

Morphometrics

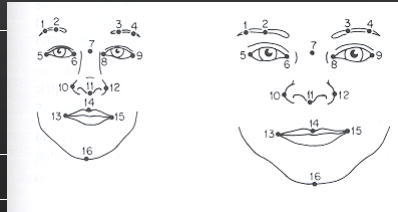
cs598b, geometric modeling
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Morphometrics

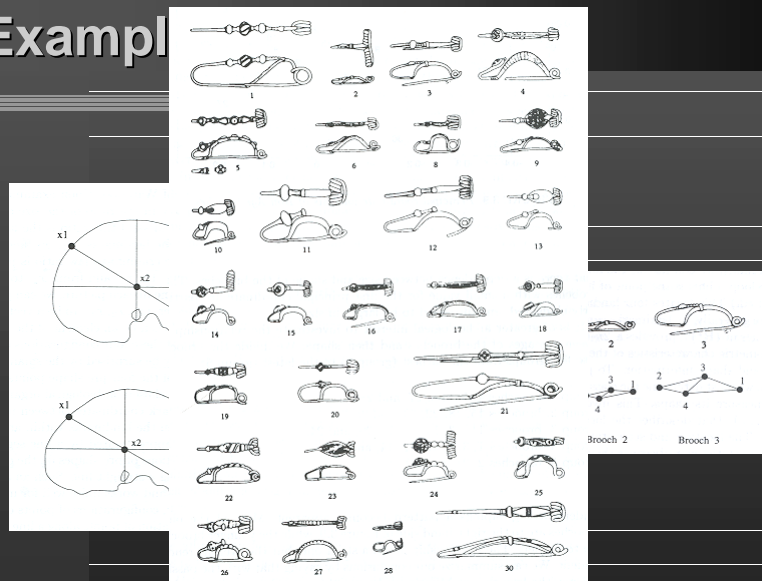
- the study of covariances of biological form
- specialized application of multivariate statistics
- (Fred L. Bookstein, “Morphometric tools for landmark data”)

Landmarks

- feature points
- use to compare changes in biological shape
- carefully chosen

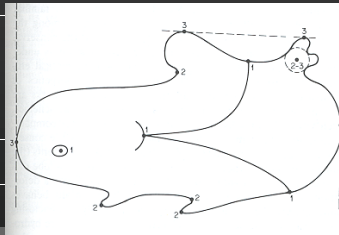


Example



Landmarks, II

- **example candidates:**
 - juxtaposition
 - maximum curvature
 - extremal points (e.g. endpoints of diameters, centroids, ...)

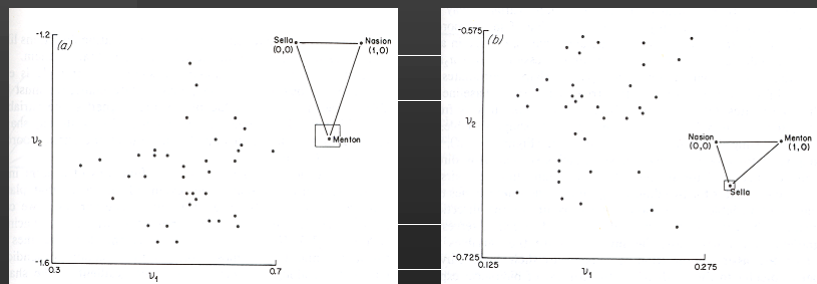


Shape coordinates

- **again, we want coordinates independent of translation, scale and rotation**
- **choose representative set of landmarks as basis**
- **e.g. in triangle: use one edge as “baseline”**

Scatter of shape coordinates

- if we choose a different baseline, the scatter differs mainly by translation, rotation and scale



More shape statistics

- Christopher Small, “The Statistical Theory of Shape”
- shapes as equivalence classes
- shapes are points on a *shape manifold*

Pre-shape

- vector of n normalized landmarks is called *pre-shape*
- lies on $(2n - 3)$ dimensional sphere called *pre-shape space*

Orientation

- there is no continuous orientation function for every shape
- *orbit* is equivalence class of shapes w.r.t. rotation
- want to find distance metric between orbits: minimum angle between all pairs of shapes from two orbits

Shape comparison

- find landmark correspondence which minimizes a distance measure
 - Procrustean distance
 - bending energy of thin-plate spline

Procrustean distance

- view n landmarks coordinates as complex numbers:

$$\tau_j = (\tau_{j1}, \tau_{j2}, \dots, \tau_{jn})$$

- Procrustean distance:

$$d(\sigma_1, \sigma_2) = \cos^{-1} \left(\left| \sum_{k=1}^n \tau_{1k} \tau_{2k}^* \right| \right)$$

- independent of orientation