



3D Modeling

COS 426, Spring 2016
Princeton University

Syllabus



I. Image processing

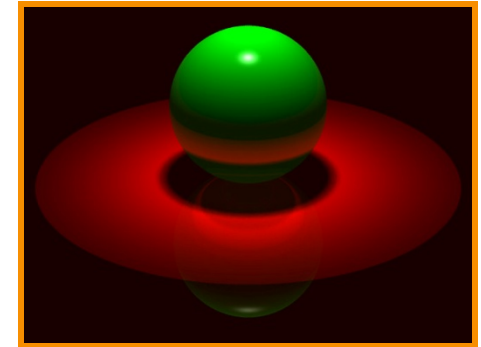
II. Modeling

III. Rendering

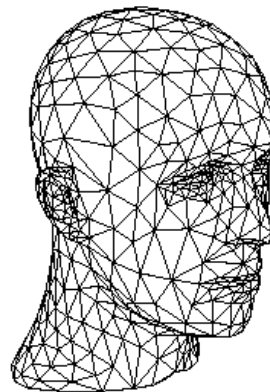
IV. Animation



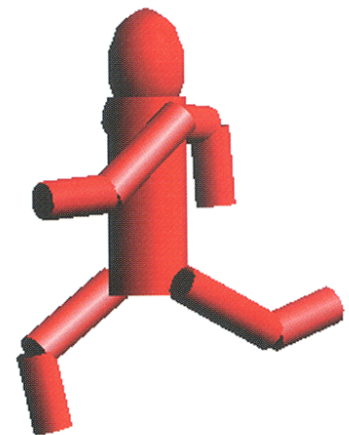
Image Processing
(Rusty Coleman, CS426, Fall99)



Rendering
(Michael Bostock, CS426, Fall99)



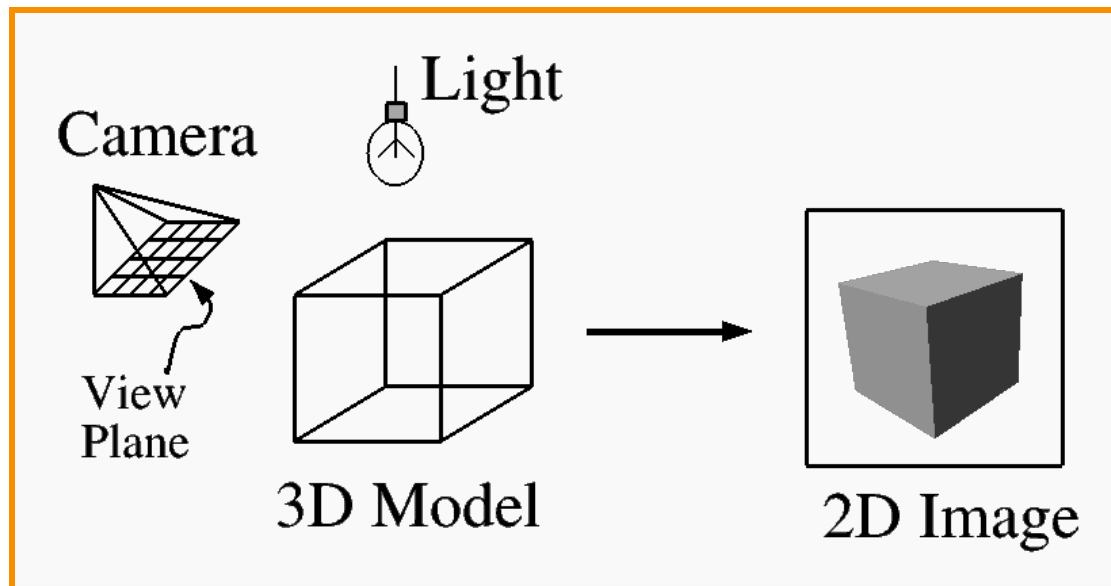
Modeling
(Denis Zorin, CalTech)



Animation
(Angel, Plate 1)

What is 3D Modeling?

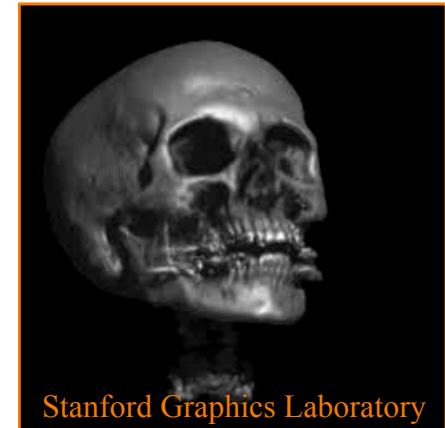
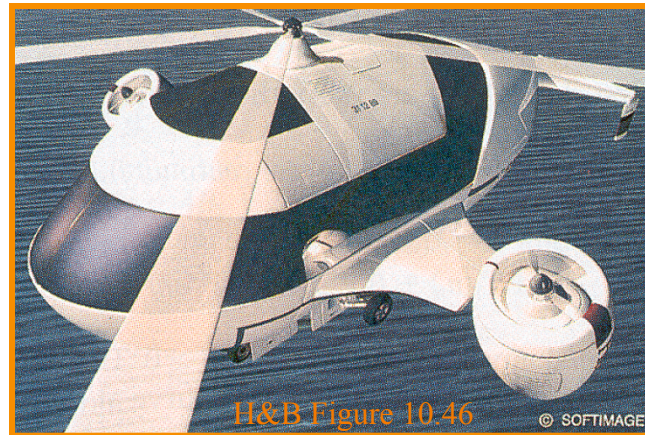
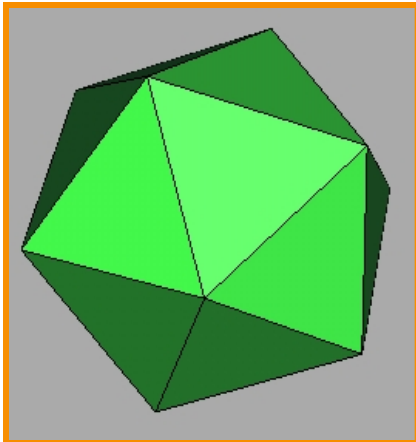
- Topics in computer graphics
 - Imaging = *representing 2D images*
 - Rendering = *constructing 2D images from 3D models*
 - Modeling = *representing 3D objects*
 - Animation = *simulating changes over time*



Modeling



- How do we ...
 - Represent 3D objects in a computer?
 - Acquire computer representations of 3D objects?
 - Manipulate computer representations of 3D objects?





Modeling Background

- Scene is usually approximated by 3D primitives
 - Point
 - Vector
 - Line segment
 - Ray
 - Line
 - Plane
 - Polygon

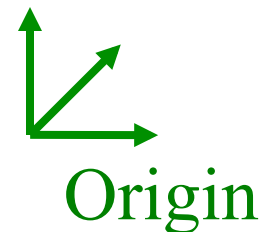
3D Point



- Specifies a location
 - Represented by three coordinates
 - Infinitely small

```
typedef struct {  
    Coordinate x;  
    Coordinate y;  
    Coordinate z;  
} Point;
```

• (x,y,z)





3D Vector

- Specifies a direction and a magnitude
 - Represented by three coordinates
 - Magnitude $||V|| = \sqrt{dx^2 + dy^2 + dz^2}$
 - Has no location

```
typedef struct {  
    Coordinate dx;  
    Coordinate dy;  
    Coordinate dz;  
} Vector;
```

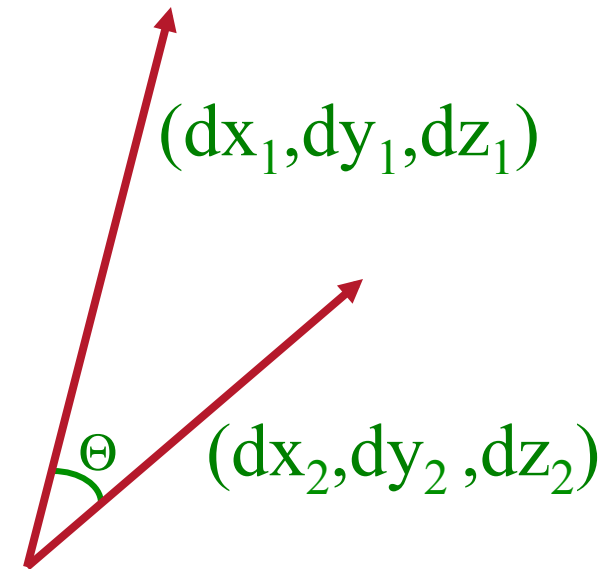
(dx, dy, dz)





3D Vector

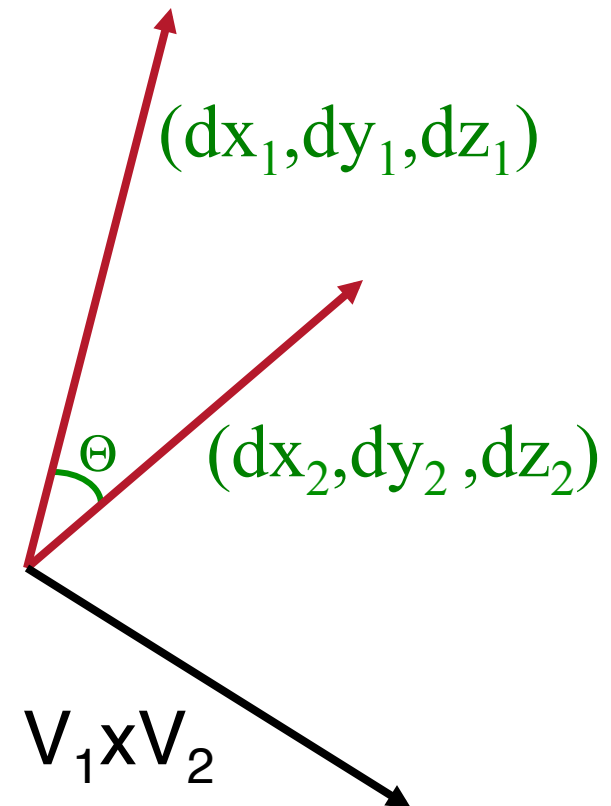
- Dot product of two 3D vectors
 - $V_1 \cdot V_2 = \|V_1\| \|V_2\| \cos(\Theta)$





3D Vector

- Cross product of two 3D vectors
 - $V_1 \times V_2 =$ vector perpendicular to both V_1 and V_2
 - $\|V_1 \times V_2\| = \|V_1\| \|V_2\| \sin(\Theta)$

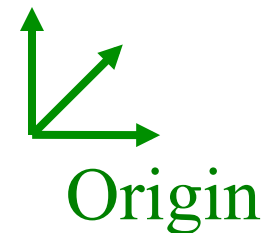




3D Line Segment

- Linear path between two points
 - Parametric representation:
 - » $P = P_1 + t (P_2 - P_1), \quad (0 \leq t \leq 1)$

```
typedef struct {  
    Point P1;  
    Point P2;  
} Segment;
```

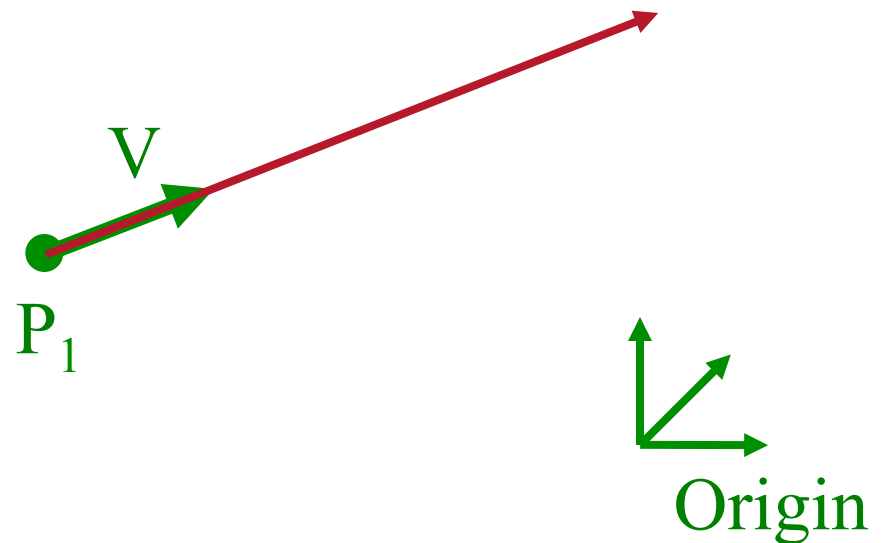




3D Ray

- Line segment with one endpoint at infinity
 - Parametric representation:
 - » $P = P_1 + t V, \quad (0 \leq t < \infty)$

```
typedef struct {  
    Point P1;  
    Vector V;  
} Ray;
```

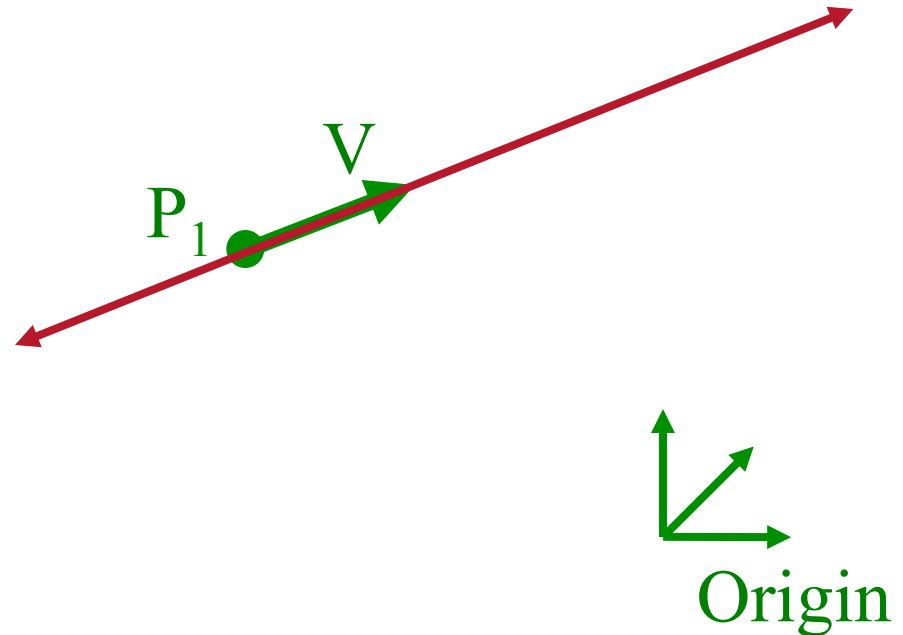




3D Line

- Line segment with both endpoints at infinity
 - Parametric representation:
 - » $P = P_1 + t V, \quad (-\infty < t < \infty)$

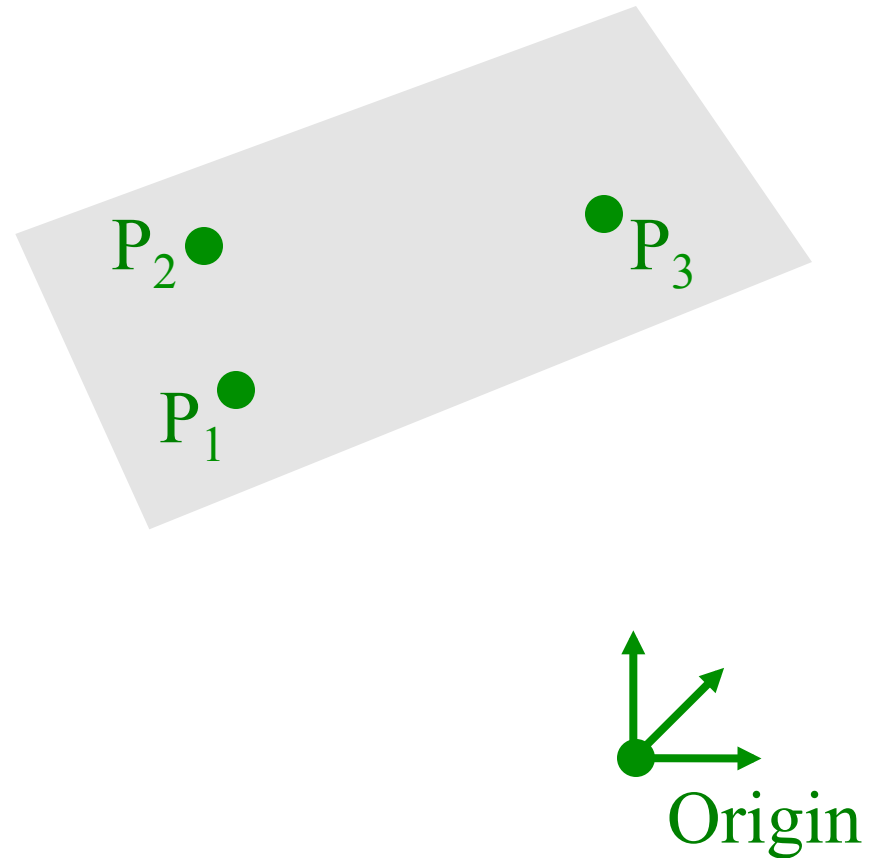
```
typedef struct {  
    Point P1;  
    Vector V;  
} Line;
```



3D Plane



- A linear combination of three points



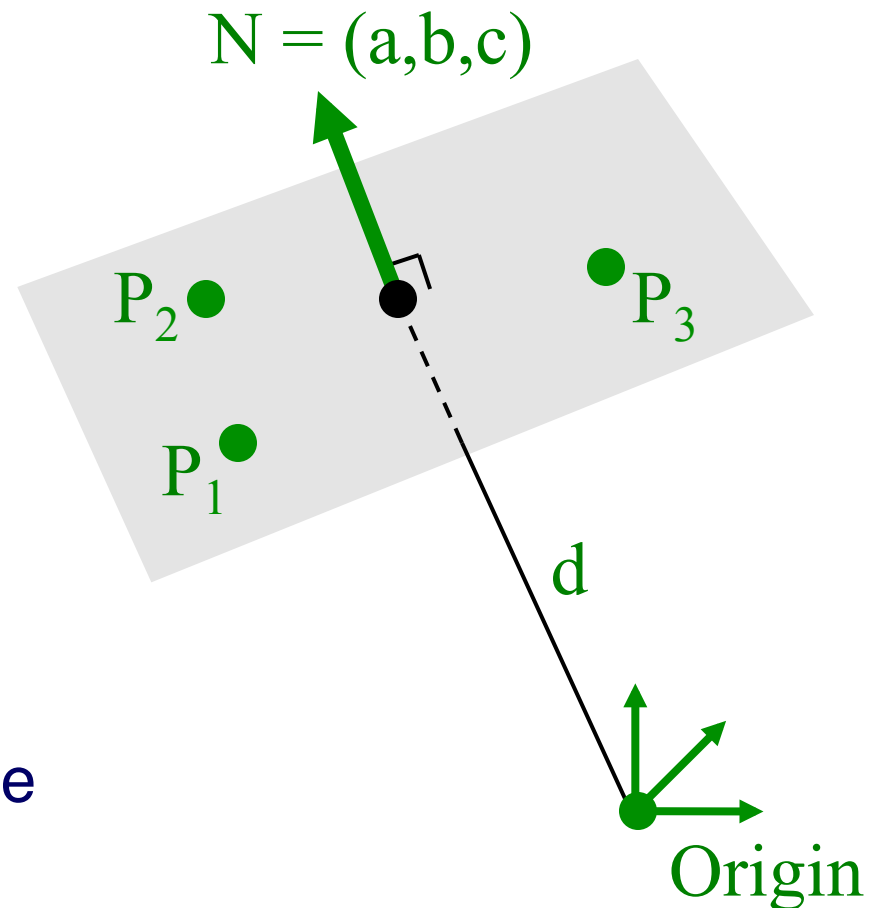


3D Plane

- A linear combination of three points
 - Implicit representation:
 - » $P \cdot N - d = 0$, or
 - » $ax + by + cz + d = 0$

```
typedef struct {  
    Vector N;  
    Distance d;  
} Plane;
```

- N is the plane “normal”
 - » Unit-length vector
 - » Perpendicular to plane

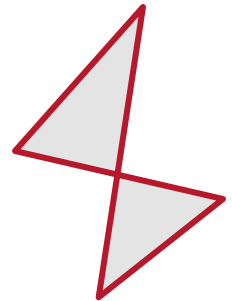
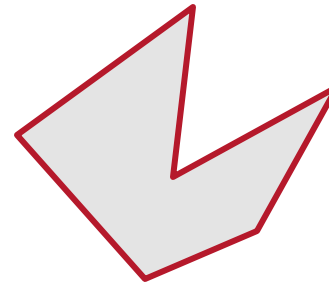
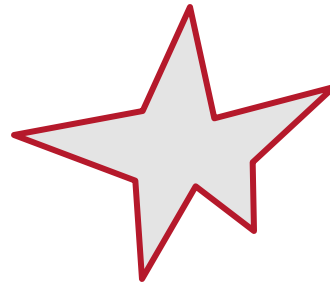
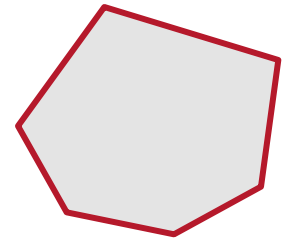
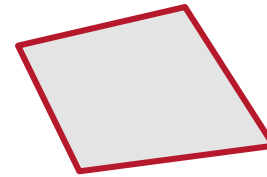
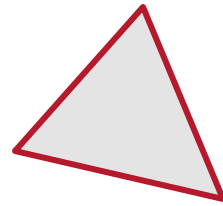




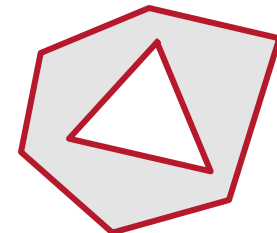
3D Polygon

Set of points “inside” a sequence of coplanar points

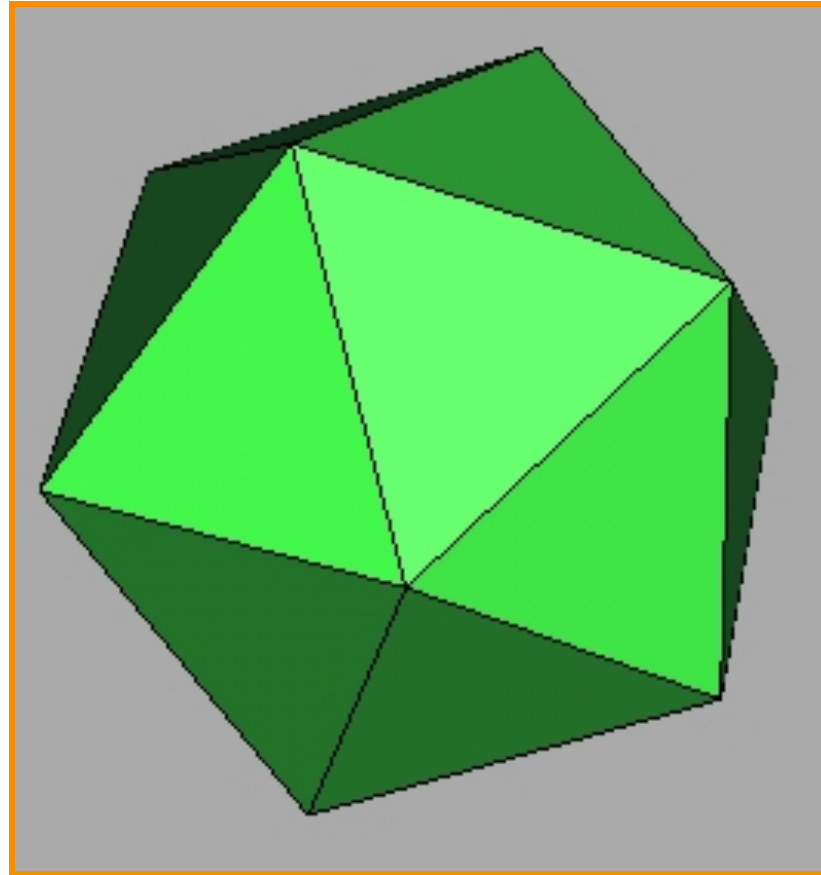
```
typedef struct {  
    Point *points;  
    int npoints;  
} Polygon;
```



Points are in counter-clockwise order

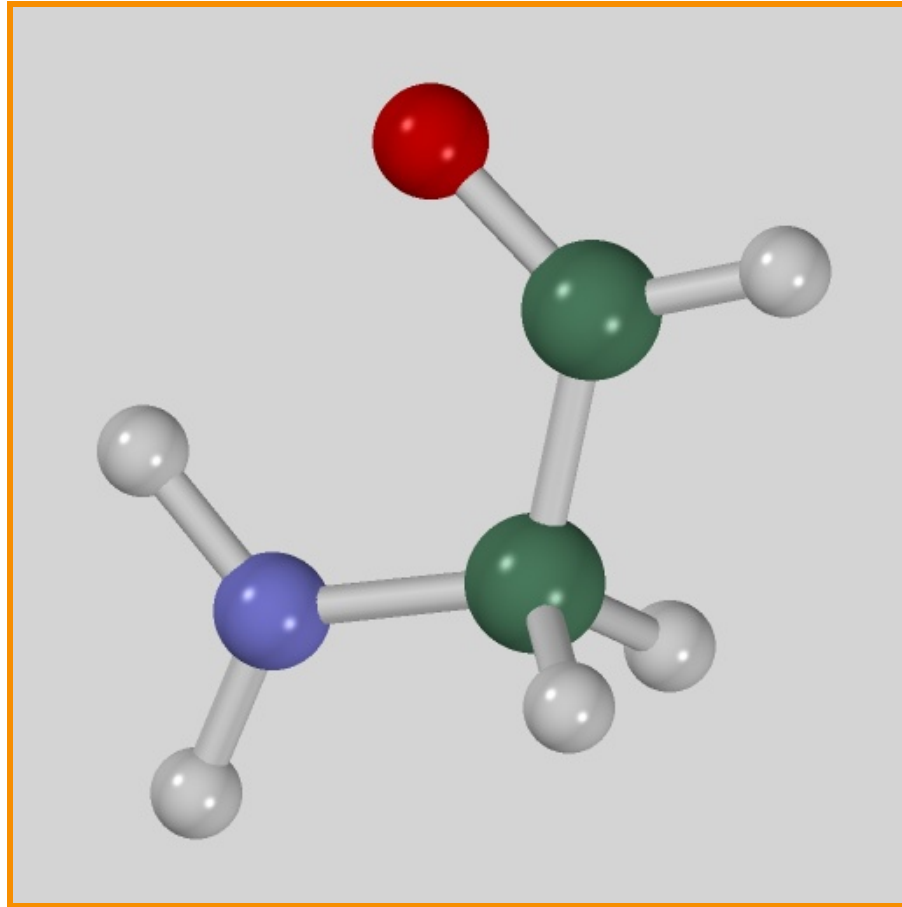


3D Object Representations



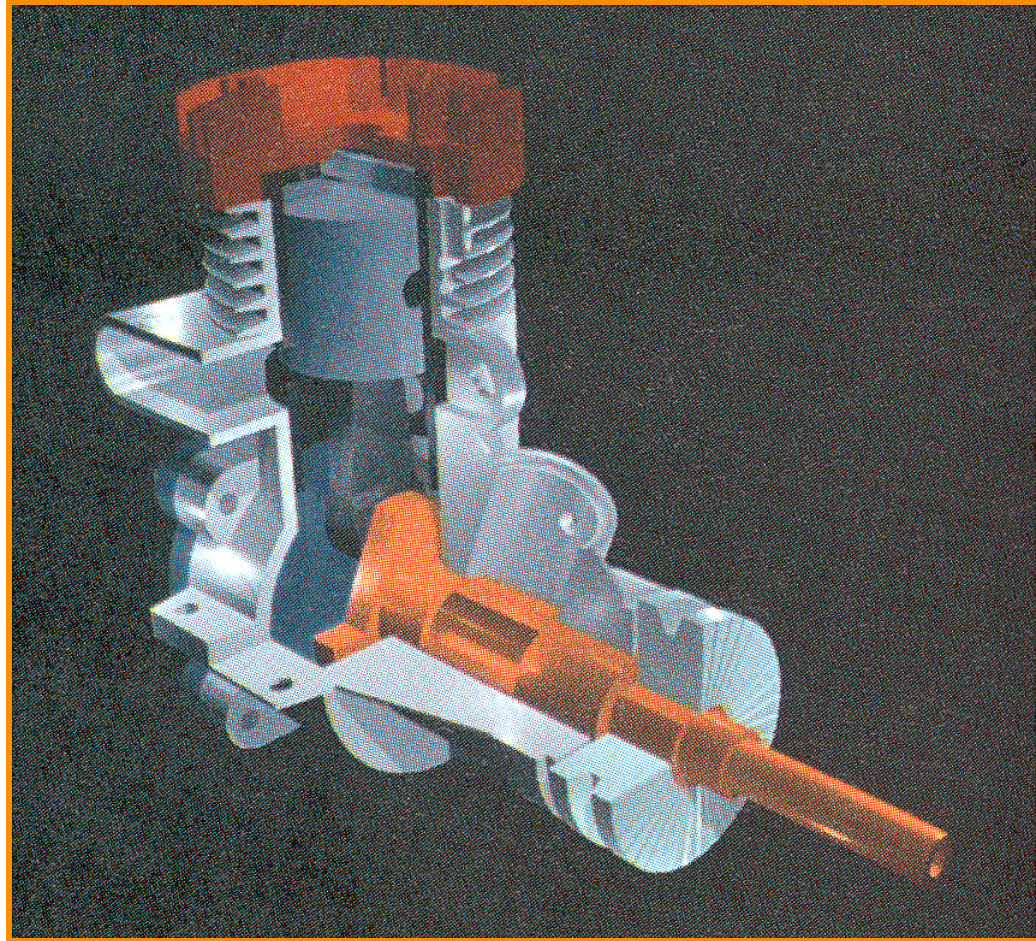
How can this object be represented in a computer?

3D Object Representations



How about this one?

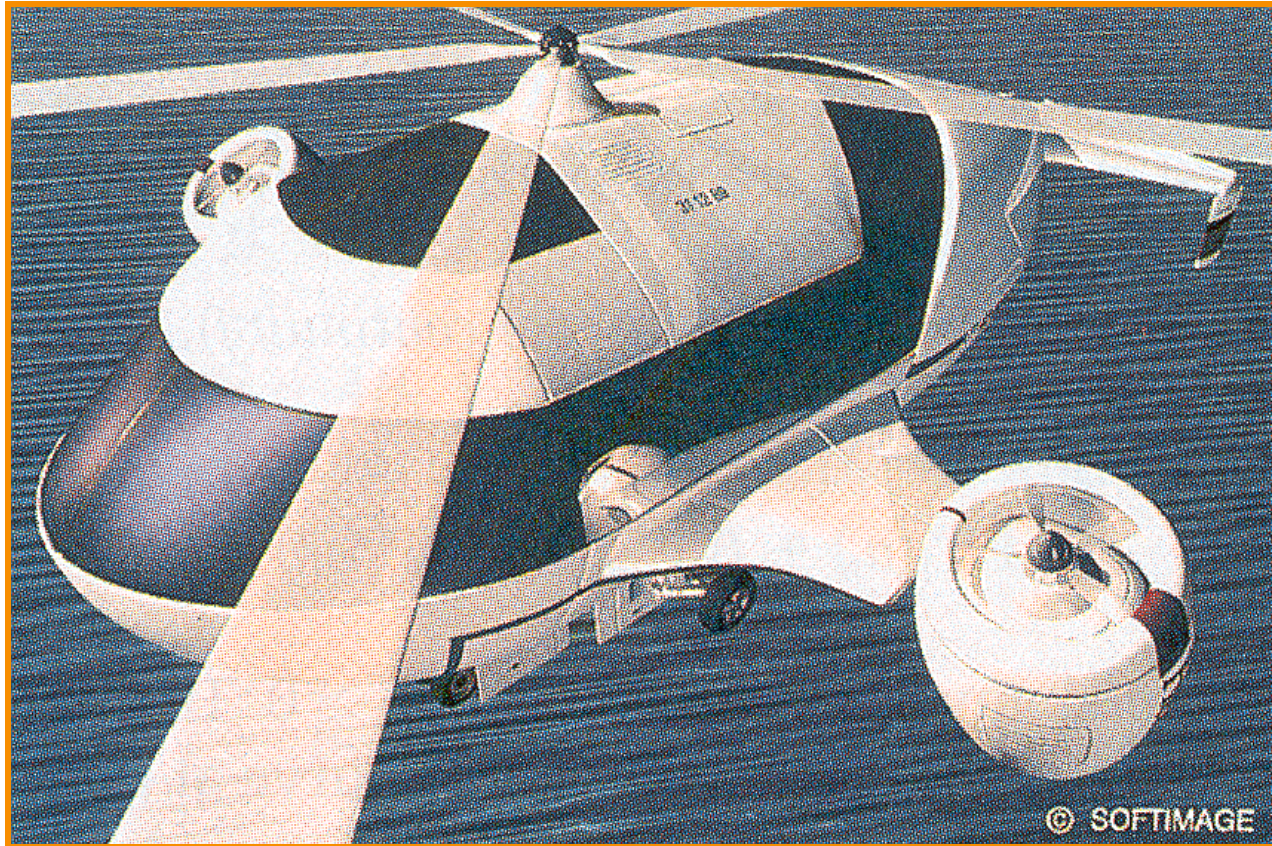
3D Object Representations



This one?

H&B Figure 9.9

3D Object Representations



H&B Figure 10.46

This one?

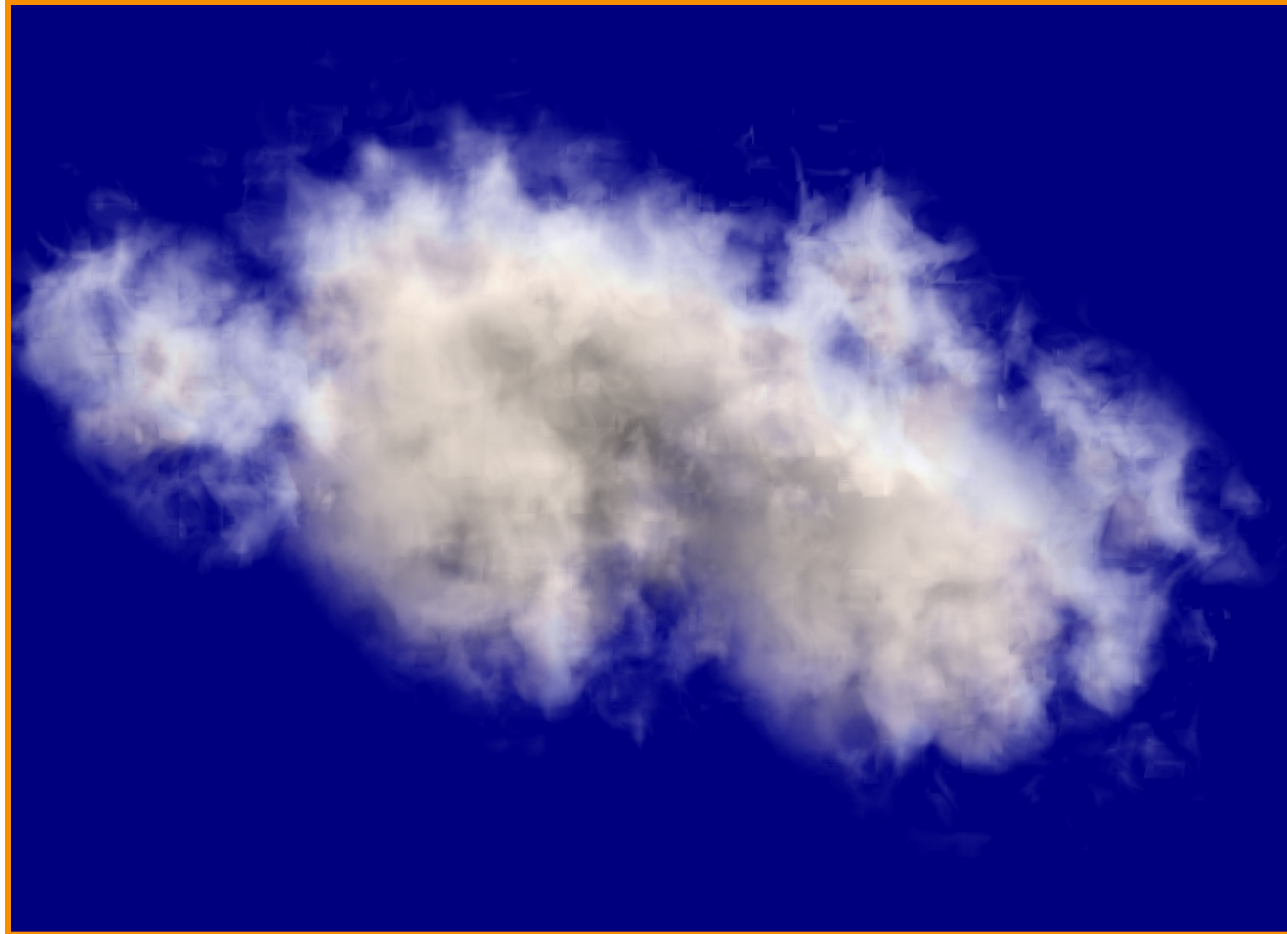
3D Object Representations



This one?

Stanford Graphics Laboratory

3D Object Representations



This one?

3D Object Representations



- Points
 - Range image
 - Point cloud
- Surfaces
 - Polygonal mesh
 - Subdivision
 - Parametric
 - Implicit
- Solids
 - Voxels
 - BSP tree
 - CSG
 - Sweep
- High-level structures
 - Scene graph
 - Application specific

Equivalence of Representations



- Thesis:
 - Each representation has enough expressive power to model the shape of any geometric object
 - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
 - Computers and programming languages are Turing-equivalent, but each has its benefits...

Why Different Representations?



Efficiency for different tasks

- Acquisition
- Rendering
- Manipulation
- Animation
- Analysis

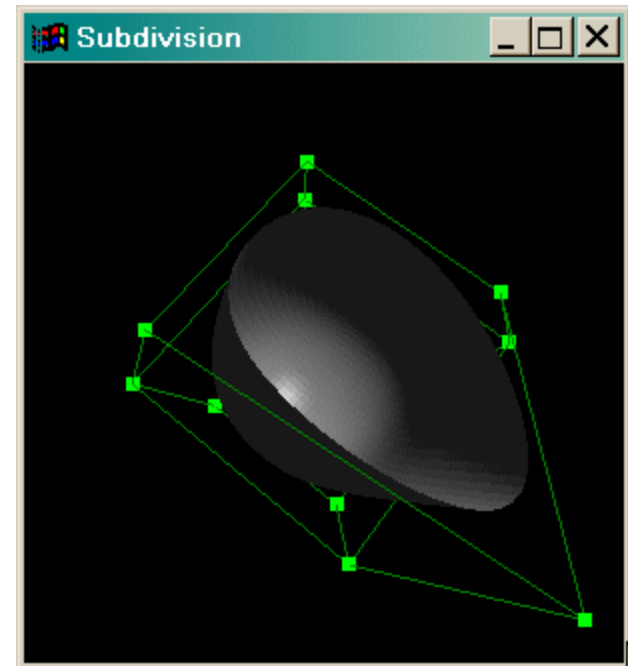
Data structures determine algorithms

Why Different Representations?



Desirable properties depend on intended use

- Easy to acquire
- Accurate
- Concise
- Intuitive editing
- Efficient editing
- Efficient display
- Efficient intersections
- Guaranteed validity
- Guaranteed smoothness
- etc.



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3D Object Representations

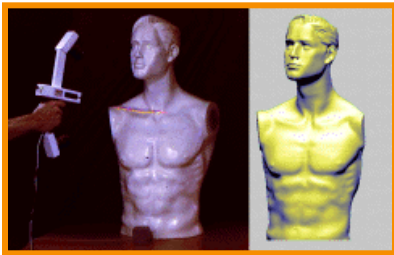


- **Points**
 - Range image
 - Point cloud
- **Surfaces**
 - Polygonal mesh
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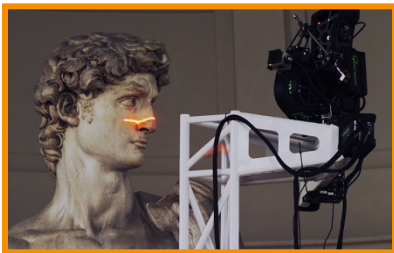
Range Image

Set of 3D points mapping to pixels of depth image

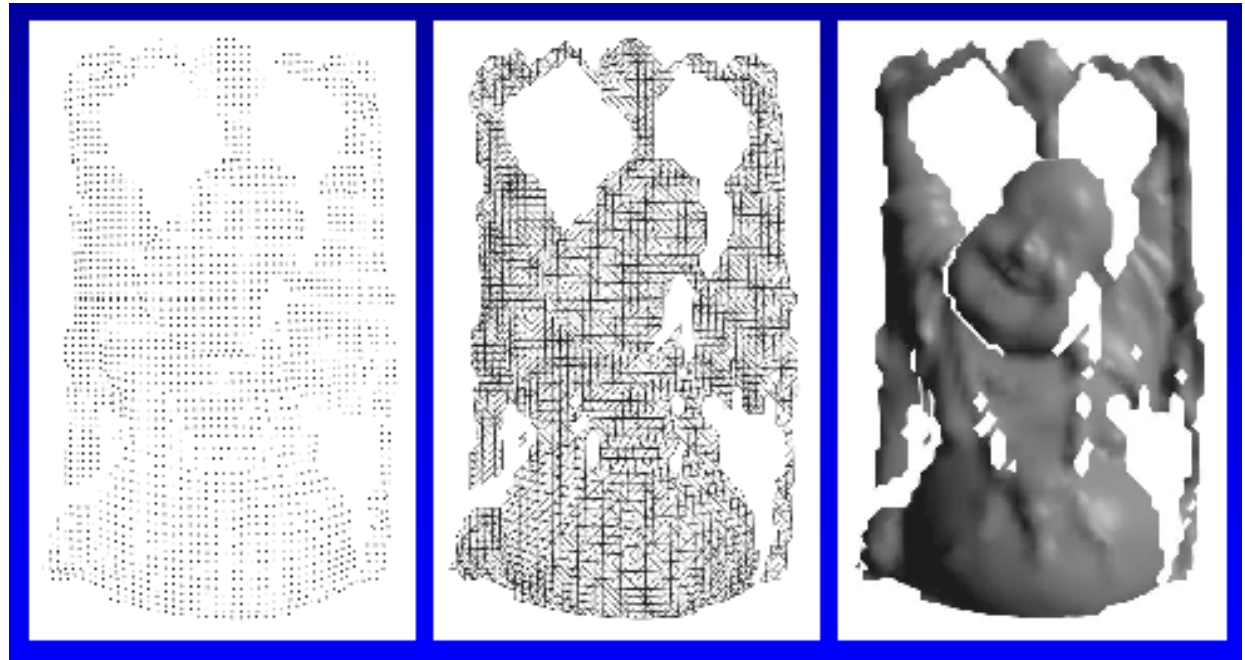
- Can be acquired from range scanner



Cyberware



Stanford



Range Image

Tessellation

Range Surface

Point Cloud

Unstructured set of 3D point samples

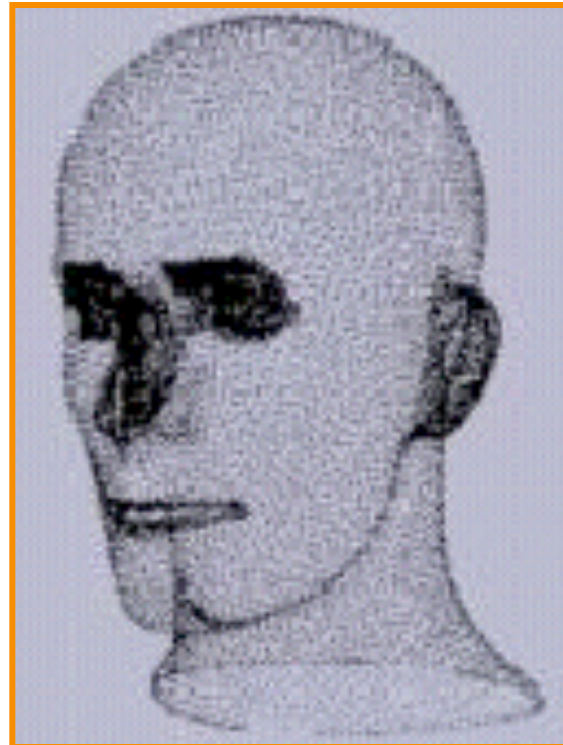
- Acquired from range finder, computer vision, etc



Polhemus



Microscribe-3D



Hoppe



Hoppe

3D Object Representations

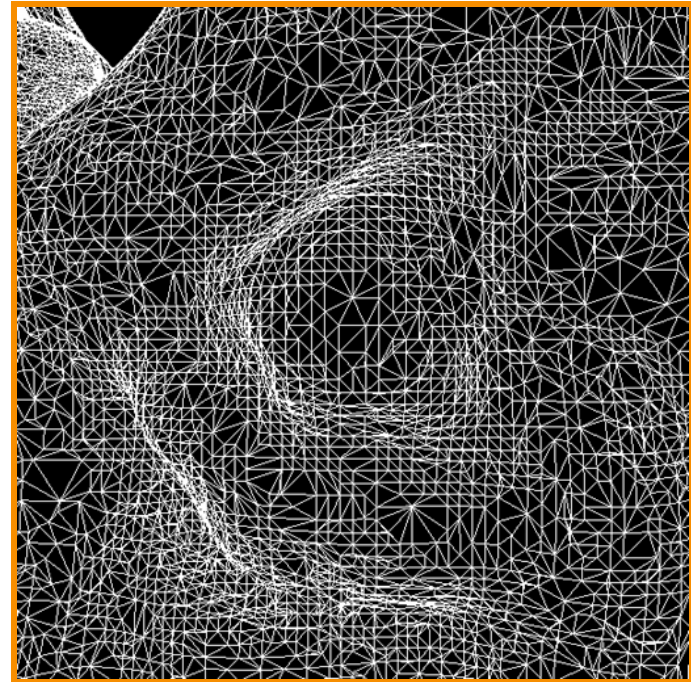


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Polygonal Mesh



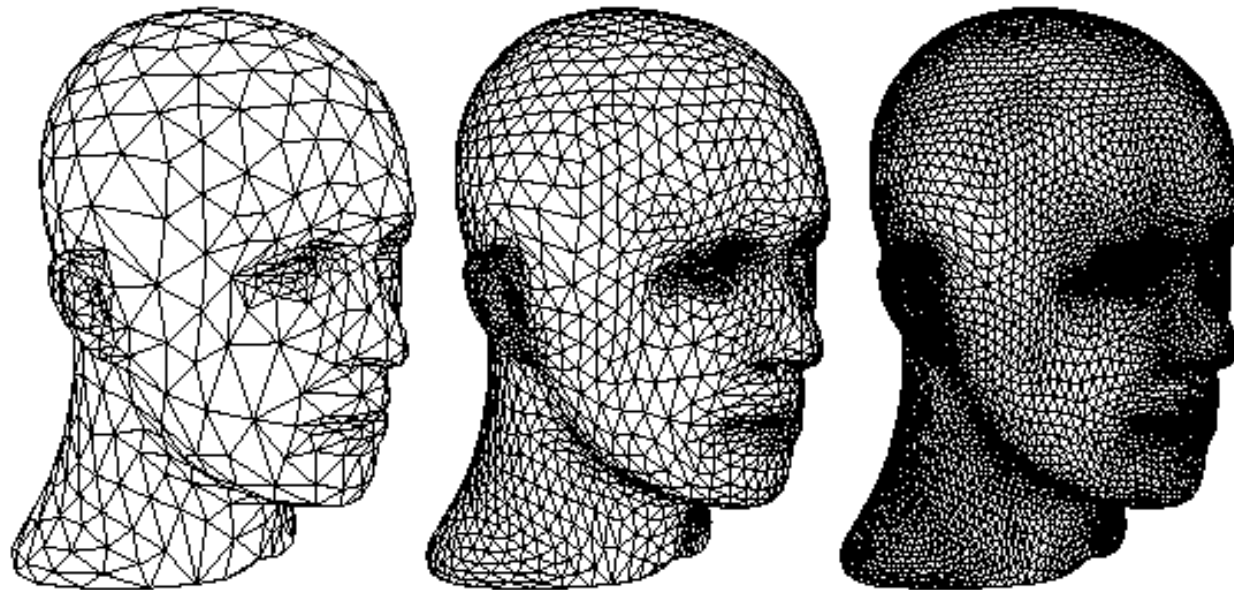
Connected set of polygons (often triangles)



Subdivision Surface

Coarse mesh & subdivision rule

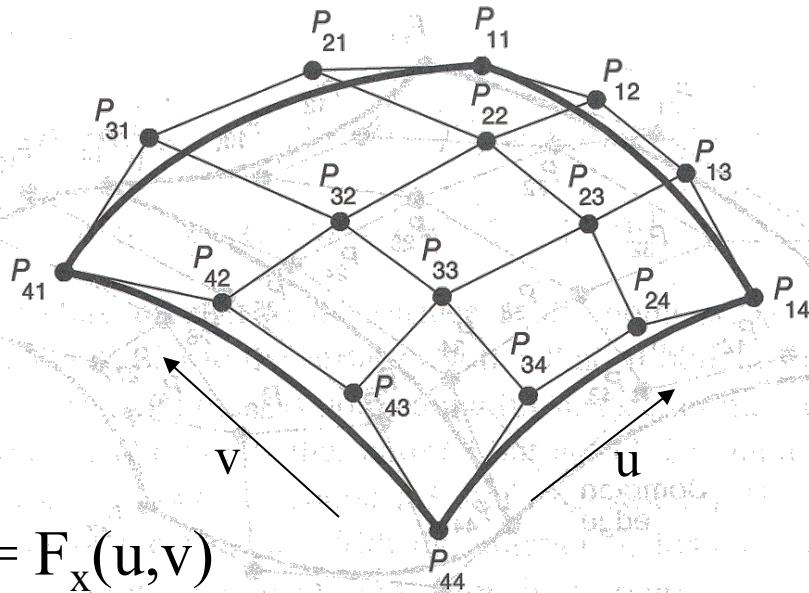
- Smooth surface is **limit** of sequence of refinements



Parametric Surface

Tensor-product spline patches

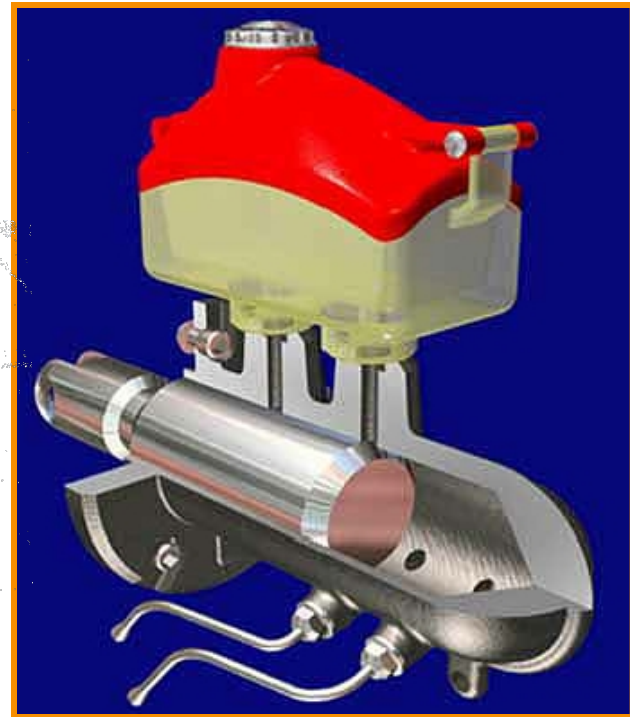
- Each patch is parametric function
- Careful constraints to maintain continuity



$$x = F_x(u,v)$$

$$y = F_y(u,v)$$

$$z = F_z(u,v)$$

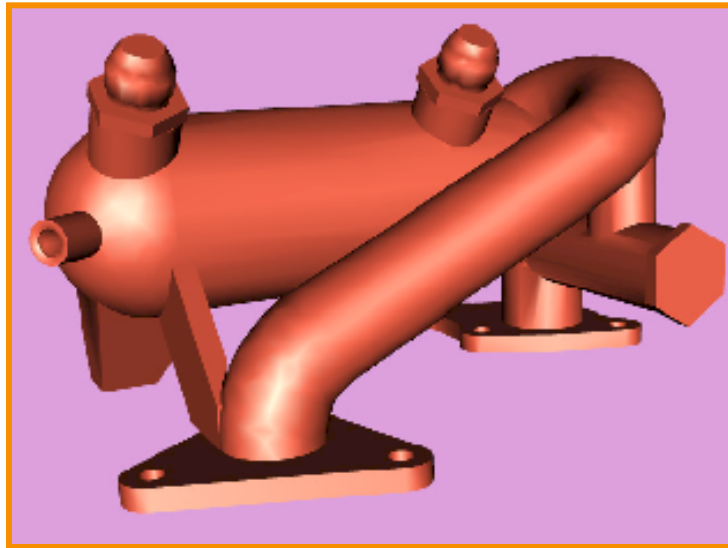


FvDFH Figure 11.44

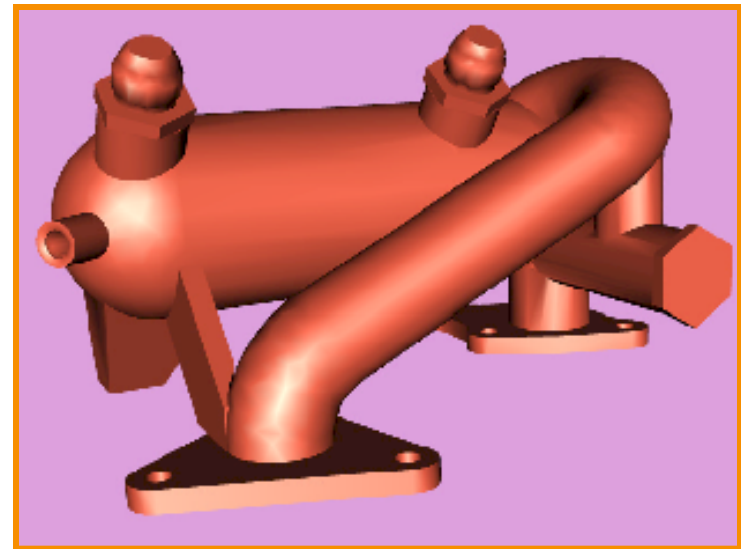


Implicit Surface

Set of all points satisfying: $F(x,y,z) = 0$



Polygonal Model



Implicit Model

3D Object Representations



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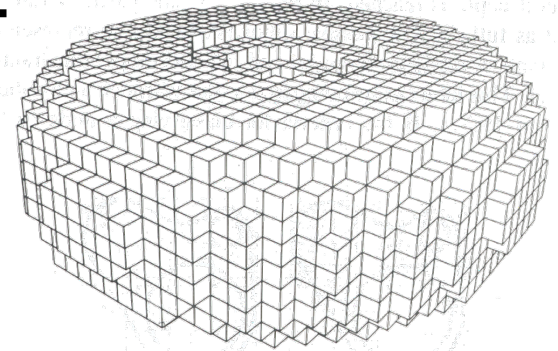
Voxel grid



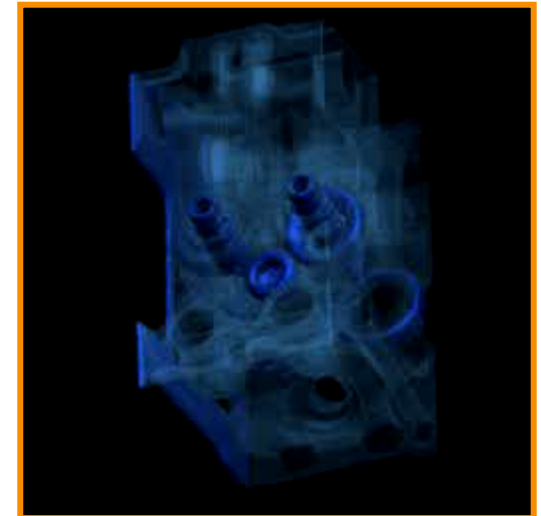
Uniform volumetric grid of samples:

- Occupancy
(object vs. empty space)
- Density
- Color
- Other function
(speed, temperature, etc.)

- Often acquired via
simulation or from
CAT, MRI, etc.



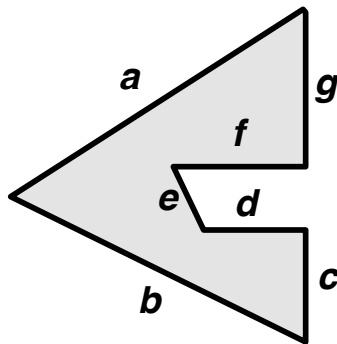
FvDFH Figure 12.20



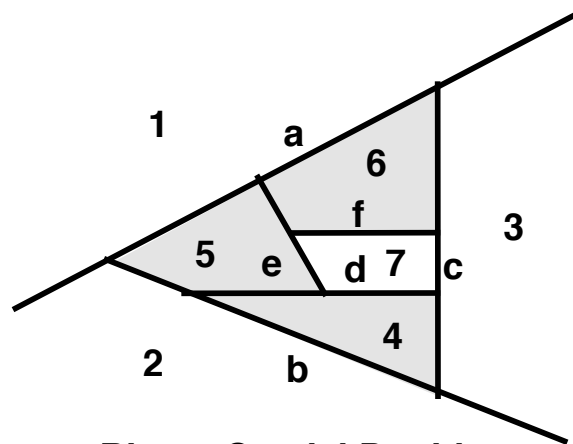
BSP Tree

Hierarchical **B**inary **S**pace **P**artition with solid/empty cells labeled

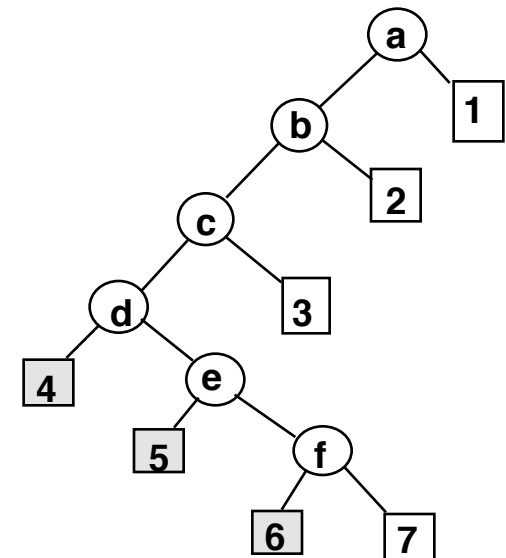
- Constructed from polygonal representations



Object



Binary Spatial Partition

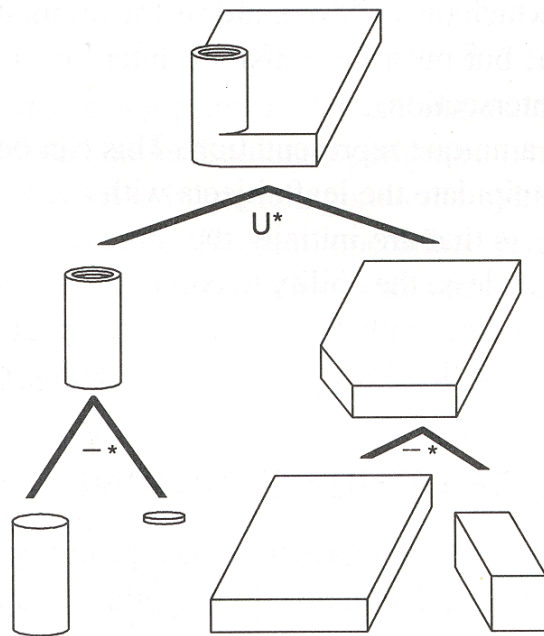


Binary Tree

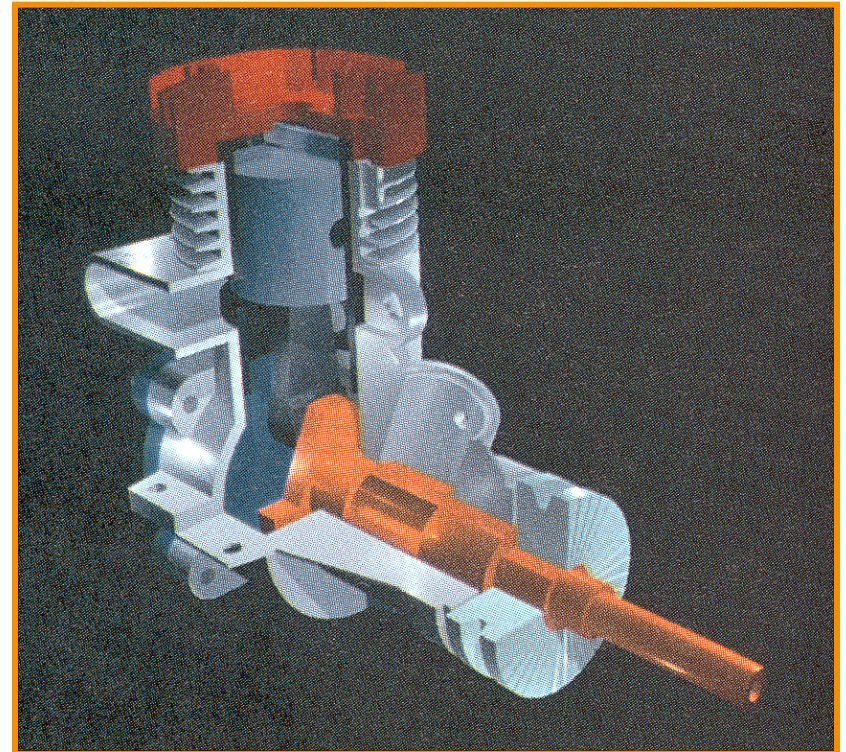
CSG



Constructive **S**olid **G**eometry: set operations (union, difference, intersection) applied to simple shapes



FvDFH Figure 12.27

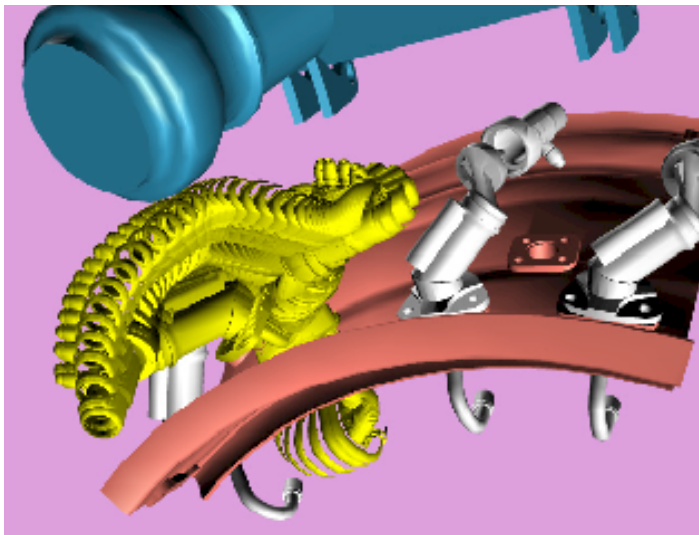


H&B Figure 9.9

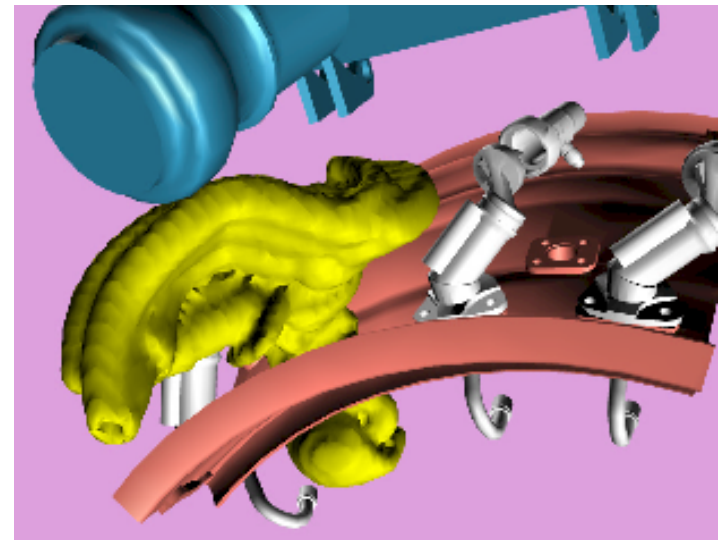
Sweep



Solid swept by curve along trajectory



Removal Path



Sweep Model

3D Object Representations



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Scene Graph



Union of objects at leaf nodes

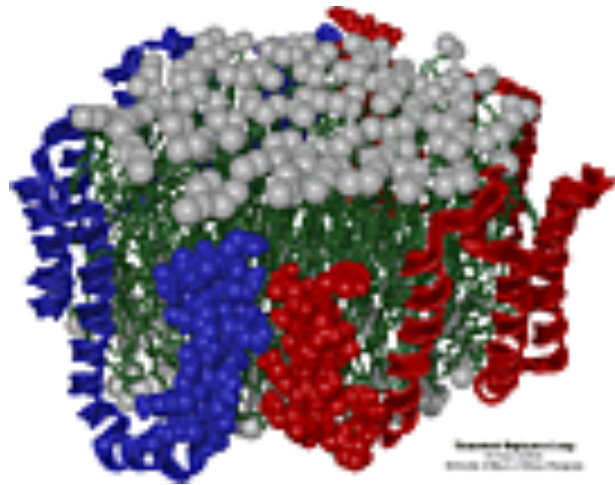


Bell Laboratories

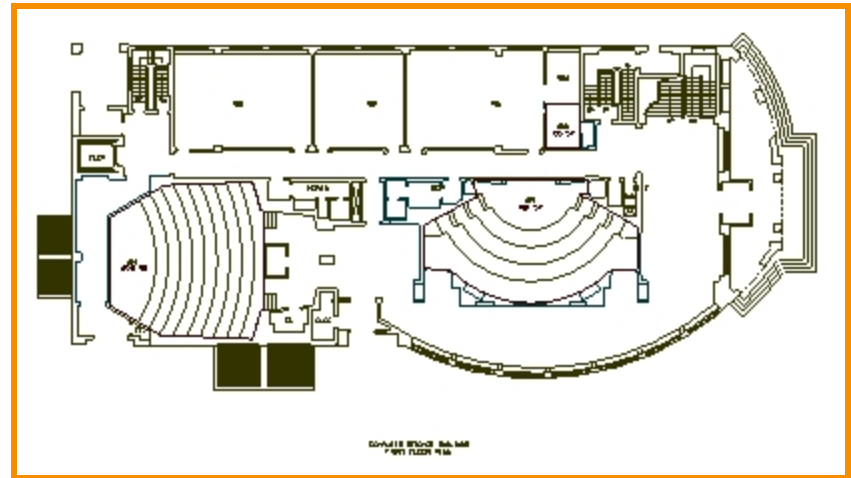


avalon.viewpoint.com

Application Specific

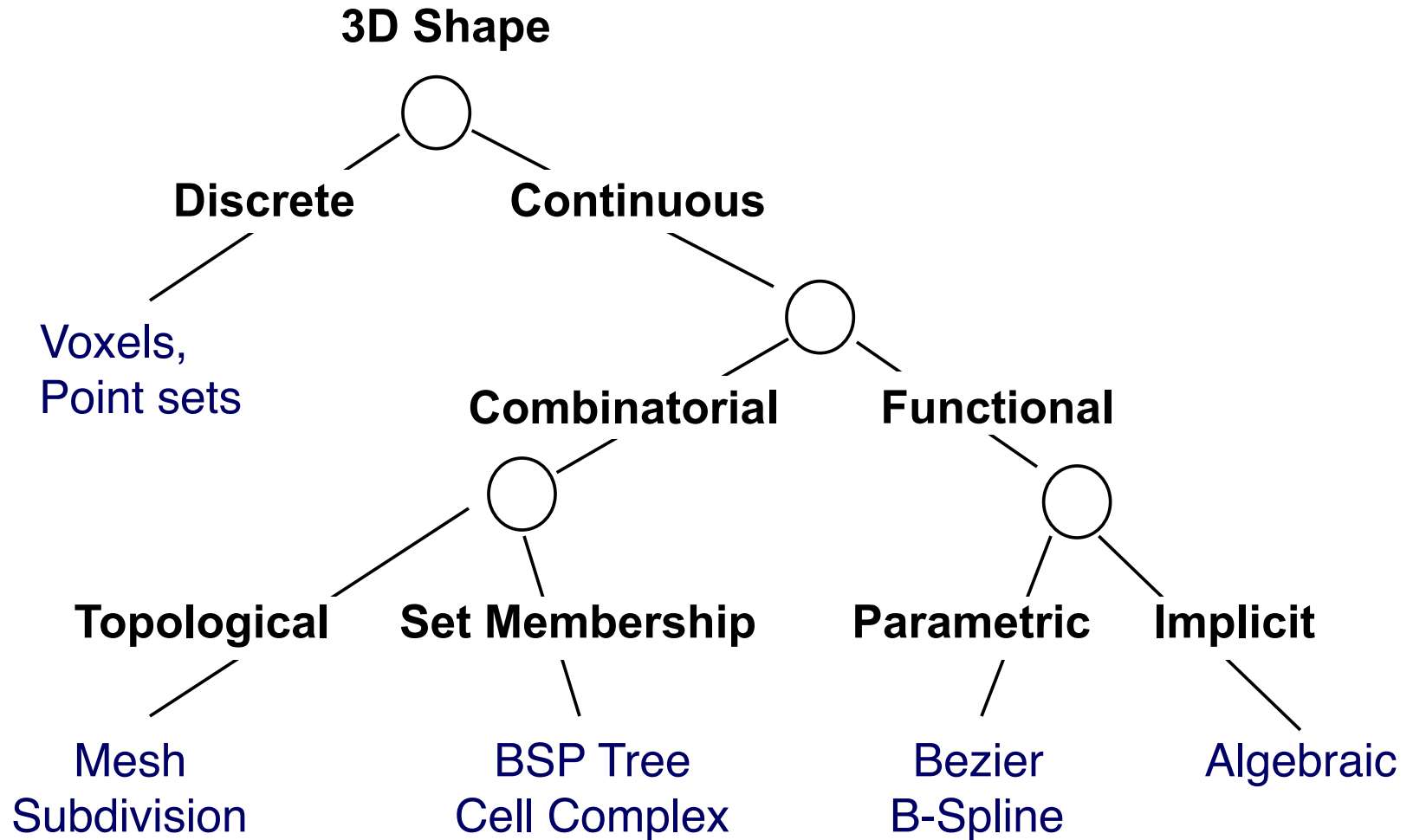


Apo A-1
*(Theoretical Biophysics Group,
University of Illinois at Urbana-Champaign)*



Architectural Floorplan
(CS Building, Princeton University)

Taxonomy of 3D Representations



Equivalence of Representations



- Thesis:
 - Each representation has enough expressive power to model the shape of any geometric object
 - It is possible to perform all geometric operations with any fundamental representation
- Analogous to Turing-equivalence
 - Computers and programming languages are Turing-equivalent, but each has its benefits...



Computational Differences

- **Efficiency**
 - Representational complexity (e.g. surface vs. volume)
 - Computational complexity (e.g. $O(n^2)$ vs $O(n^3)$)
 - Space/time trade-offs (e.g. tree data structures)
 - Numerical accuracy/stability (e.g. degree of polynomial)
- **Simplicity**
 - Ease of acquisition
 - Hardware acceleration
 - Software creation and maintenance
- **Usability**
 - Designer interface vs. computational engine

Modeling Operations



What can we do with a 3D object representation?

- Edit
- Transform
- Smooth
- Render
- Animate
- Morph
- Compress
- Transmit
- Analyze
- etc.



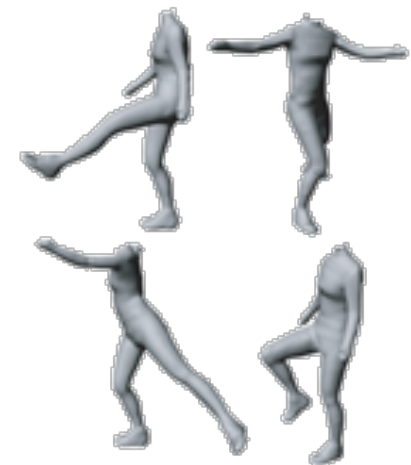
Digital Michelangelo



Pirates of the Caribbean



Thouis "Ray" Jones



Sand et al.



Upcoming Lectures

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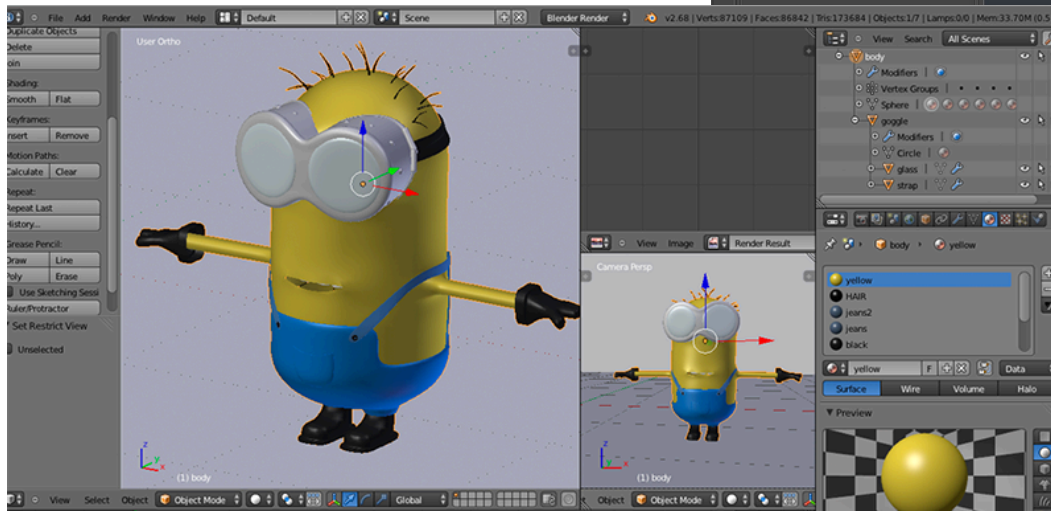
Bonus Today...



Nora gives a brief introduction to modeling in Maya.

Maya free for students:
www.autodesk.com/education

[getintopc.com]



Blender free for everyone:
www.blender.org

[www.eliwhitney.org]