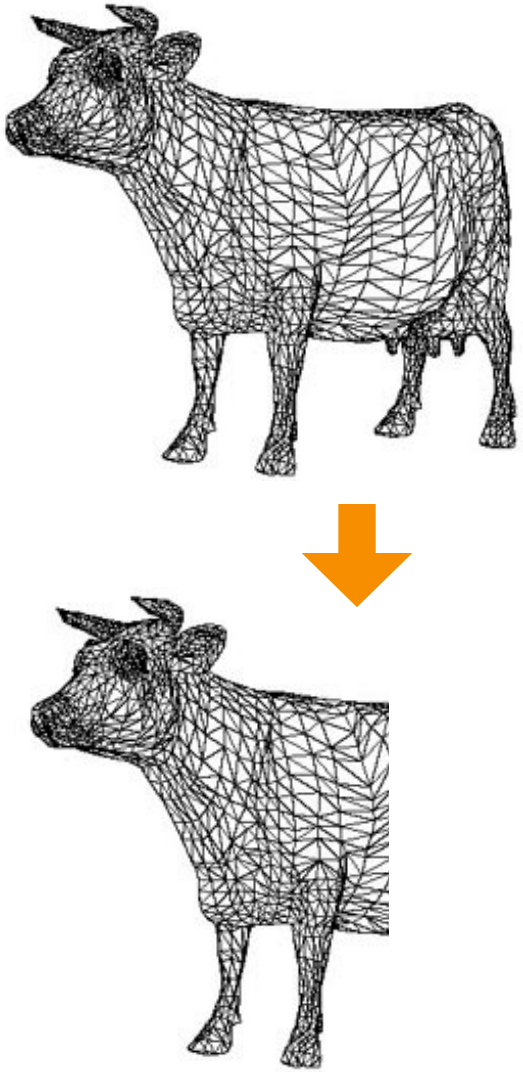

# Polygonal Mesh Processing



Remeshing
- Subdivide
- Resample
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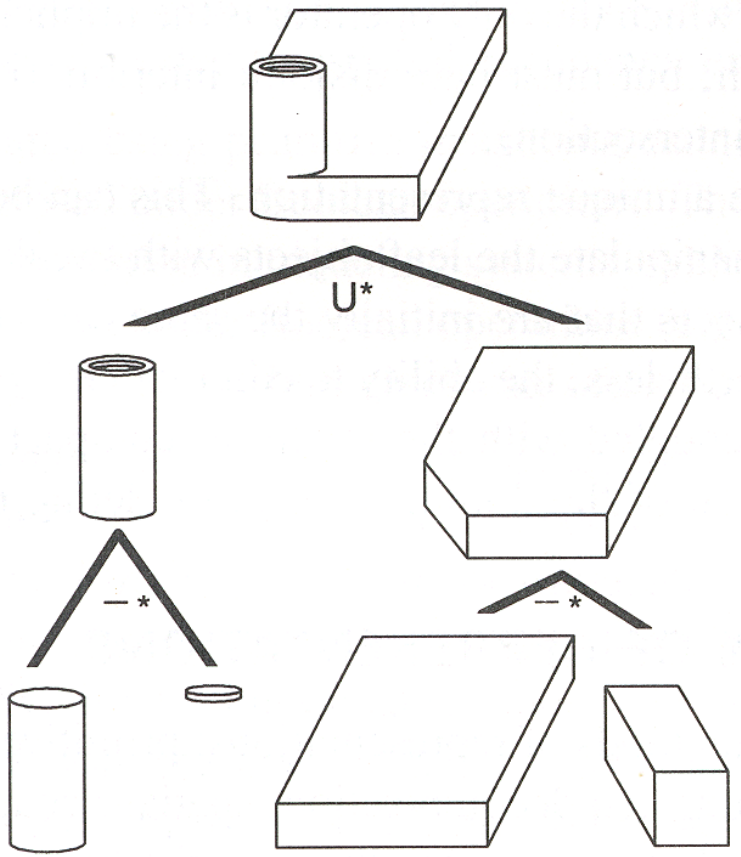
Topological fixup
- Fill holes
- Fix self-intersections

Boolean operations
- Crop
- ➢ Subtract




FvDFH Figure 12.27

# Outline

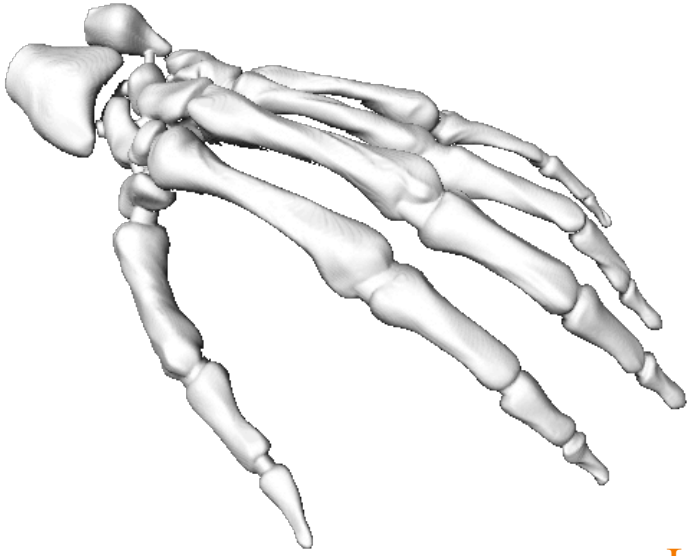

Acquisition

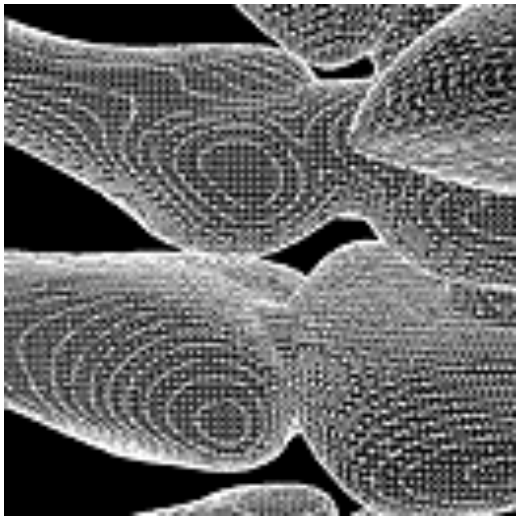
Processing

Representation

# Polygon Mesh Representation

Important properties of mesh representation?



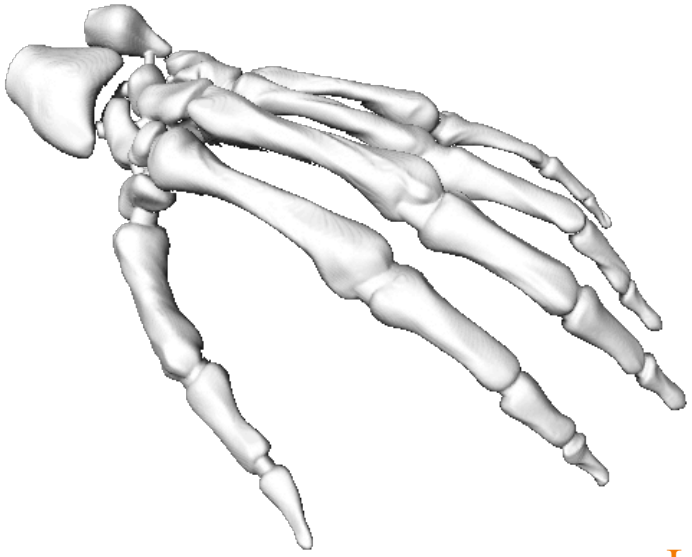


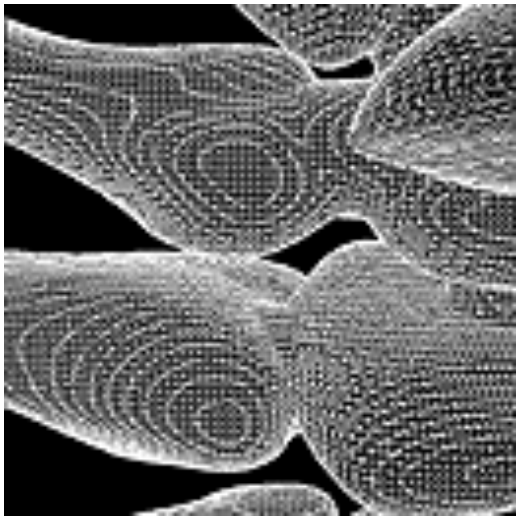
Large Geometric Model Repository
Georgia Tech

# Polygon Mesh Representation

Important properties of mesh representation?

- Efficient traversal of topology
- Efficient use of memory
- Efficient updates



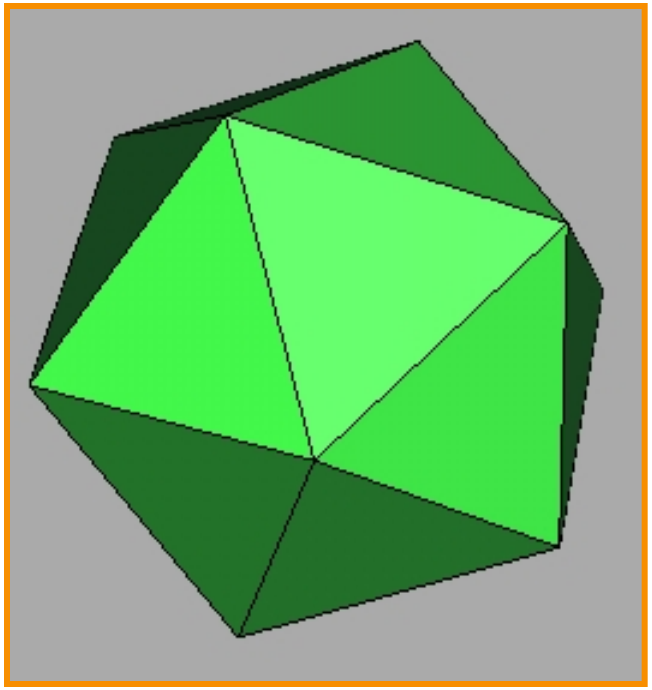


Large Geometric Model Repository
Georgia Tech

# Polygon Mesh Representation


Possible data structures

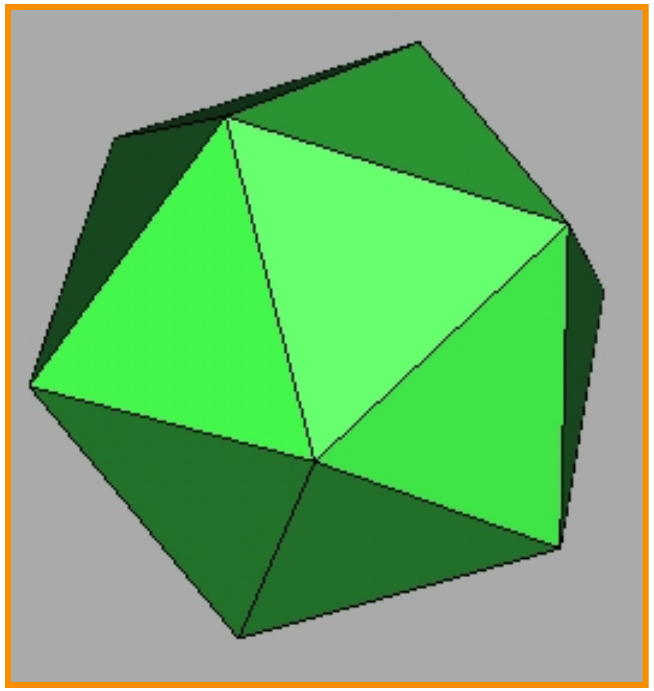
- List of independent faces
- Vertex and face tables
- Adjacency lists
- Winged edge
- Half edge
- etc.

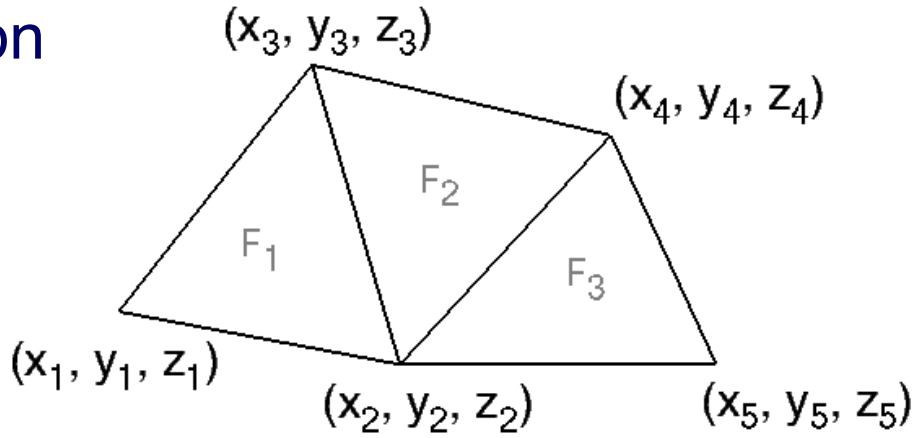


# Independent Faces

Each face lists vertex coordinates

- Redundant vertices
- No adjacency information



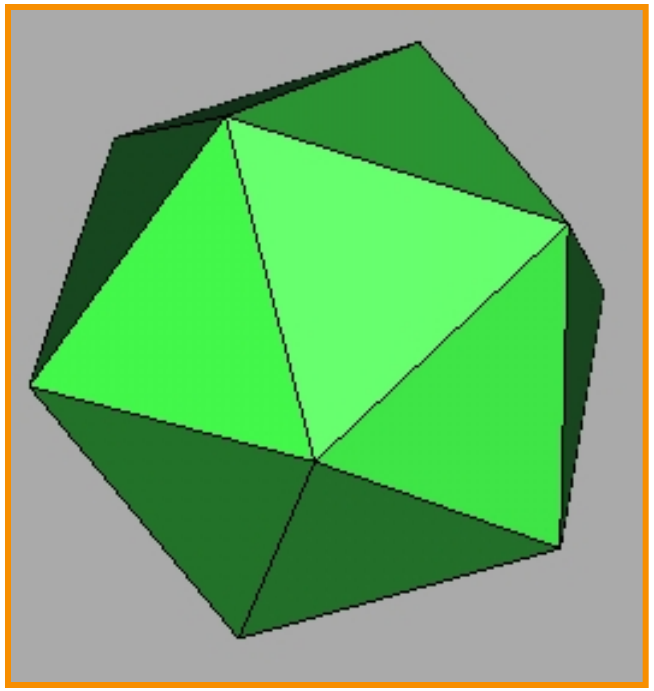


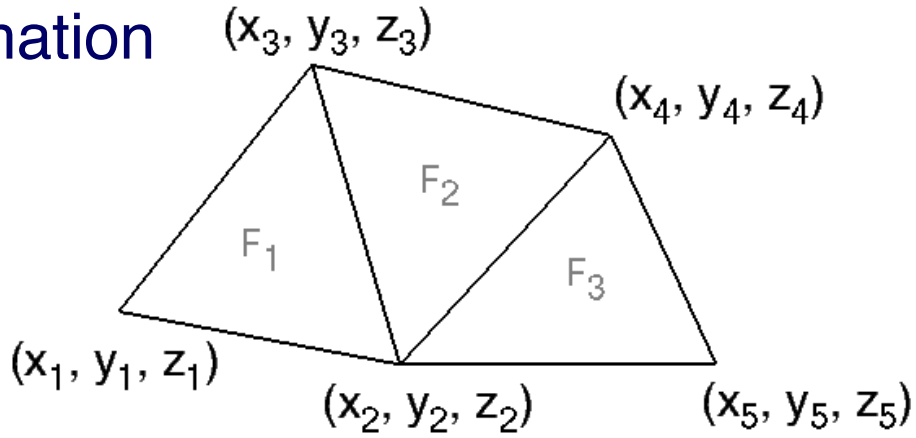

| FACE TABLE | |
|---|---|
| $F_1$ | $(x_1, y_1, z_1)$ $(x_2, y_2, z_2)$ $(x_3, y_3, z_3)$ |
| $F_2$ | $(x_2, y_2, z_2)$ $(x_4, y_4, z_4)$ $(x_3, y_3, z_3)$ |
| $F_3$ | $(x_2, y_2, z_2)$ $(x_5, y_5, z_5)$ $(x_4, y_4, z_4)$ |

# Vertex and Face Tables

Each face lists vertex references

- Shared vertices
- Still no adjacency information





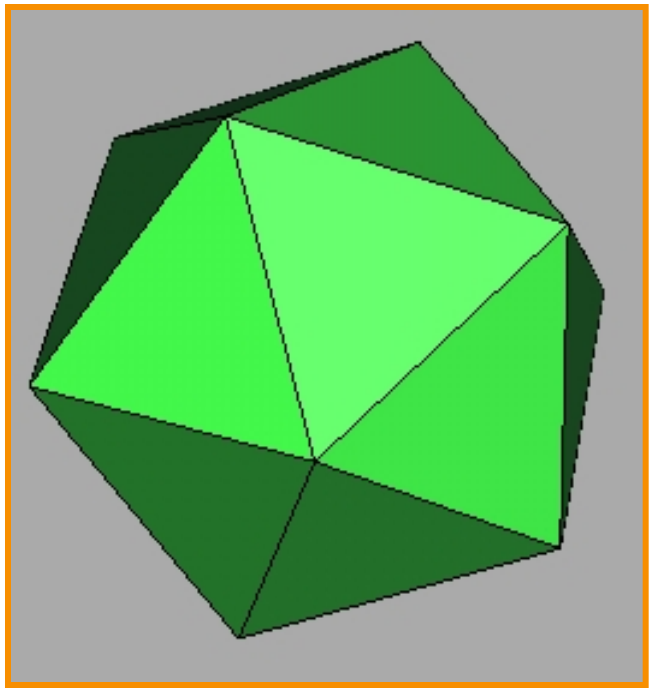

| VERTEX TABLE | | | |
|---|---|---|---|
| $V_1$ | $X_1$ | $Y_1$ | $Z_1$ |
| $V_2$ | $X_2$ | $Y_2$ | $Z_2$ |
| $V_3$ | $X_3$ | $Y_3$ | $Z_3$ |
| $V_4$ | $X_4$ | $Y_4$ | $Z_4$ |
| $V_5$ | $X_5$ | $Y_5$ | $Z_5$ |

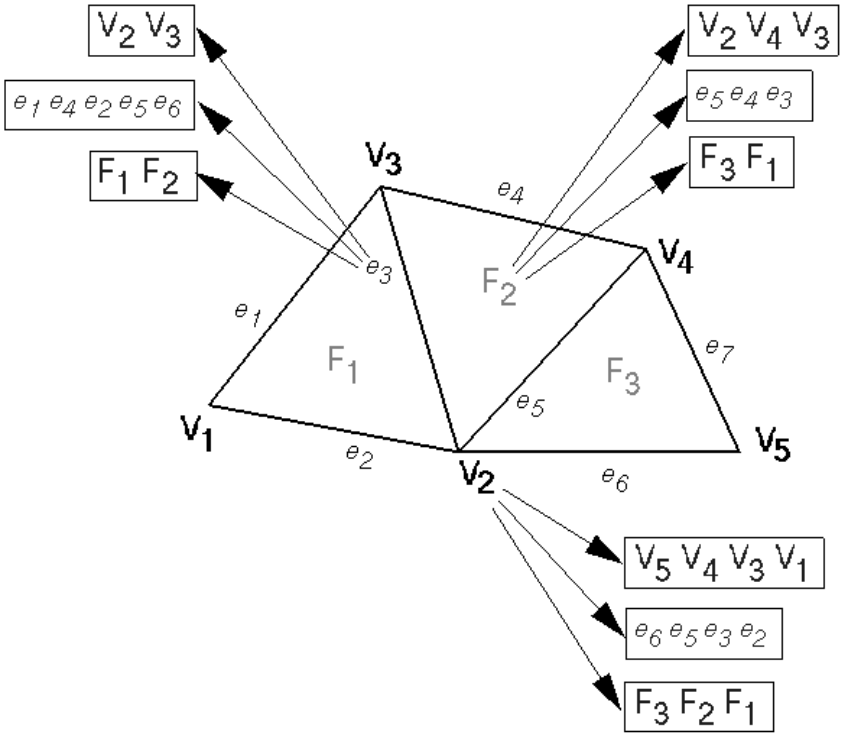
| FACE TABLE | | | |
|---|---|---|---|
| $F_1$ | $V_1$ | $V_2$ | $V_3$ |
| $F_2$ | $V_2$ | $V_4$ | $V_3$ |
| $F_3$ | $V_2$ | $V_5$ | $V_4$ |

# Adjacency Lists

Store all vertex, edge, and face adjacencies

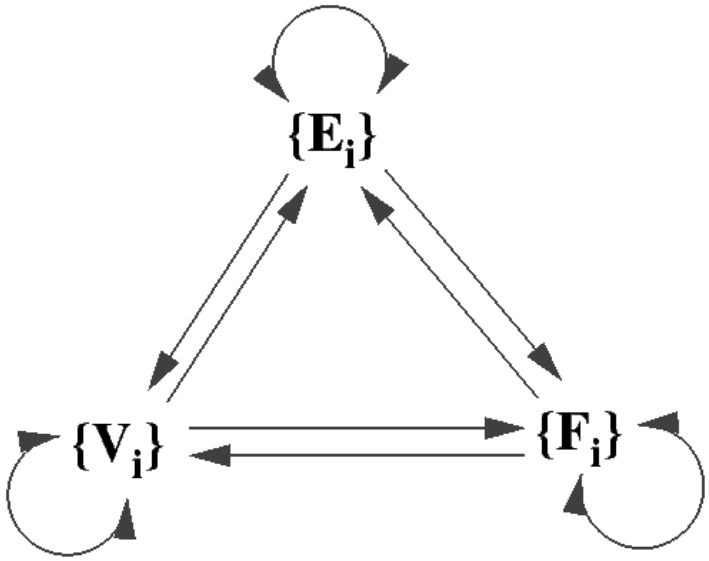
- Efficient adjacency traversal
- Extra storage

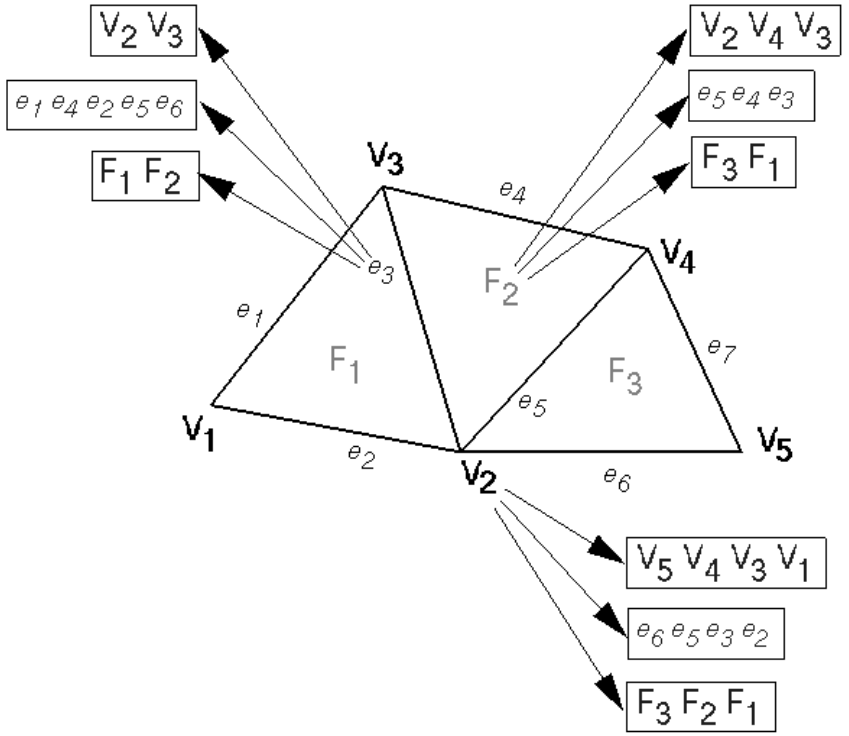




# Partial Adjacency Lists

Can we store only some adjacency relationships and derive others?

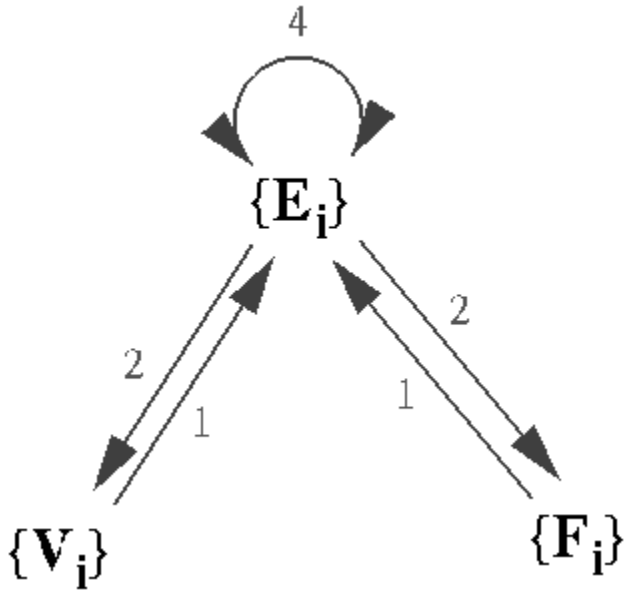


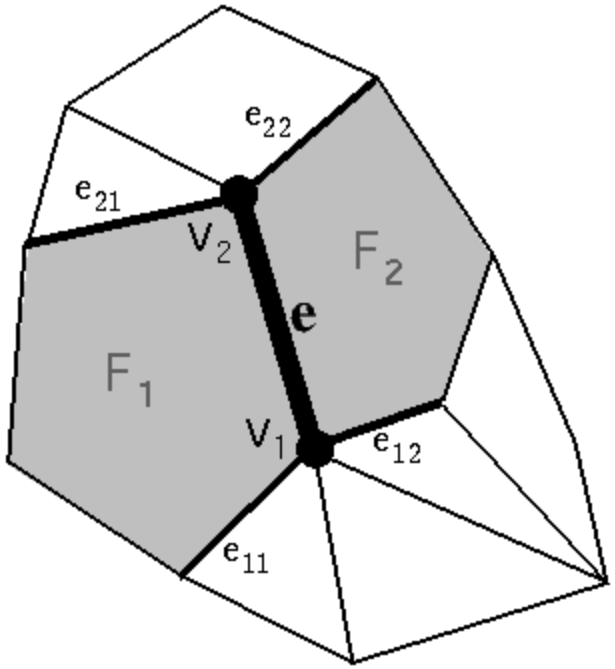



# Winged Edge

Adjacency encoded in edges

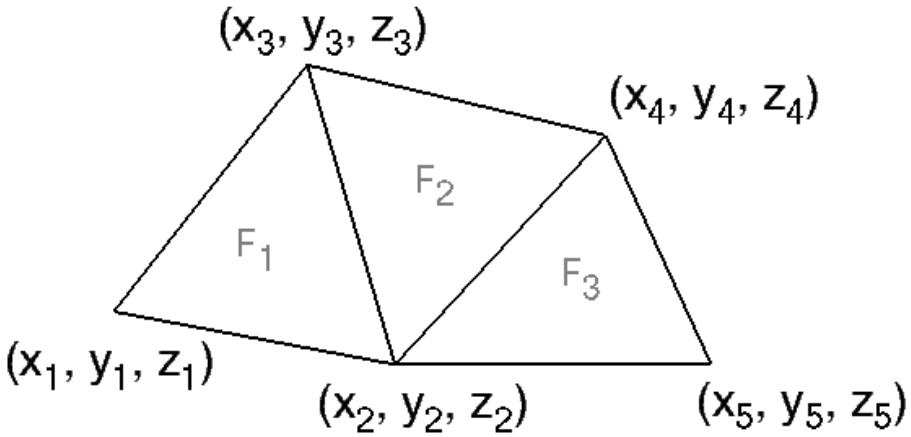
- All adjacencies in O(1) time
- Little extra storage (fixed records)
- Arbitrary polygons



# Winged Edge

Example:




**VERTEX TABLE**

| | | | | |
|---|---|---|---|---|
| $V_1$ | $X_1$ | $Y_1$ | $Z_1$ | $e_1$ |
| $V_2$ | $X_2$ | $Y_2$ | $Z_2$ | $e_6$ |
| $V_3$ | $X_3$ | $Y_3$ | $Z_3$ | $e_3$ |
| $V_4$ | $X_4$ | $Y_4$ | $Z_4$ | $e_5$ |
| $V_5$ | $X_5$ | $Y_5$ | $Z_5$ | $e_6$ |

**EDGE TABLE**

| | | | | | 11 | 12 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|
| $e_1$ | $V_1$ | $V_3$ | | $F_1$ | $e_2$ | $e_2$ | $e_4$ | $e_3$ |
| $e_2$ | $V_1$ | $V_2$ | $F_1$ | | $e_1$ | $e_1$ | $e_3$ | $e_6$ |
| $e_3$ | $V_2$ | $V_3$ | $F_1$ | $F_2$ | $e_2$ | $e_5$ | $e_1$ | $e_4$ |
| $e_4$ | $V3$ | $V_4$ | | $F_2$ | $e_1$ | $e_3$ | $e_7$ | $e_5$ |
| $e_5$ | $V_2$ | $V_4$ | $F_2$ | $F_3$ | $e_3$ | $e_6$ | $e_4$ | $e_7$ |
| $e_6$ | $V_2$ | $V_5$ | $F_3$ | | $e_5$ | $e_2$ | $e_7$ | $e_7$ |
| $e_7$ | $V_4$ | $V_5$ | | $F_3$ | $e_4$ | $e_5$ | $e_6$ | $e_6$ |

**FACE TABLE**

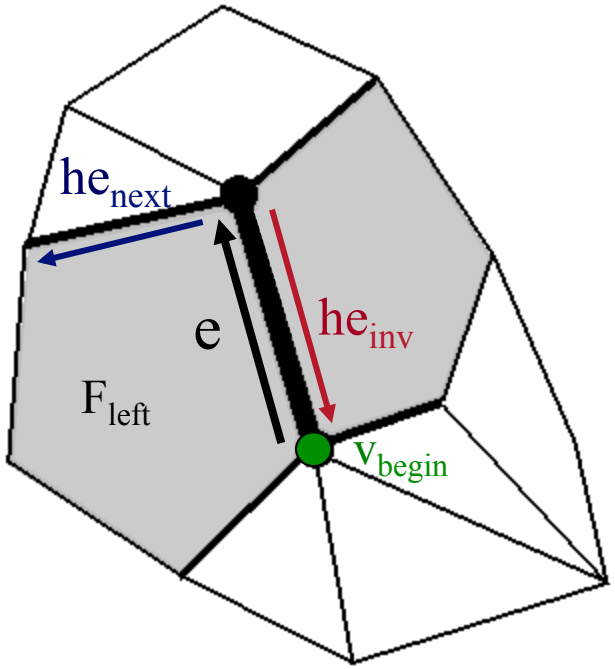
| | |
|---|---|
| $F_1$ | $e_1$ |
| $F_2$ | $e_3$ |
| $F_3$ | $e_5$ |

# Half Edge

Adjacency encoded in edges

- All adjacencies in O(1) time
- Little extra storage (fixed records)
- Arbitrary polygons

Similar to winged-edge, except adjacency encoded in half-edges



# Summary


Polygonal meshes

- Most common surface representation
- Fast rendering

Processing operations

- Must consider irregular vertex sampling
- Must handle/avoid topological degeneracies

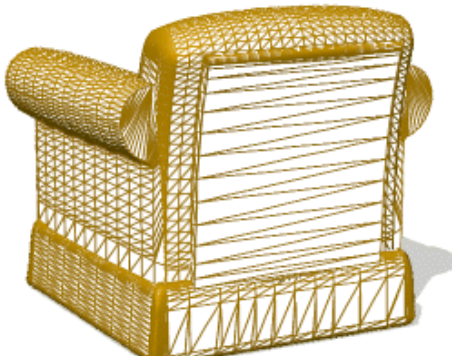
Representation

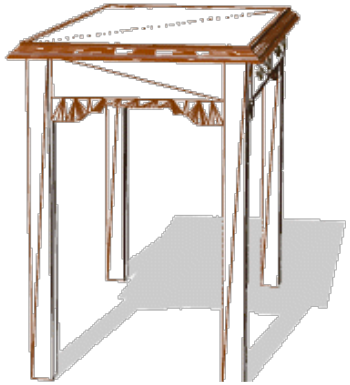
- Which adjacency relationships to store depend on which operations must be efficient

# 3D Polygonal Meshes

Properties

- ? Efficient display
- ? Easy acquisition
- ? Accurate
- ? Concise
- ? Intuitive editing
- ? Efficient editing
- ? Efficient intersections
- ? Guaranteed validity
- ? Guaranteed smoothness
- ? etc.



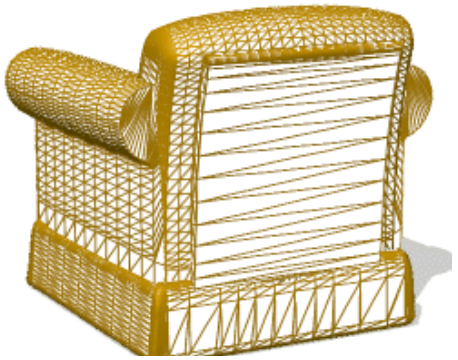


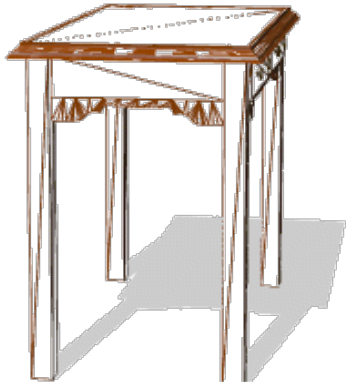
Viewpoint

# 3D Polygonal Meshes

Properties

- ☺ Efficient display
- ☺ Easy acquisition
- ☹ Accurate
- ☹ Concise
- ☹ Intuitive editing
- ☹ Efficient editing
- ☹ Efficient intersections
- ☹ Guaranteed validity
- ☹ Guaranteed smoothness





Viewpoint