COS 522 Complexity — Homework 4.

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Total of 165 points

Exercise 1 (20 + 20 points)**.** Do Exercise 11.3 (probabilistic 7/8 approximation to 3SAT). For 20 more points, show the deterministic algorithm (Exercise 11.4).

Exercise 2 (20 points)**.** Do Exercise 11.13 (polynomial time algorithm for 2SAT).

Exercise 3 (40 points)**.** Do Exercise 22.2, and then use it (along the proof of Lemma 22.8) to show that there is some $\epsilon > 0$, such that unless $\mathbf{P} = \mathbf{NP}$, there is no algorithm that on input an n-vertex graph G always outputs a number between $\alpha(G)/n^{\epsilon}$ and $\alpha(G)n^{\epsilon}$.

Exercise 4 (30 points)**.** Do Exercise 22.16 (Max 3SAT is hard to approximate even when restricting to formulas where each variable appears in at most 5 clauses— can replace 5 with any other constant if it helps you). See the footnote for a fuller hint than the one appearing in the book[1]

Optional exercises. The following three exercises are completely optional, and you can feel free to skip them if you don't have time. They are just calculations. However, such calculations often come in handy, and if you work them out yourself, you are more likely to remember them.

Exercise 5 (10 points)**.** Do exercise 22.1. Can you see how does the bound improve when you're guaranteed that $|S| < \epsilon n$ rather than only $|S| < n/2$?

Exercise 6 (10 points)**.** Do exercise 22.3. (Bounding the statistical distance of Binomial distributions with close parameter.)

Exercise 7 (10 points)**.** Do Exercise 22.4. For extra 5 points, generalize your proof to show also the Paley-Zygmund Inequality: if Z is a non-negative round variable and $\epsilon > 0$, then $\Pr[Z \geq \epsilon \,\mathrm{E}[Z]] \geq (1-\epsilon)^2 \frac{\mathrm{E}[Z]^2}{\mathrm{E}[Z^2]}$.

[1] **Hint:** First, try to prove that this problem is just NP complete. The idea is to introduce multiple copies of each variable so that no variable appears too often, and place all possible equality constraints between various copies. Show that if we allowed weights on the constraints (i.e., the value of the formula would be not be the number of satisfied constraints but the sum of their weights, where the weights are normalized to sum to 1), then you can ensure that the reduction maps a formula with value $1-\epsilon$ into a formula with value at most $1-\epsilon'$ for some $\epsilon' > 0$ depending only on ϵ. Then show you can avoid weights by replacing the complete graph of equality constraints with a degree 3 expander graph.