Memory; Sequential & Clocked Circuits; Finite State Machines

COS 116, Spring 2010

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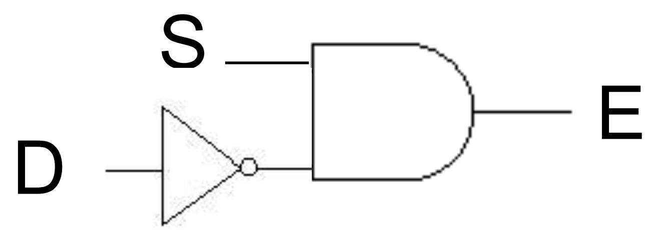
Recap: Boolean Logic

Boolean Expression E = S AND $\overline{D}$

Boolean Circuit



Truth table:

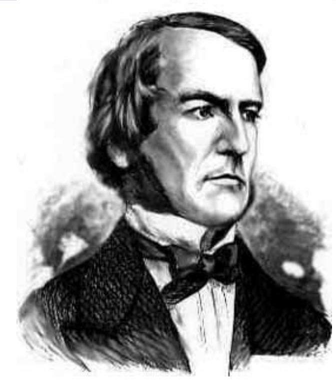
Value of E for every possible D, S.

TRUE=1; FALSE= 0.

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

Truth table has 2^k rows if the number of variables is k

Boole's reworking of Clarke's "proof" of existence of God (see handout)



- General idea: Try to prove that Boolean expressions E_1, E_2, …, E_k cannot simultaneously be true
- <u>Method</u>: Show $E_1 \cdot E_2 \cdot \ldots \cdot E_k = 0$
- Discussion: What exactly does Clarke's "proof" prove? How convincing is such a proof to you?

Also: Do Google search for "Proof of God's Existence."

Combinational circuit for binary addition?

	25	11001
+	29	11101
	54	110110

- Want to design a circuit to add any two *N*-bit integers.

Is the truth table method useful for N=64?

Modular design

Have small number
of basic components.

Put them together to achieve
desired functionality

Basic principle of modern industrial design;
recurring theme in next few lectures.

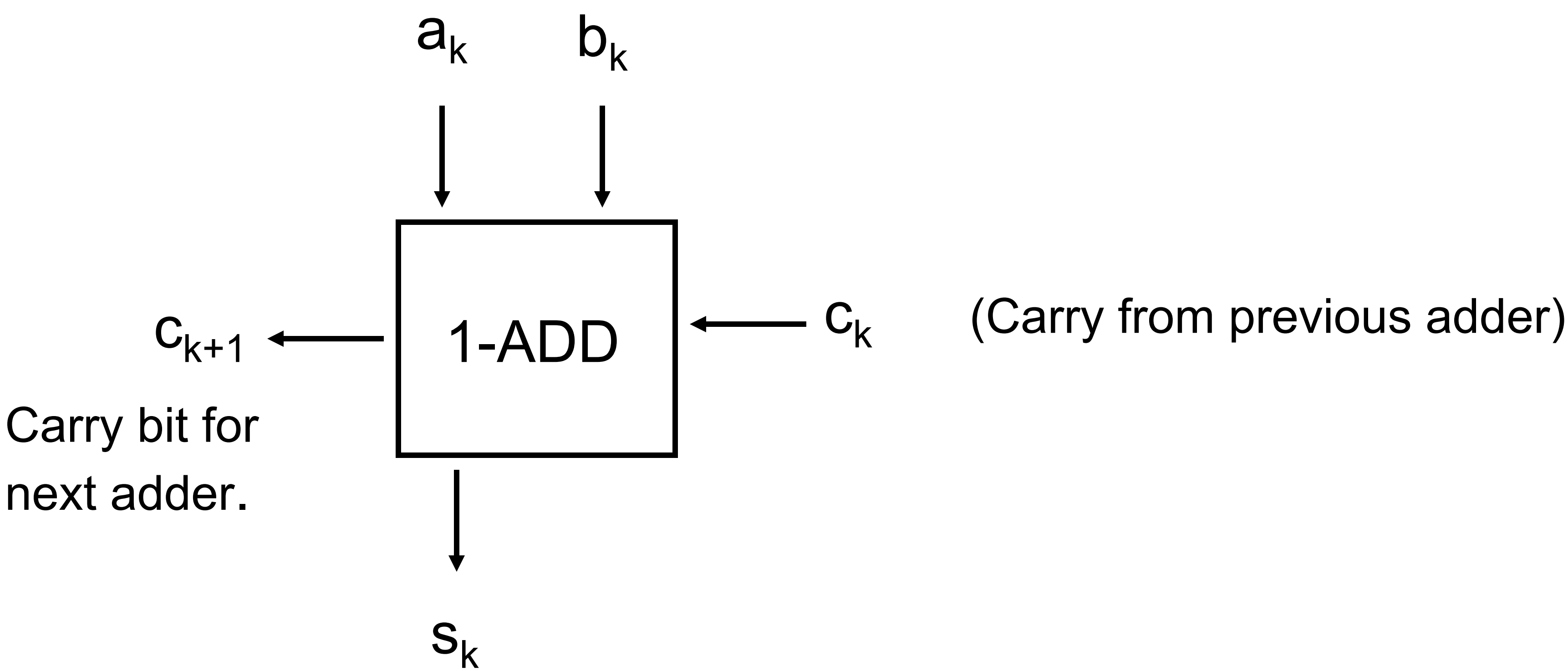
Modular design for N-bit adder

Carry bits

$$
\begin{array}{ccccccc}
 & & c_{N-1} & c_{N-2} & \cdots & c_1 & c_0 \\
 & & a_{N-1} & a_{N-2} & \cdots & a_1 & a_0 \\
+ & & b_{N-1} & b_{N-2} & \cdots & b_1 & b_0 \\
\hline
 & s_N & s_{N-1} & s_{N-2} & \cdots & s_1 & s_0
\end{array}
$$

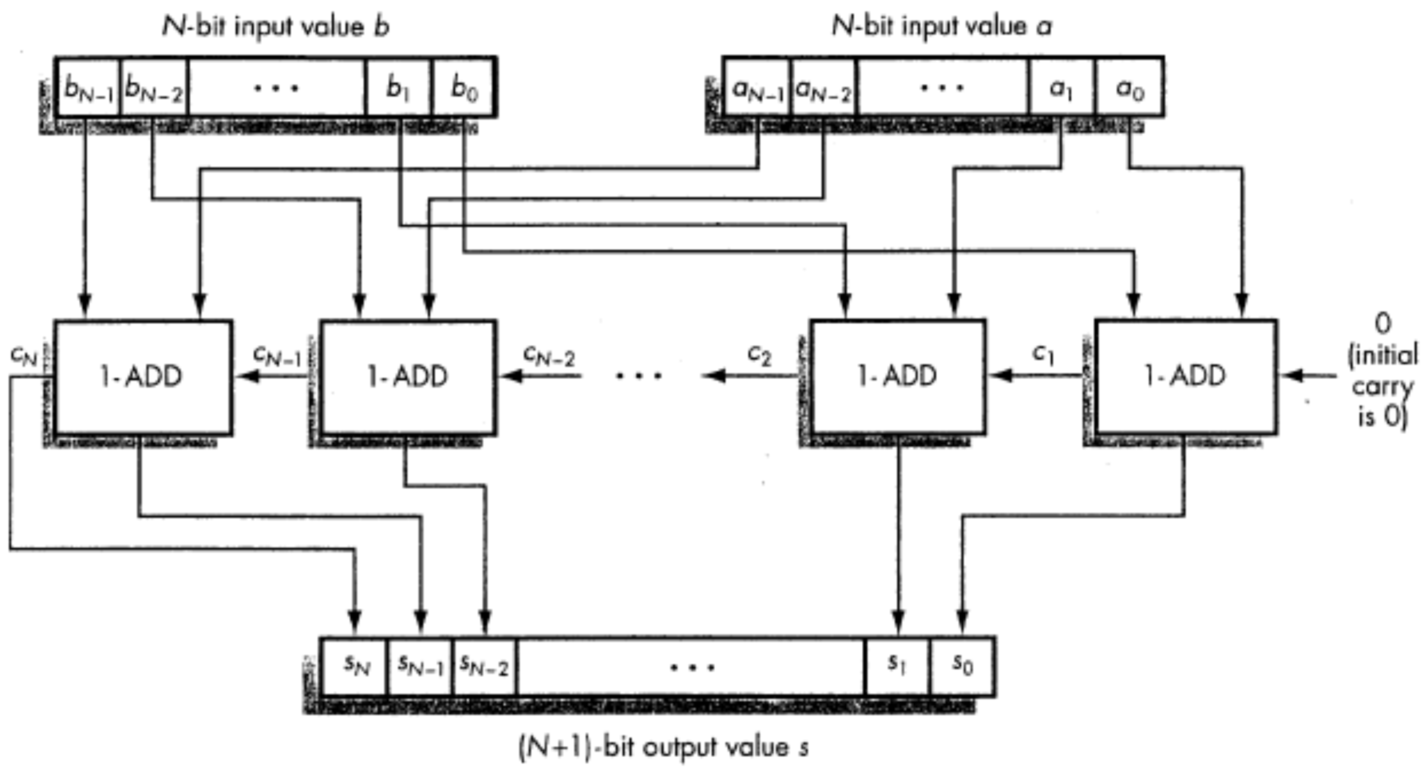
Suffices to use *N* 1-bit adders!

1-bit adder



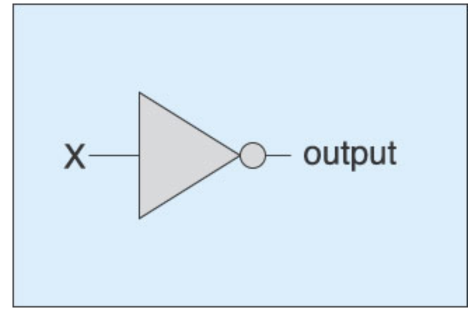
Do yourself: Write truth table, circuit.

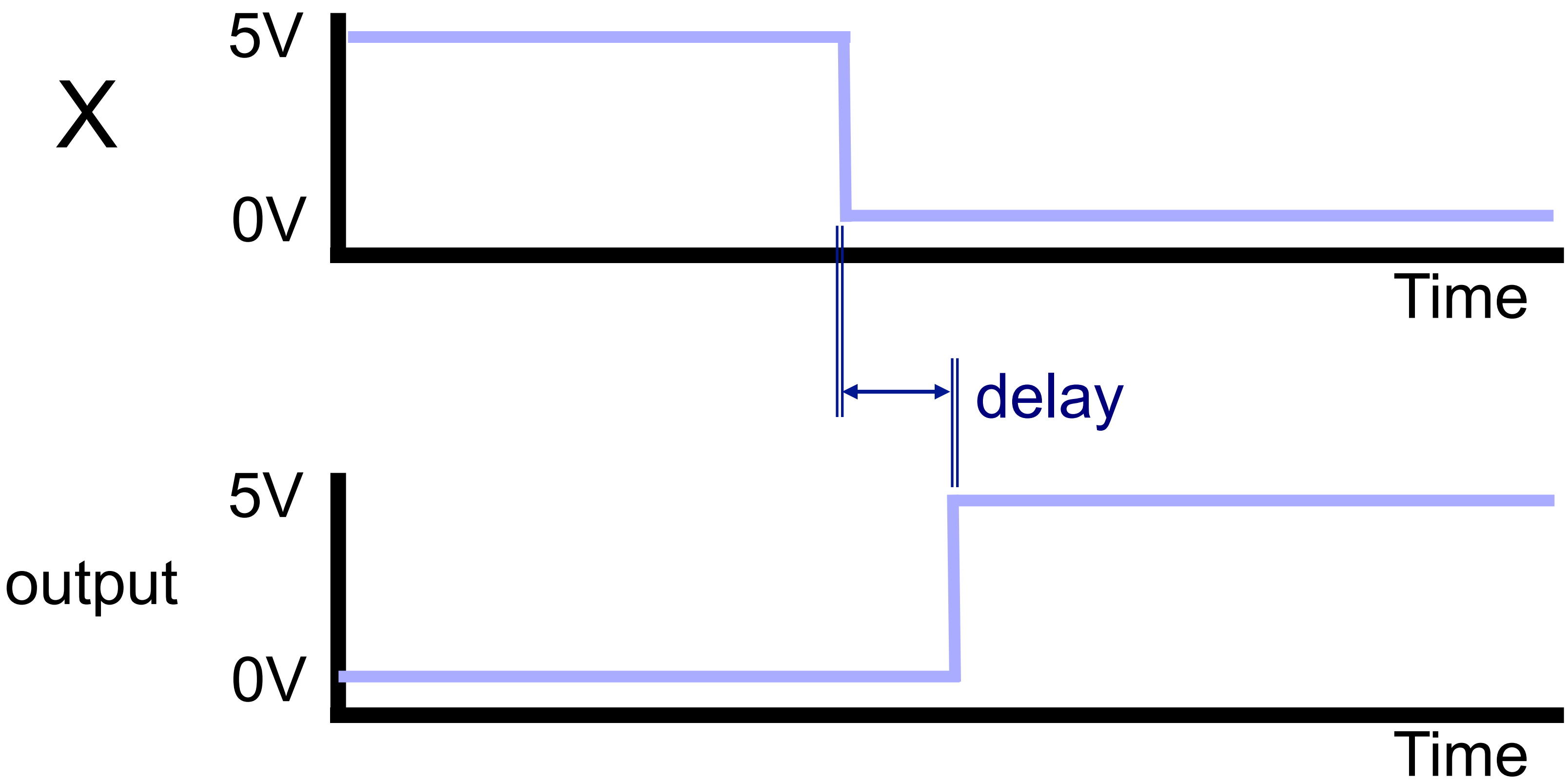
A Full Adder (from handout)



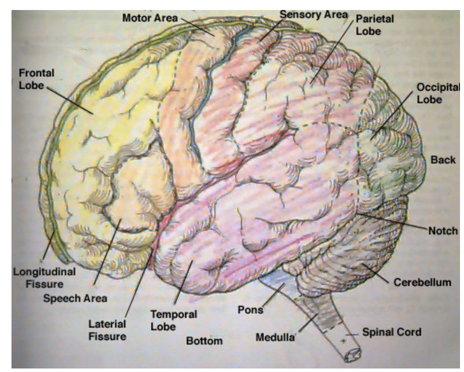
Timing Diagram

NOT gate





Memory



Rest of this lecture:
How boolean circuits have “memory”.

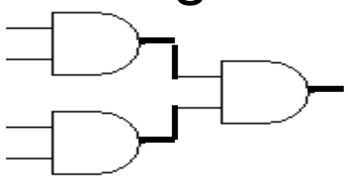
What do you understand by ‘memory”?

How can you tell that a 1-year old child has it?

Behaviorist’s answer: His/her actions depend upon past events.

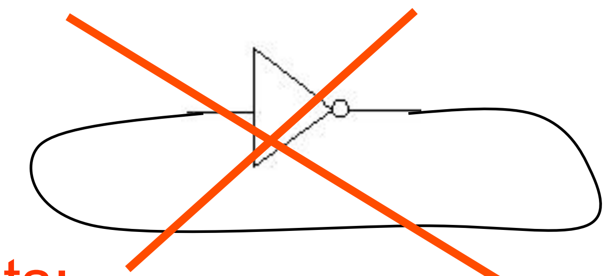
Why combinational circuits have no "memory"

- Boolean gates connected by wires



Wires: transmit voltage
(and hence value)

- Important: no loops allowed

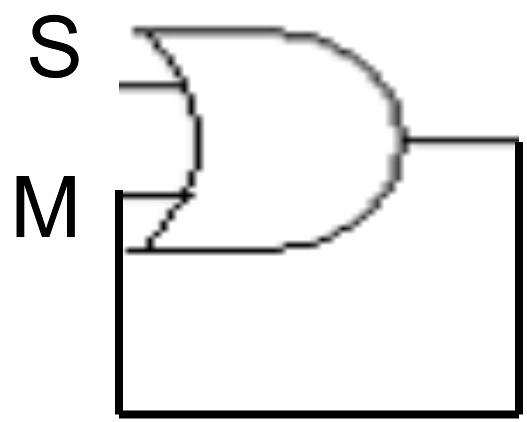


Output is determined by current inputs;
no "memory" of past values of the inputs.

Today: Circuits with loops; aka "Sequential Circuits"

Matt likes Sue but he doesn't like changing his mind

- Represent with a circuit: Matt will go to the party if Sue goes or if he already wanted to go



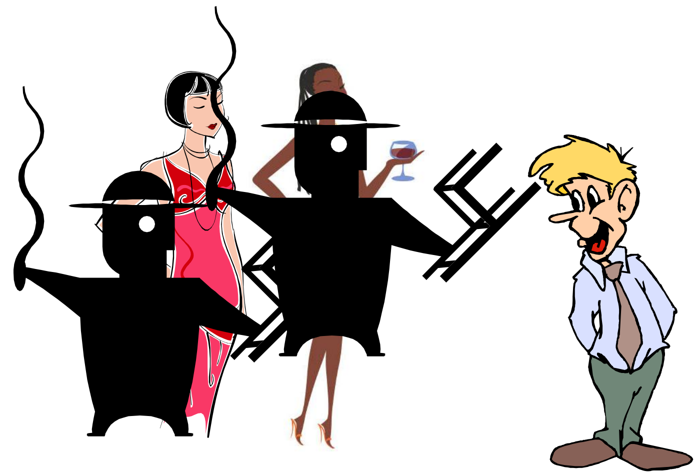
Is this well-defined?

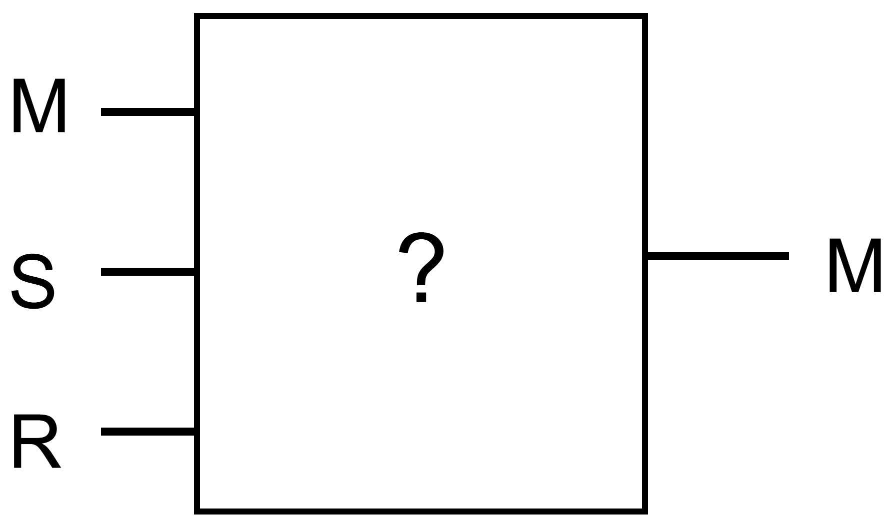
Sequential Circuits

- Circuits with AND, OR and NOT gates.
- Cycles ***are*** allowed (ie outputs can feed back into inputs)
- Can exhibit “memory”.
- Sometimes may have “undefined” values

Enter Rita

- Matt will go to the party if Sue goes OR if the following holds: if Rita does not go *and* he already wanted to go.

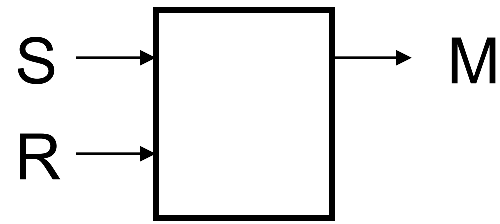


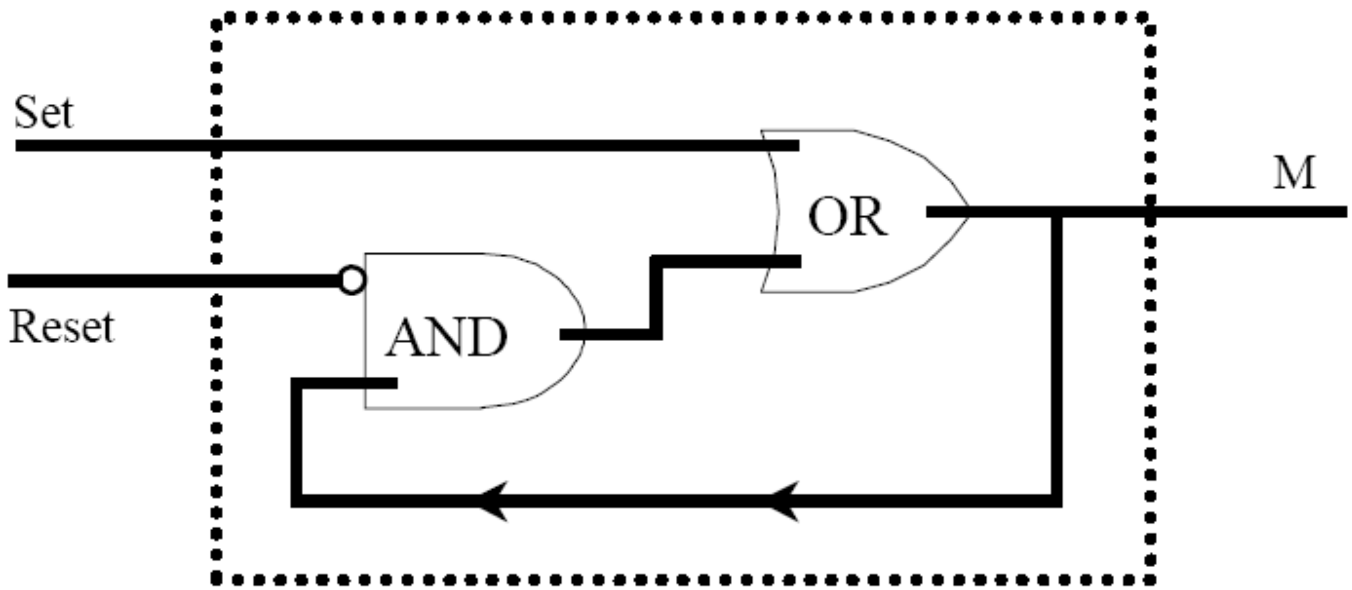


R, S: "**control**" inputs

What combination of R, S changes M?

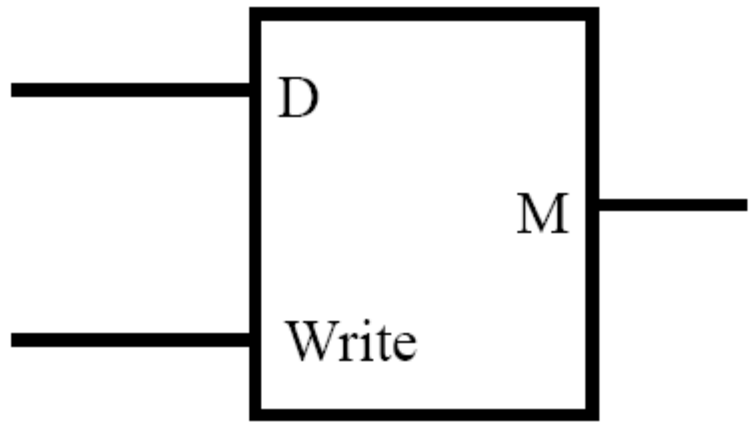
R-S Flip-Flop





- M becomes 1 if Set is turned on
- M becomes 0 if Reset is turned on
- Otherwise (if both are 0), M just remembers its value

A more convenient form of memory

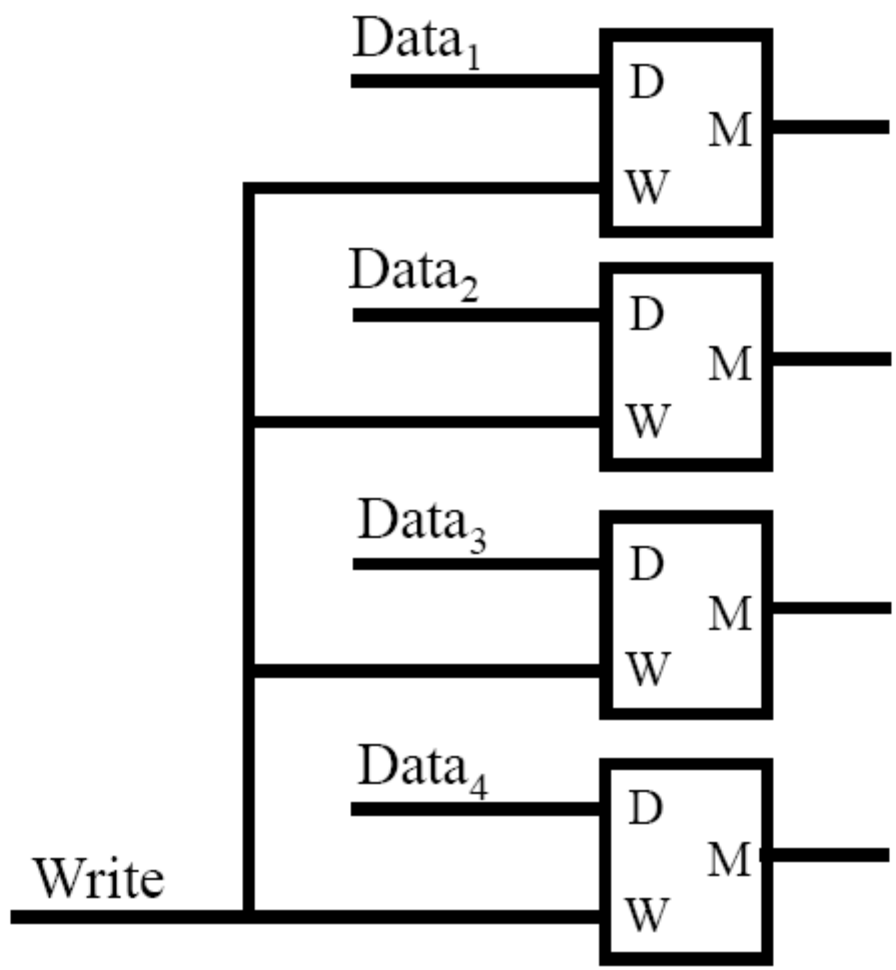


No "undefined" outputs ever!

- If Write = 0, M just keeps its value. (It ignores D.)
- If Write = 1, then M becomes set to D

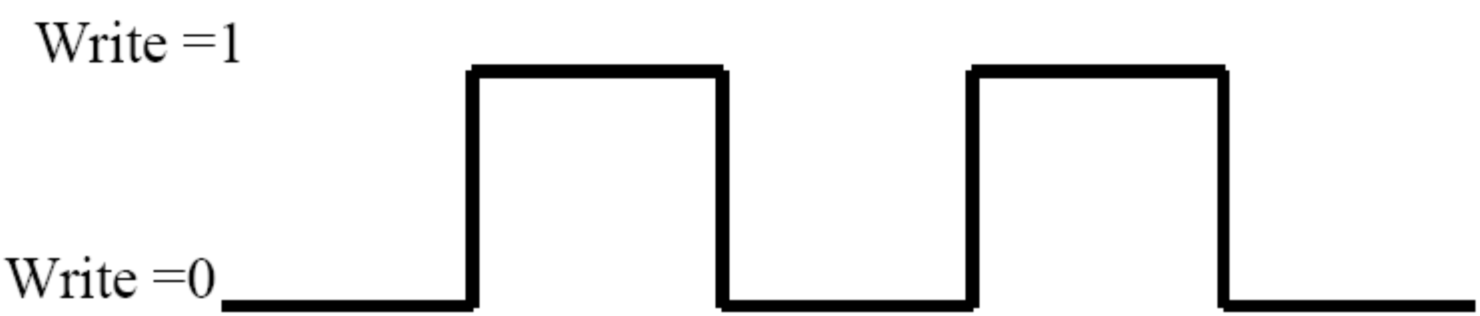
Fact: "Data Flip-Flop" or "D flip flop"; can be implemented using R-S flip flops.

"Register" with 4 bits of memory



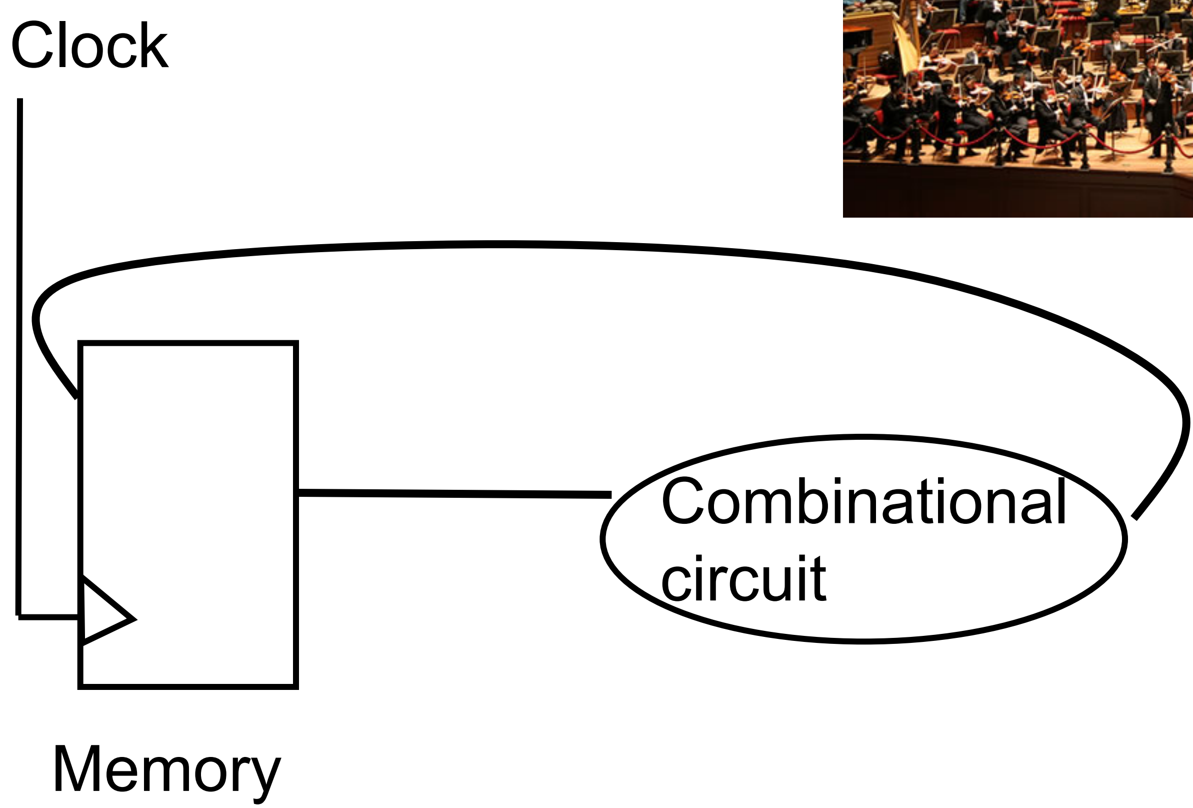
What controls the "Write" signal?

- Often, the system clock!
- "clock" = device that sends out a fluctuating voltage signal that looks like this



"Computer speed" often refers to the clock frequency (e.g. 2.4GHz)

The "symphony" inside a computer

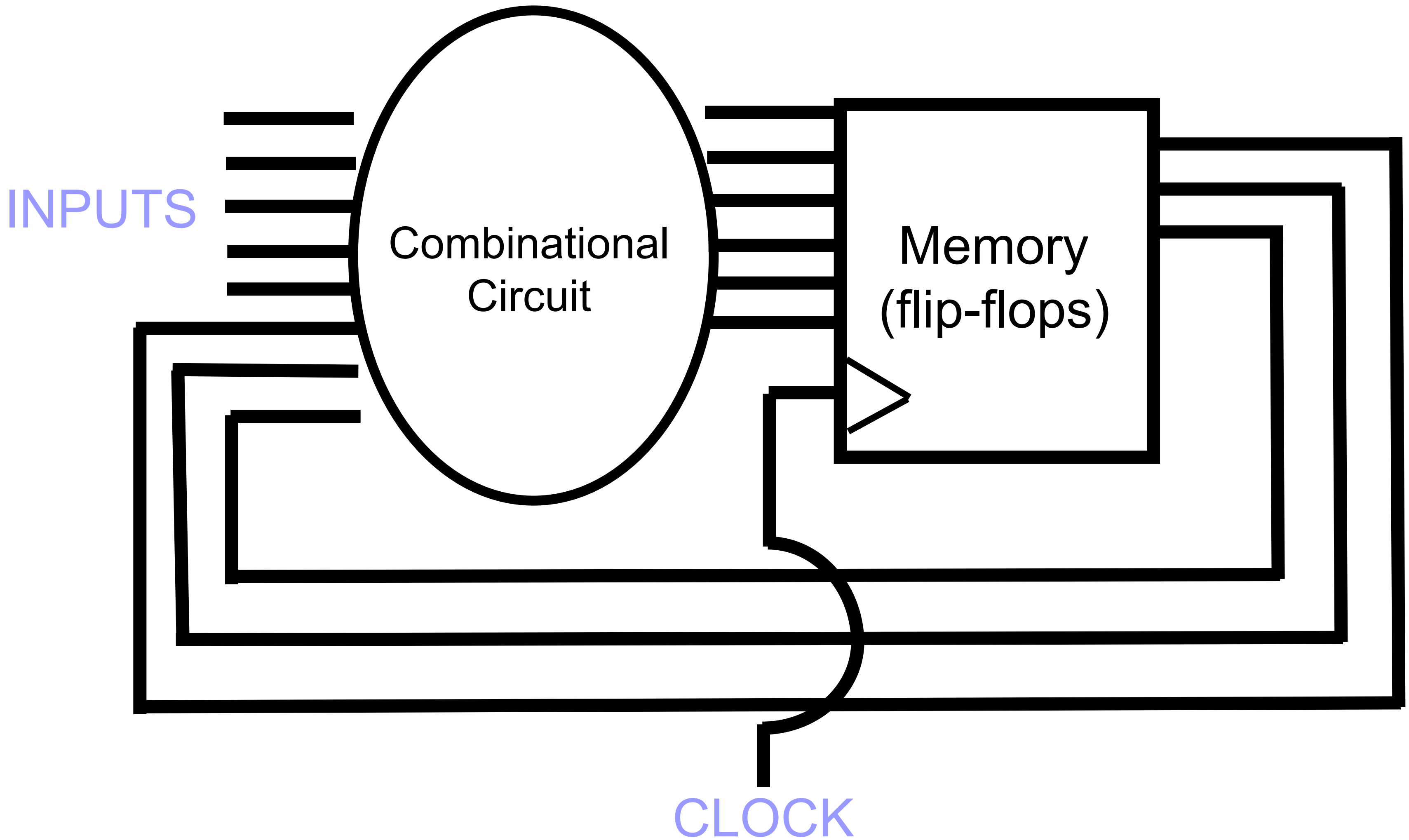


Clocked Sequential Circuit (aka Synchronous Circuits)

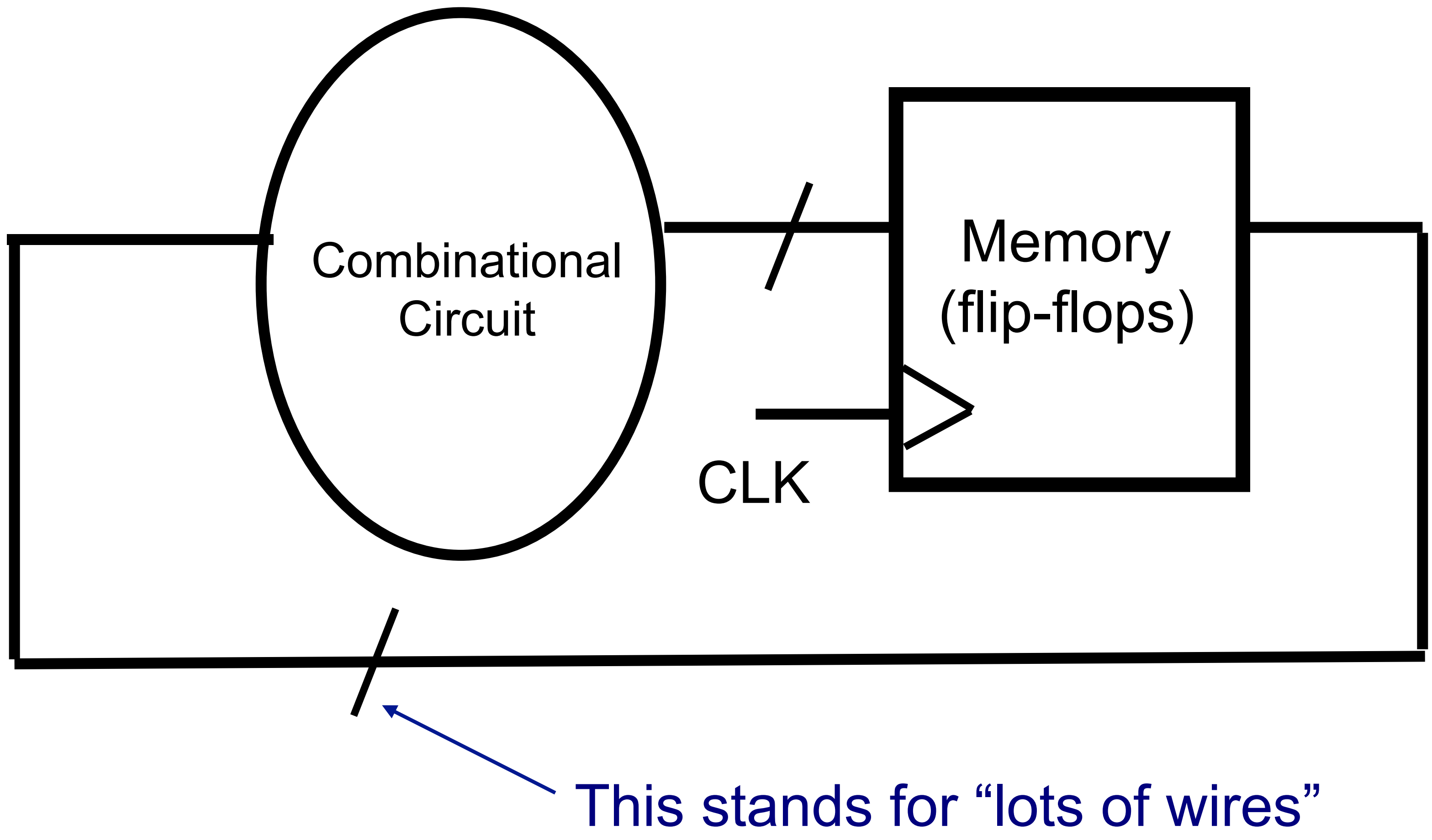
Clocked Sequential Circuits

Synchronous Sequential Circuit

(aka Clocked Sequential Circuit)



Shorthand



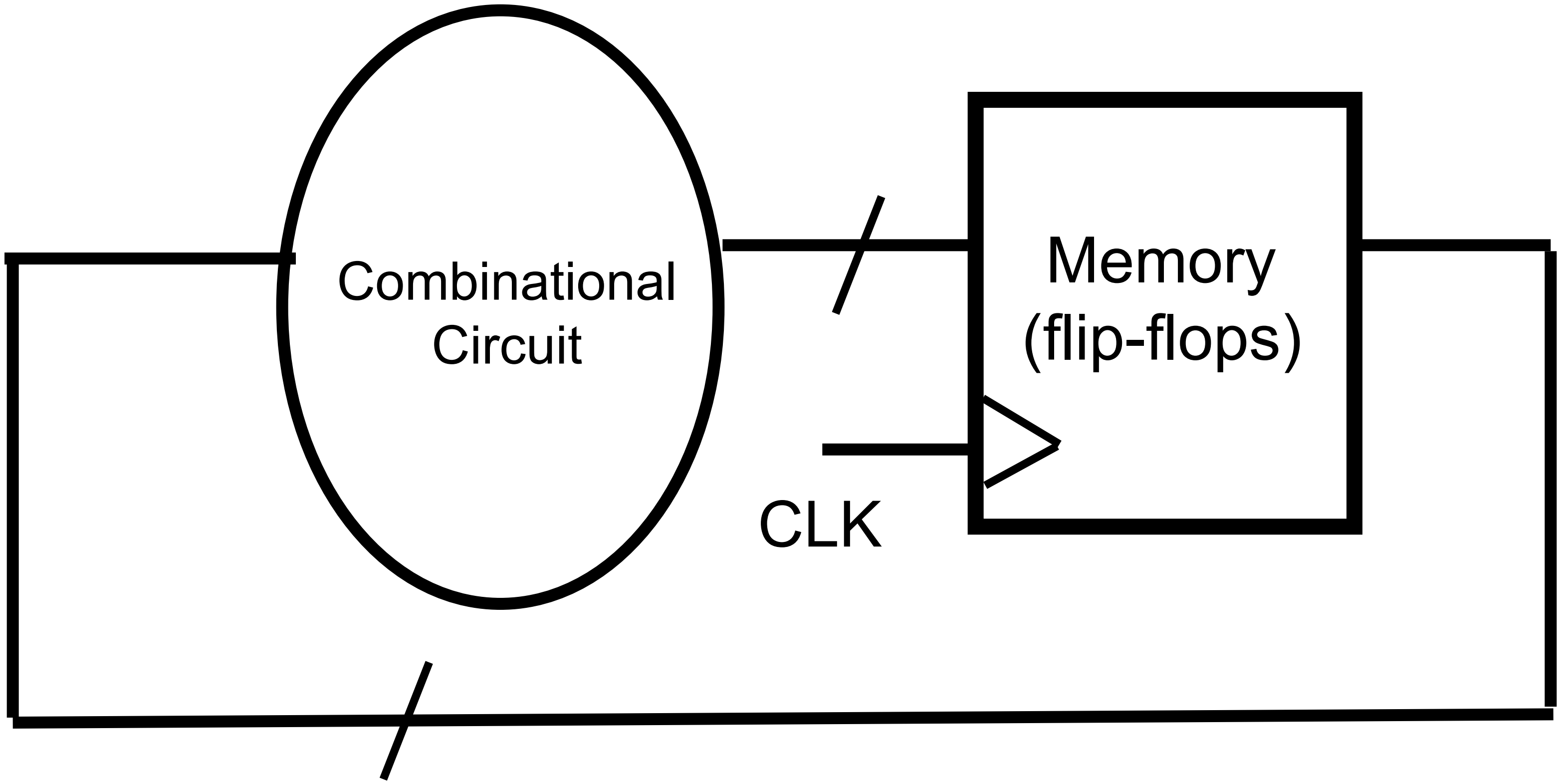
Combinational
Circuit
Memory
(flip-flops)
CLK
This stands for "lots of wires"

Clock Speeds

Heinrich Hertz
1857-94

1974	Intel 8080	2 MHz (Mega = Million)
1981	Original IBM PC	4.77 MHz
1993	Intel Pentium	66 MHz
2005	Pentium 4	3.4 GHz (Giga = Billion)

What limits clock speed?



Delays in combinational logic (remember the adder)

During 1 clock cycle of Pentium 4, light travels: **4 inches**

Next two lectures…

- Computer organization: CPUs and RAM
- Lessons from computer architecture.

- Guest lecturer: Szymon Rusinkiewicz