“It ain’t no good if it ain’t snappy enough.” (Efficient Computations)

COS 116
2/21/2006
Instructor: Sanjeev Arora

Discussion

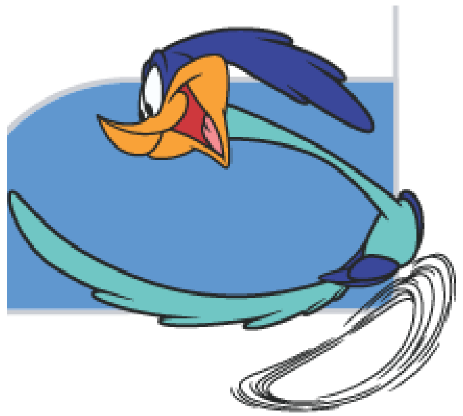
1. What different ways does Brooks describe for a robot to “orient” itself? Did your experiments with Scribbler give you insight into any of them?

2. In what ways (according to Brian Hayes) is the universe like a cellular automaton?

Bureaucratic stuff

- New blogging assignment for this week. (see handout).
- Reminder for this week’s lab: Come with robots,cables.
- Make sure you understand pseudocode. Precept Wed 7-8pm in Computer Science 102
- Scribbler Lab again in 2 weeks: make robot do some Kind of fine art (music, dance, sketching, etc.)
- Super cool guest lecturer on Thurs. (computer music)

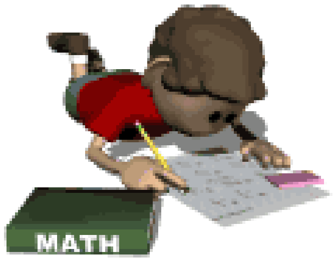
Question: How do we measure the “speed” of an algorithm?



- Ideally, should be machine and technology independent

“Running time” of an algorithm

- Definition: the number of “elementary operations” performed by the algorithm



- Elementary operations: +, -, *, /, assignment, evaluation of conditionals

- “Speed” of computer: number of elementary steps it can perform per second (NB. Simplified Definition.)

Example: Find Min

- *n* items, stored in array *A*
- Variables are *i*, *best*
- *best* ← 1
- for *i* = 2 to *n* do

```
{
      if ($A[\,i\,] < A[best]$) then
      { $best \leftarrow i$ }
}
```

2 operations per iteration:
1 comparison, 1 assignment

Uses at most 2(*n* – 1) + 1 operations

Number of iterations   Initialization

Selection Sort

```
Do for $i$ = 1 to $n – 1$
{
    Find cheapest bottle among those numbered $i$ to $n$
                                        ← About 2($n – i$) steps
    Swap that bottle and the $i$'th bottle.
                                        ← 3 steps
}
```

- For the i'th round, takes at most $2(n – i) + 3$
- To figure out running time, need to figure out how to sum up $(n – i)$ for $i = 1$ to $n – 1$

Gauss's trick : Sum of (n – i) for i = 1 to n – 1

$$\begin{aligned} S &= 1 + 2 + \dots + (n-2) + (n-1) \\ +\quad S &= (n-1) + (n-2) + \dots + 2 + 1 \\ 2S &= \underbrace{n + n + \dots + n + n}_{n-1 \text{ times}} \end{aligned}$$

$$S = n(n-1)/2$$

- So total time for selection sort is $\leq n(n-1) + 3n$

“20 Questions”: Guess the number I have in mind; you can only ask yes/no questions

- My number is an integer from 1 to *n*
- Binary Search Algorithm: First Question: “Is number ≥ *n* / 2?”
- Answer halves the range!

$$\underbrace{1, 2, \dots, \frac{n}{2} - 1,}_{\text{Repeat}} \underbrace{\frac{n}{2}, \frac{n}{2} + 1, \dots, n}_{\text{Repeat}}$$

Exercise: Express as pseudocode.

Brief detour: Logarithms (CS view)

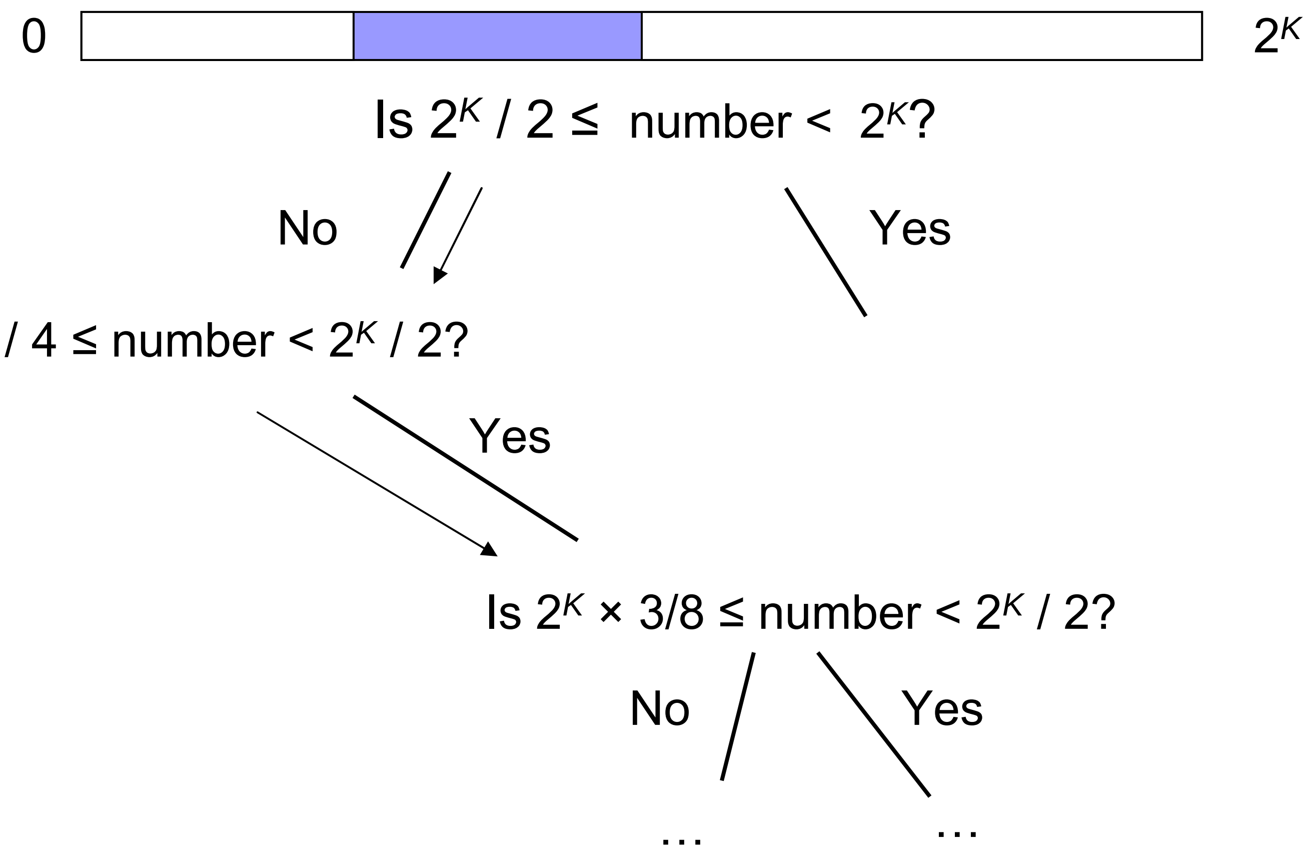
John Napier

- $\log_2 n = K$ means $2^{K-1} \leq n < 2^K$
- In words: K is the number of times you need to divide n by 2 in order to get a number ≤ 1

	n= 8	n= 1024	n= 1048576	n=8388608
$\log_2 n$	3	10	20	23
n	8	1024	1048576	8388608
n^2	64	1048576	1099511627776	70368744177664

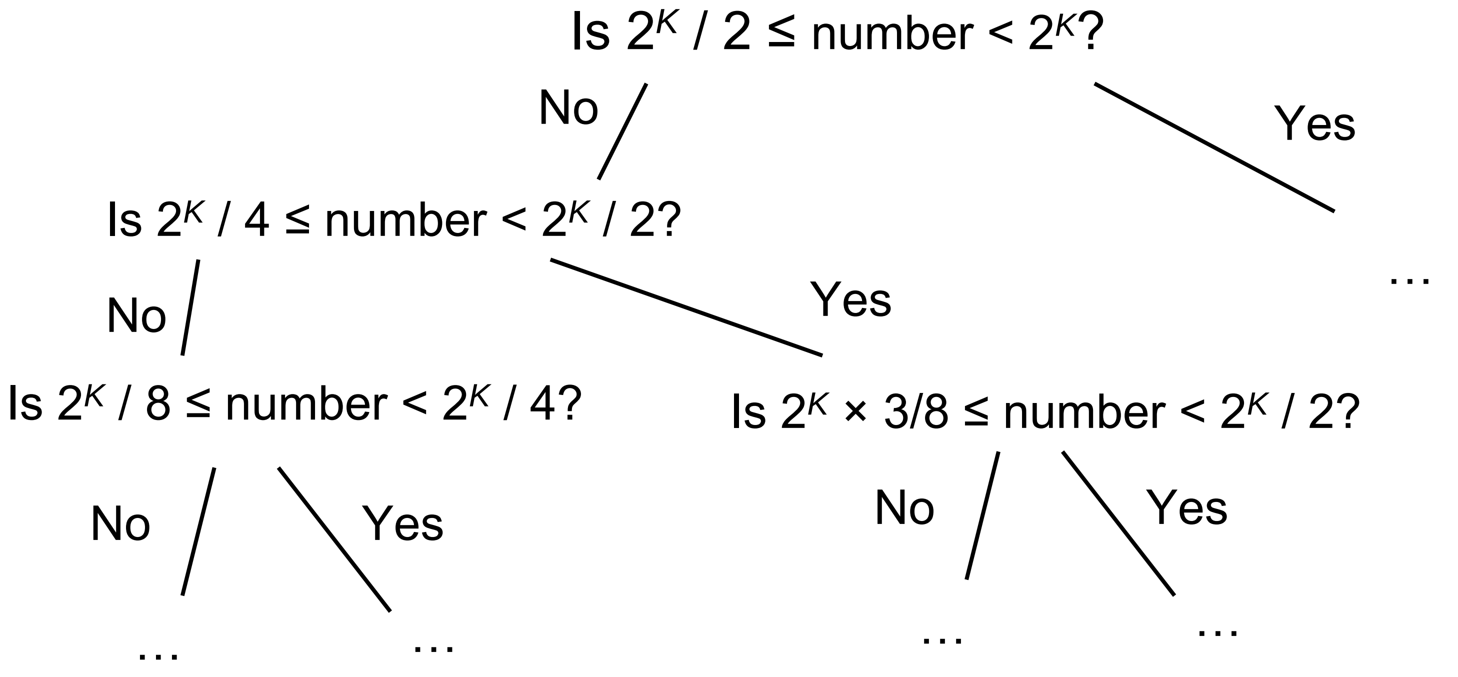
Binary search and binary representation of numbers

- Say we know $0 \leq \text{number} < 2^K$


0
2^K
Is $2^K / 2 \leq$ number $< 2^K$?
No
Yes
Is $2^K / 4 \leq$ number $< 2^K / 2$?
No
Yes
Is $2^K \times 3/8 \leq$ number $< 2^K / 2$?
No
Yes
…
…

Binary representations (cont'd)

- In general, each number uniquely represented by a sequence of yes/no answers to these questions.
- Correspond to paths down this tree:


Is $2^K / 2 \leq$ number $< 2^K$?
No
Yes
Is $2^K / 4 \leq$ number $< 2^K / 2$?
…
No
Yes
Is $2^K / 8 \leq$ number $< 2^K / 4$?
Is $2^K \times 3/8 \leq$ number $< 2^K / 2$?
No
Yes
No
Yes
…
…
…
…

Binary representation of *n* *(more standard definition)*

If

$$n = 2^k b_k + 2^{k-1} b_{k-1} + \ldots + 2\, b_2 + b_1$$

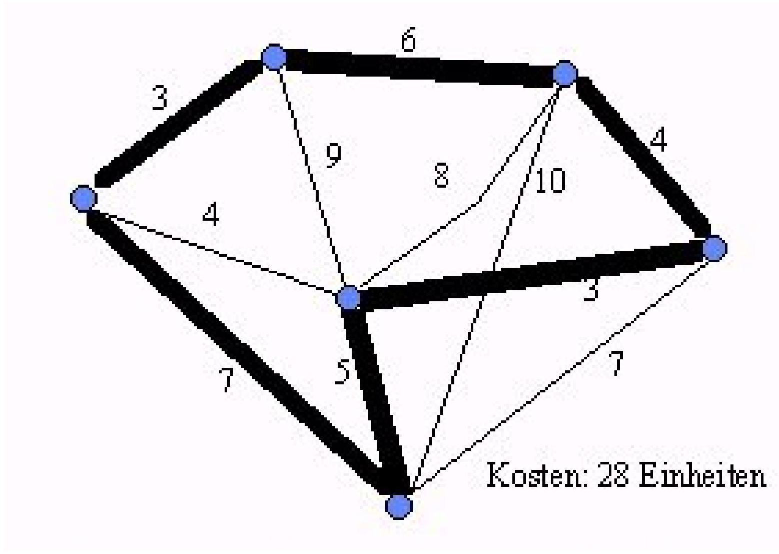
where the *b*'s are either 0 or 1)

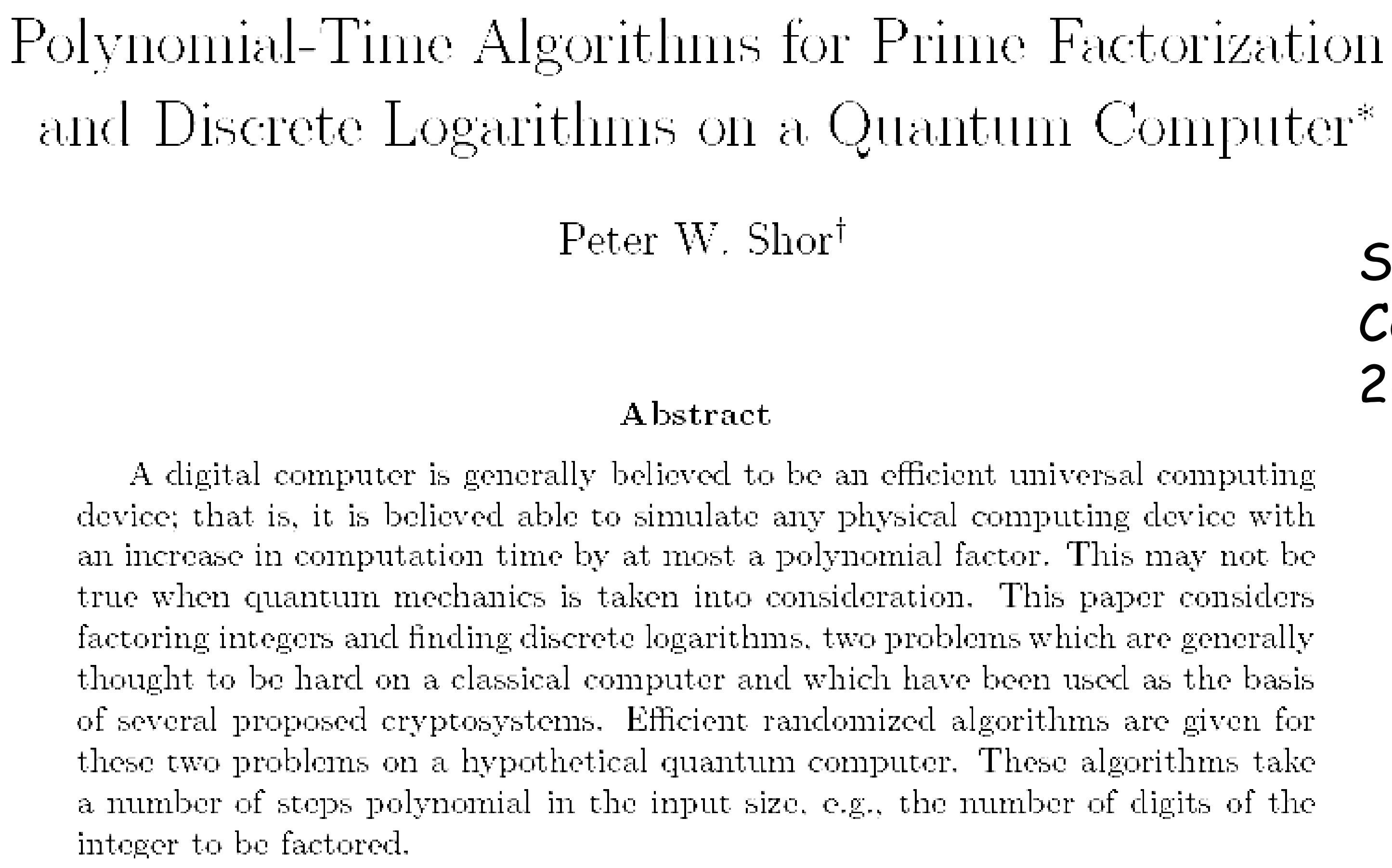
Then binary representation of n

$\lfloor n \rfloor_2 = b_k\, b_{k-1} \ldots b_2\, b_1$

Efficiency of Effort: A lens on the world

- QWERTY keyboard
- "UPS Truck Driver's Problem" (a.k.a. Traveling Salesman Problem or TSP)
- Handwriting Recognition
- CAPTCHA's
- Quantum Computing





Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor†

SIAM J. Computing 26(5) 1997

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

Can n particles do 2^n “operations” in a single step?
Or is Quantum Mechanics not quite correct.?