COS 226 Lecture 9: Hashing

Symbol Table, Dictionary
- records with keys
- INSERT
- SEARCH

Balanced trees, randomized trees
- use O(lgN) comparisons

Is lgN required?
- (no, and yes)

Are comparisons necessary?
- (no)

Hashing: basic plan

Save keys in a table, at a location determined by the key
KEY-INDEXED TABLE

HASH FUNCTION
- method for computing table index from key

COLLISION RESOLUTION STRATEGY
- algorithm and data structure to handle
  two keys that hash to the same index

Time-space tradeoff
- No space limitation:
  trivial hash function with key as address
- No time limitation:
  trivial collision resolution: sequential search
- Limitations on both time and space
  hashing

Hash function for short keys

Treat key as integer, use PRIME table size M
- h(K) = K mod M

Ex: four-character keys, table size 101

```
bin   01100001011000100110001101100100
hex      6   1   6   2   6   3   6   4
ascii        a       b       c       d
```

Key ``abcd'' hashes to 11

```
0x61626364 = 1633831724
16338831724 % 101 = 11
```

Key ``dcba'' hashes to 57

```
0x64636261 = 1684234849
1633883172 % 101 = 57
```

Key ``abbc'' also hashes to 57

```
0x61626263 = 1633837667
1633837667 % 101 = 57
```

Obvious point:
- huge number of keys, small table: most collide!

Hash function for long keys (strings)

Same function: h(K) = K mod M

Need multiprecision arithmetic calculation
- Use Horner's method

Ex: (check with 4 chars; works for any length)

```
hex      6   1   6   2   6   3   6   4
ascii        a       b       c       d
```

```
0x61626364 = 256*(256*(256*97+98)+99)+100
```

take mod after each multiplication:

```
256*97+98   = 24930 % 101 = 84
256*84+99   = 21603 % 101 = 90
256*90+100  = 23140 % 101 = 11
```

String hash function implementation

```
int hash(char *v, int M)
  { int h, a = 117;
    for (h = 0; *v != ’ ’; v++)
      h = (a*h + *v) % M;
    return h;
  }
```

Scramble by replacing 256 by 117

Uniform hashing:

- use a different random value for each digit

Collisions

N keys, table size M

How many insertions until the first collision?

BIRTHDAY PARADOX (classical probability theory)

- Assume hash function "random"
- Expected insertions to first collision (table size M):

M	sqrt(pi M/2)
100	12
1000	40
10000	125

Option 1: Allow N >> M

- put keys hashing to i in a list
- about N/M keys per list

Option 2: Keep N < M

- put keys somewhere in table
- complex collision pattern

Collisions (continued)

Experiment 1:

- generate random probes between 0 and 100
- 84 35 45 32 89 1 58 16 38 69 5 90 16 53 61 ...
- collision at 13th as predicted

Experiment 2:

- use hash function to scatter 4-char keys

bcba	47	ccad	1	baca	26	abad	4
bddc	43	bdac	83	dbcb	24	cada	85
dabc	85	dabb	84	dbab	17	dabd	86
dbdb	78	dcbd	60	dbdd	80		
babb	74	bccc	2	addd	39		
bcbd	50	adbc	31	bcda	55		

collision after 20 probes

- still as predicted (standard dev. not small)

Separate chaining

Simple, practical, widely used

Cuts search time by a factor of M over sequential search

Method: M linked lists, one for each table

```
.     0:    *
.     1:    L    A    A    A    *
.     2:    M    X    *
.     3:    N    C    *
.     4:    *
.     5:    E    P    E    E    *
.     6:    *
.     7:    G    R    *
.     8:    H    S    *
.     9:    I    *
.    10:    *
```

Separate chaining analysis

Insert cost: 1

Avg. search cost (successful): N/2M

Avg. search cost (unsuccessful): N/M

Classical balls-and-urns ``occupancy`` problem
- Probability that some list length is > t(N/M)
  exponentially small in t
- Long lists unlikely PROVIDED hash is random
- [Analysis doesn't account for bugs or bad hashes]

M large: CONSTANT avg. search time
- independent of how keys are distributed (!)

Keep lists sorted?
- increases insert time to N/2M
- cuts unsuccessful search time to N/2M

Linear Probing

No links, keep everything in table

Method: start linear search at hash position
- (stop when empty position hit)

Still get O(1) avg. search time if table sparse

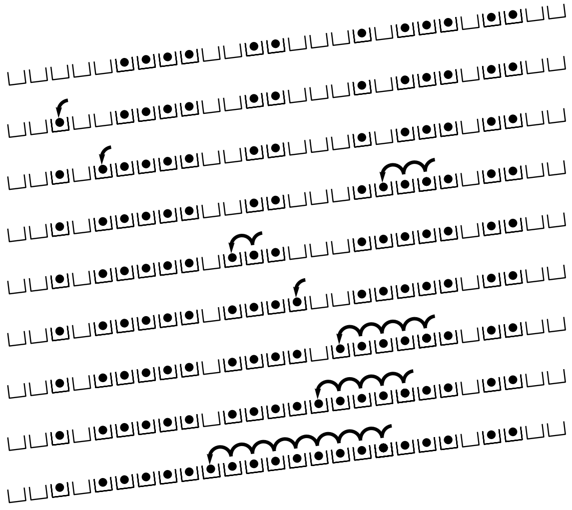
Very sparse table: like separate chaining

As table fills up: CLUSTERING occurs
- (infinite loop on full table)

Linear probing code

```
void STinit(int max)
  { int i;
    N = 0; M = 2*max;
    st = malloc(M*sizeof(Item));
    for (i = 0; i < M; i++) st[i] = NULLitem;
  }
void STinsert(Item item)
  { Key v = key(item);
    int i = hash(v, M);
    while (!null(i)) i = (i+1) % M;
    st[i] = item; N++;
  }
Item STsearch(Key v)
  { int i = hash(v, M);
    while (!null(i))
      if eq(v, key(st[i])) return st[i];
      else i = (i+1) % M;
    return NULLitem;
  }
```

Linear probing example



Linear probing analysis

CLUSTERING

- bad phenomenon: items clump together
- long clusters tend to get longer
- avg. search cost grows to M as table fills

Precise analysis very difficult.

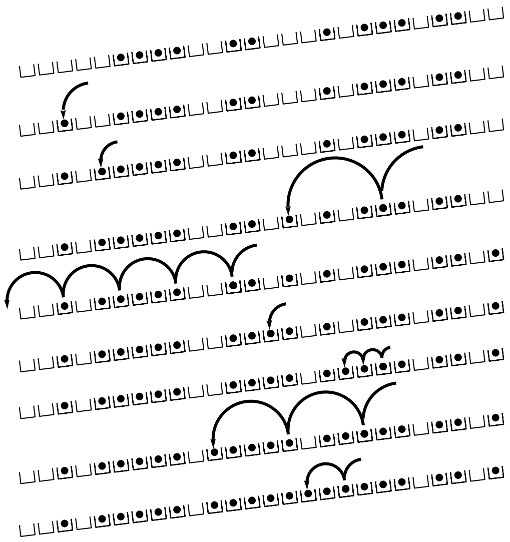
THM (Knuth):

- Insert cost: approx. (1+ 1/(1-N/M)^2)/2
- Search cost (hit): approx. (1+ 1/(1-N/M))/2
- Search cost (miss): same as insert

Too slow when table gets 70%-80% full

Double Hashing

Avoid clustering by using 2nd hash to compute skip for search



Double Hashing analysis

Extremely difficult

THM: (Guibas-Szemeredi) Nearly equivalent to random probe ideal

- Insert cost: approx. 1/(1-N/M)
- Search cost (hit): approx. ln(1+N/M)/(N/M)
- Search cost (miss): same as insert

Not too slow until table gets 90%-95% full

Amortized analysis of algorithms

Measure running time for X operations by

- (total cost of all X operations)/ X

Ex:

- insert N elements in a heap:
  (lg1 + lg2 + ... + lgN) / N = lgN + O(1)

Ex:

- insert N elements in a binomial queue:
  (1*N/2 + 2*N/4 + 3*N/8 +...)/N < 2

Worst case for a SEQUENCE of operations

- guarantee bound on TOTAL
  (same as cost per operation)
- individual operation may be slow

Dynamic hashing

Hashing:

- grow table while keeping search cost $O(1)$
- when number of keys in table doubles
    rebuild to double the size of the table

Ex: separate chaining

- avg search cost < 2
- $4M$ keys in table of size M
- proof by induction: amortized cost < 2
    cost to build: $x*4M$
    cost to rebuild to new table size $2M$: $4M$
    amortized cost of first $8M$ insertions:
        $(x*4M + 4M + 4M)/8M$
        $x/2 + 1 < x$

Same argument works for other basic ADTs!

Ex: stacks, queues in arrays, double hashing

Separate chaining vs. double hashing

Space for separate chaining w/ rehashing

- $4M$ keys (or links to keys)
- M table links (approx same size as keys)
- $4M$ links in nodes
- Total space: $9M$ words for $4M$ items
- Avg search time: 2

Double hashing in same space

- $4M$ items, table size $9M$
- avg search time: $1/(1-4/9) = 1.8$ (10% faster)

Double hashing in same time

- $4M$ items, avg search time 2
- space needed: $8M$ words $(1/(1-4/8) = 2)$ (11% less)

Separate chaining advantages

- idiot-proof (doesn't break)
- no large chunks of memory (is that good?)

Other ST ADT operations

DELETION

- Separate chaining: trivial
- Linear probing: rehash keys in cluster
    or use indirect method (see below)
- Double hashing: no easy direct method
    mark deleted nodes as "deadwood"
    rebuild periodically to clear deadwood

SORT, FIND kth largest

- Separate chaining w/ sorted lists
- Linear probing/double hashing
    have to do full sort

JOIN

- Separate chaining: easy
- Linear probing/double hashing:
    rehash whole table

Reasons not to use hashing

Hashing achieve ST ADT implementation goal

- search and insert in constant time.

Why use anything else?

- no performance guarantee
- too much arithmetic on long keys
- takes extra space
- doesn't support all ADT ops efficiently
- compare abstraction works for partial order
    (searching without keys)

Other hashing variants

Perfect hashing

- fixed set of keys
- hash function with no collisions
- good hack for small tables
- not practical for large tables
- totally static

Coalesced hashing

- properly account for link space
- mix hash table, storage allocation

Ordered hashing

- cut costs in half as with ordered lists

Brent's variation

- guarantee constant search cost
- up to M insert cost