CSG-based 3-D Object Recognition

Robert Osada
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Main Tasks

- Reconstruct model from data
  - Segment data into surface patches
  - Derive precedence graph
- Match models
  - Setup correspondence network
  - Define energy functional
  - Optimize

Segmentation

- Paper
  - [P. Besl, R. Jain] Segmentation Through Variable-Order Surface Fitting (IEEE 1988)
- Goals
  - Decompose surface into patches
    - Each patch is a low-order bivariate polynomial $f = a_{00} + a_{10}x + a_{01}y + a_{11}xy + \ldots$
  - Algorithm based <u>only</u> on the assumption of *surface coherence*
  - Minimize total approximation error and number of regions

Segmentation

- Algorithm – initial guess
  - Compute partial derivatives
    - Smoothing and convolution operators
  - Compute mean curvature and Guassian curvature
    - Rotation, translation, scaling, and parameterization invariant

Segmentation

- Use signs of two curvatures to label surface types
  - $3^2=9$ types

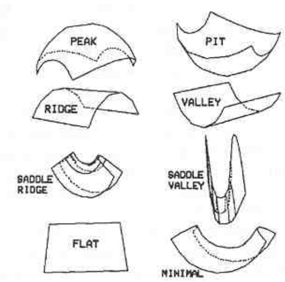


Fig. 1. Eight fundamental surface types from surface curvature sign.

	K>0	K=0	K<0
H<0	Peak	Ridge	Saddle Ridge
H=0	?	Flat	Minimal Surface
H>0	Pit	Valley	Saddle Valley

[P. Besl]

Segmentation

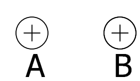
- Algorithm – iterative growth
  - Isolate largest connected component of same type
  - Erode region to small seed region
  - Fit plane
  - Grow region
    - Adaptively add/remove points based on error thresholds
    - Raise surface order if necessary
- Results
  - Successful in variety (40) of test <u>images</u>

Precedence Graphs

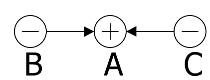
- Paper
  - [T. Chen, W. Lin] A Neural Network Approach to CSG-Based 3-D Object Recognition (IEEE 1994)
- Definition
  - Nodes correspond to positive or negative primitives
  - Arcs indicate order of participation (precedence) of primitives
    - Directed edges represent difference operator
    - No representation of union or intersection

Precedence Graphs

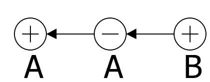
- Examples
  - $A \cup B$



  - $A - (B \cup C)$



  - $A \cap B = A - (A\text{-}B)$



- Uniqueness?
  - Maybe not, but less constrictive than CSG's

Precedence Graphs

- Additional geometric information
  - Undirected edges represent positive objects belonging to same object
    - Connected components define sub-objects
- Derivation
  - Assume quadratic patches
    - $ax^2+by^2+cz^2+dxy+eyz+fzx+gx+hy+iz = 1$
  - Translate and rotate coordinates
    - $a'x'^2+b'y'^2+c'z'^2+g'= 1$

Precedence Graphs

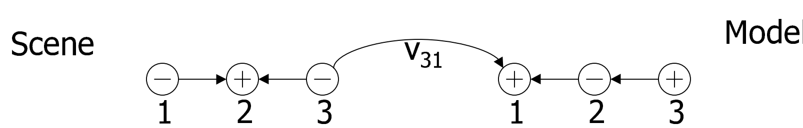
- Derivation
  - Combine planes to form primitives
    - Ellipsoid, cylinder, cone, box, plane
    - All primitives are convex, so if surface is concave then primitive is negative
  - Establish relations between primitives
    - If a primitive is inside an oppositely signed primitive, add a directed edge
    - If two positive primitives are neighbors, add an undirected edge

Matching

- Network
- Constraints
- Energy
- Optimization
- Similarity rate

Matching Network

- Two-dimensional array. v_{xi} represents the degree of supporting the proposition that primitive x in the scene matches primitive i in the model
  - $0 <= v_{xi} <= 1$
  - Lift concept of matching from discrete domain to fuzzy domain



Matching Constraints

- Validity of match
  - A scene node can map to only one model node, and a model node cannot be mapped by more than one scene node
- Similarity of primitives
  - Same sign, same type, similar size
- Preservation of precedence graph
  - Primitives should have the same precedence
  - Gluing arcs should correspond
- Preservation of geometry
  - Minimize distances between reference points and angles between reference vectors of primitives

Matching Energy

$$E = \tfrac{A}{2} \sum_{x,i,y\neq x} v_{xi} v_{yi} + \tfrac{B}{2} \sum_{x,i} N_{xi} v_{xi} + \tfrac{C}{2} \sum_{x,i,y\neq x,\, j\neq i} M_{xiyj} v_{xi} v_{yj}$$

- Each term enforces difference matching constraints

Matching Energy

$$E = \tfrac{A}{2}\sum_{x,i,y\neq x} v_{xi}v_{yi} + \tfrac{B}{2}\sum_{x,i} N_{xi}v_{xi} + \tfrac{C}{2}\sum_{x,i,y\neq x,j\neq i} M_{xiyj}v_{xi}v_{yj}$$

- Discourages two scene primitives from mapping to the same model primitive

Matching Energy

$$E = \tfrac{A}{2}\sum_{x,i,y\neq x} v_{xi}v_{yi} + \tfrac{B}{2}\sum_{x,i} N_{xi}v_{xi} + \tfrac{C}{2}\sum_{x,i,y\neq x,j\neq i} M_{xiyj}v_{xi}v_{yj}$$

- Enforces primitive similarity constraint
  - N_{xi} compares types, sizes, and signs of scene primitive x and model primitive i
    - -1 if similar, 1 otherwise

Matching Energy

$$E = \tfrac{A}{2} \sum_{x,i,y \neq x} v_{xi} v_{yi} + \tfrac{B}{2} \sum_{x,i} N_{xi} v_{xi} + \tfrac{C}{2} \sum_{x,i,y \neq x, j \neq i} M_{xiyj} v_{xi} v_{yj}$$

- Enforces precedence and geometry constraints
  - M_{xiyj}=-1 initially
  - Set to 1 if graph topology is violated
  - Set to 1 if distances between reference points or orientations are above threshold

Optimization

- Papers
  - [S. Kirkpatrick, et al] Optimization by Simulated Annealing (Science 1983)
  - [G. Bilbro, et al] Optimization by Mean Field Annealing (Advances in Neural Information Processing Systems, 1989)
- Iterative algorithms
  - Can optimize non-linear functions, but… get stuck in local optimum

Optimization Theory

- Simulated annealing
  - Crystallization Analogy
    - Carefully lower temperature (annealing)
  - Algorithm
    - Given objective function and concise description of configuration, generate rearrangements of configuration
    - If $\Delta E<=0$, accept new configuration, if $\Delta E>0$, accept with probability $\sim\exp(-\Delta E/T)$
    - T follows annealing schedule
  - Results
    - Temperatures prevents algorithm from getting stuck, provides extra control, finds better solutions
    - Applied to a wide variety of NP-complete problems, proves a "natural framework for heuristic design"

Optimization Theory

- Mean field annealing
  - Similar to simulated annealing, but replaces discrete degrees of freedom with averages values
  - Mean field = effective field
    - How does a variable/parameter/spin effect the energy?
    - Calculate the interaction of a single variable and the rest of the system
  - Update to "expected value" based on mean field approximation
    - Details in paper… thermodynamic background helps
  - Claim that equilibrium is achieved 1-2 orders faster in certain applications

Matching Optimization

$$E = \tfrac{A}{2}\sum_{x,i,y\neq x} v_{xi}v_{yi} + \tfrac{B}{2}\sum_{x,i} N_{xi}v_{xi} + \tfrac{C}{2}\sum_{x,i,y\neq x,j\neq i} M_{xiyj}v_{xi}v_{yj}$$

$$= \sum_{x,i} v_{xi}\left(\tfrac{A}{2}\sum_{y\neq x} v_{yi} + \tfrac{B}{2}\sum_{x,i} N_{xi} + \tfrac{C}{2}\sum_{x,i,y\neq x,j\neq i} M_{xiyj}v_{yj}\right)$$

$$= \sum_{x,i} v_{xi}E_{xi}$$

- Isolate interaction a single variable and the rest of the system

Matching Optimization

- Pick random row, update v_{xi} according to mean field annealing
- Repeat until convergence

$$v_{xi} \propto \exp(-\tfrac{E_{xi}}{T})$$

$$v_{xi} = \exp(-\tfrac{E_{xi}}{T}) / \sum_{j} \exp(-\tfrac{E_{xj}}{T})$$

Matching Similarity Rate

- After optimization, consider objects matched if $v_{xi}>e_1$, not matched if $v_{xi}<e_2$, undecided otherwise
- Estimate 3D transforms between matched primitives
- Match error

$$-\frac{\#match}{\#primitives}+w_1 Err_{rot}+w_2 Err_{trans}$$

Ideas

- Differential geometry used to help segment surfaces
- Fuzzy correspondences between primitives
- Energy functions combines matching constraints
- Standard optimization techniques used to find best matching