What computers just cannot do

COS 116
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Epimenides Paradox

- *Κρ□τες □ε□ ψε□σται*
- “Cretians, always liars!”
- But Epimenides was a Cretian!’

  (can be resolved…)

- More troubling: “This sentence is false.”

Recap from last time

...	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	...

- Turing-Post computational model:
  - Greatly simplified model
  - Infinite tape, each square either 0/1
  - Program = finite sequence of instructions (only 6 types!)
  - Unlike pseudocode, no conditionals or loops, only “GOTO”
  - code(*P*) = binary representation of program *P*

Motivation

Simplify!

(Get to the heart of the matter)

Doubling program

1. PRINT 0
2. GO LEFT
3. GO TO STEP 2 IF 1 SCANNED
4. PRINT 1
5. GO RIGHT
6. GO TO STEP 5 IF 1 SCANNED
7. PRINT 1
8. GO RIGHT
9. GO TO STEP 1 IF 1 SCANNED
10. STOP

Halting

...	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	...

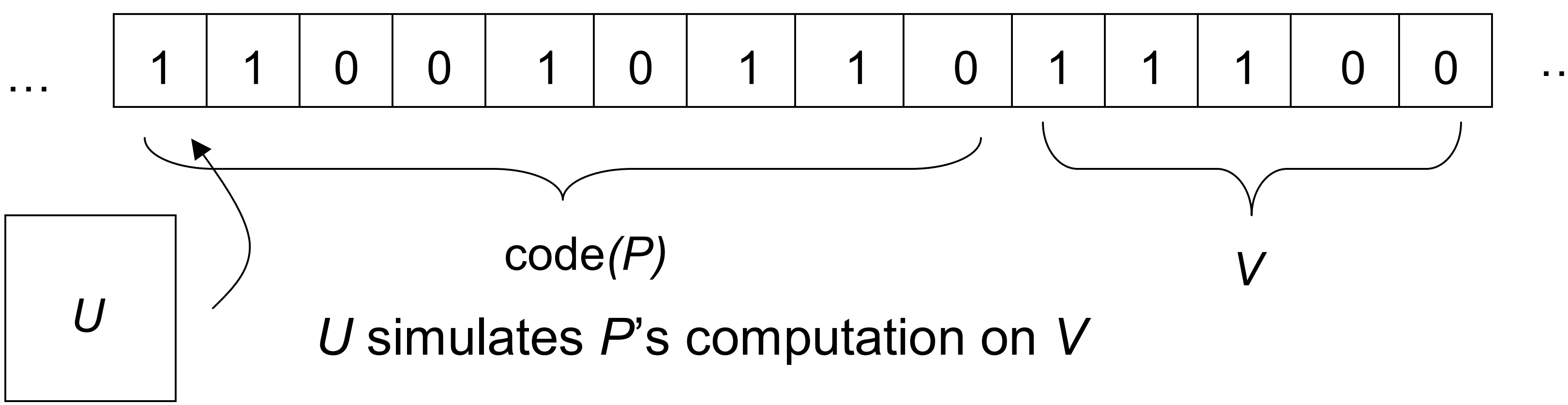
Program

1. PRINT 0
2. GO LEFT
3. GO TO STEP 2 IF 1 SCANNED
4. PRINT 1
5. GO RIGHT
6. GO TO STEP 5 IF 1 SCANNED
7. PRINT 1
8. GO RIGHT
9. GO TO STEP 1 IF 1 SCANNED
10. STOP

Program halts on this input data if STOP is executed in a finite number of steps

Some facts

- Fact 1: Every pseudocode program can be written as a T-P program, and vice versa
- Fact 2: There is a universal T-P program



Discussion

Is there a universal pseudocode program ?

How would you write it?

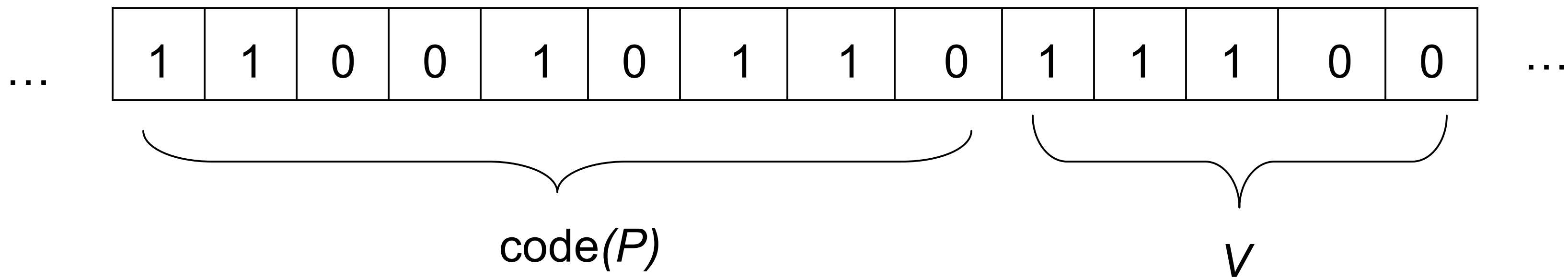
Composing programs P_1, P_2

Desired: A T-P program that, on input V:

- First runs P_1 on V
- If the previous step halts, runs P_2 on the new tape contents

Ideas??

Halting Problem



- Decide whether *P* halts on *V* or not
- Cannot be solved! Turing proved that no Turing-Post program can solve the Halting Problem

Proof (by contradiction)

- Suppose *H* is a Turing-Post program solving the Halting problem
- Use it to write a new program P_0:
  1. On input *V*, P_0 checks if *V* is the code to a Turing-Post program
  2. If not, HALT
  3. Else, use a Doubling Program to get *V,V*
  4. Run *H* on *V,V*
  5. If *H* says "doesn't halt", then HALT immediately
  6. Otherwise, go into an infinite loop

Proof (cont'd)

P_0

1. On input V, check if V is the code to a Turing-Post program
2. If not, HALT
3. Else, use a Doubling Program to get V,V
4. Run H on V,V
5. If H says "doesn't halt", then HALT immediately
6. Otherwise, go into an infinite loop

- But does P_0 halt on code(P_0)?
- If it doesn't, then at step 5 it should halt
- If it does, then it should reach step 6 and go into an infinite loop (i.e. not halt!)

Lessons to take away

- Computation is a very simple process
(can arise in unexpected places)

- Universal Program

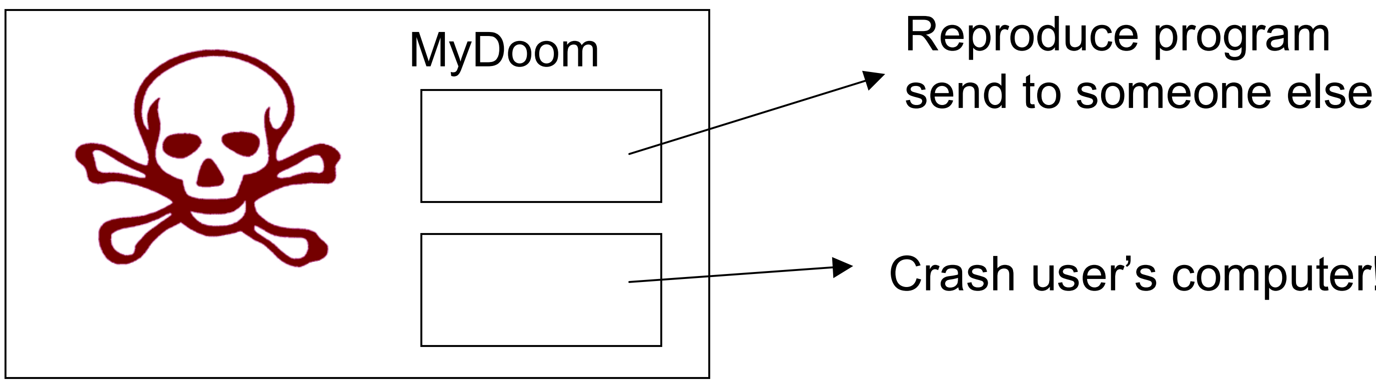
- No real boundary between hardware, software, and data

- No program that decides whether or not mathematical statements are theorems.

Age-old mystery: Self-reproduction.

How does the seed encode the whole?

Self-reproducing programs



- Fact: for every program *P*, there exists a program *P’* that has the exact same functionality except at the end it also prints code(*P’*) on the tape

Discussion next time: How to write a self-reproducing T-P program

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Hint: The idea is the following:

Write the following twice, the second time in quotes "Write the following twice, the second time in quotes"