Lecture 17. Analysis of Algorithms

- **An *algorithm* is a 'method' for solving a problem that is *independent* of a specific computer or programming language**
- ***Design*: Finding a way to solve the problem**
- ***Analysis*: Determining the algorithm's cost in machine-independent terms, e.g. lg *N***
- **Need to make a program faster?**

  Get a new machine

  - **Costs $$$ or more**
  - **Makes 'everything' run faster**
  - **But, it may — or *may not* — have much impact on a specific problem**

  Get a new algorithm

  - **Costs ¢ or less**
  - **Can make or break a specific problem by allowing it to be solved at all**
  - **But, it may have *little or no* impact on 'everything'**

Sublist Sum Problem

- **Given a list of numbers, find the contiguous sublist that has the largest sum**

31	-41	59	26	-53	58	97	-93	-23	84	
						187				
31	*31*									
31	-41	59	*49*							
31	-41	59	26	*75*						
31	-41	59	26	-53	*22*					
31	-41	59	26	-53	58	*80*				
31	-41	59	26	-53	58	97	*177*			
31	-41	59	26	-53	58	97	-93	*84*		
31	-41	59	26	-53	58	97	-93	-23	*61*	
31	-41	59	26	-53	58	97	-93	-23	84	*145*

- **Easy if all the numbers are nonnegative; tricky when some numbers are negative**
- **Sums must be positive; negative sublist sums are taken to be zero**

A Simple Brute-Force Solution

- **Try all possible sublists of n integers: `x[lb..ub]` for all `lb`, ub from 0 to n**

```
void sublist(int x[], int n) {
    int lb, ub, l, r, max = 0;

    for (lb = 0; lb < n; lb++)
        for (ub = lb; ub < n; ub++) {
            int i, sum = 0;
            for (i = lb; i <= ub; i++)
                sum += x[i];
            if (sum > max) {
                max = sum;
                l = lb;
                r = ub;
            }
        }
    printf("x[%d..%d] = %d\n", l, r, max);
}
```

```
% lcc -I/u/cs126/include sublistn3.c /u/cs126/lib/libmisc.a
% echo 31 -41 59 26 -53 58 97 -93 -23 84 | a.out
x[2..6] = 187
```

Profiling

- **Program *profiles* help understand execution *frequencies*; use `lcc -b` and `bprint`**

```
% lcc -b -I/u/cs126/include sublistn3.c /u/cs126/lib/libmisc.a
% echo 31 -41 59 26 -53 58 97 -93 -23 84 | a.out
x[2..6] = 187
% bprint
...
    ❶   for (<1>lb = 0; <11>lb < n; <10>lb++)
        ❷   for (<10>ub = lb; <65>ub < n; <55>ub++) {
                int i, sum = <55>0;
            ❸   for (<55>i = lb; <275>i <= ub; <220>i++)
                    <220>sum += x[i];
                if (<55>sum > max) {
                    <6>max = sum;
                    <6>l = lb;
                    <6>r = ub;
                }
        }
        <1>printf("x[%d..%d] = %d\n", l, r, max);
```

- **For *N* = 10**

Loop	❶	**is executed**	**11 ≈ 10^1**	**times**
	❷		**65 ≈ $10^2/2$**	
	❸		**275 ≈ $10^3/3$**	

Execution time ≈ N^3, can't solve $N = 10{,}000$, since 10^{12} microseconds ≈ 11 days

A Better Algorithm

- **Don’t recompute the whole sum every time**

`x[lb] + x[lb+1] + ... + x[ub]` = `(x[lb] + ... + x[ub-1])` **`+ x[ub]`**

```
void sublist(int x[], int n) {
    int lb, ub, l, r, max = 0;

    for (lb = 0; lb < n; lb++) {
        int sum = 0;
        for (ub = lb; ub < n; ub++) {
            sum += x[ub];
            if (sum > max) {
                max = sum;
                l = lb;
                r = ub;
            }
        }
    }
    printf("x[%d..%d] =
}
```

31	-41	59	26	-53	58	97	-93	-23	84
31	-10	**49**	**75**	22	**80**	**177**	84	61	145
	-41	18	44	-9	49	146	53	30	114
		59	85	32	90	**187**	94	71	155
			26	-27	31	128	35	12	96
				-53	5	102	9	-14	70
					58	155	62	39	123
						97	4	-19	65
							-93	-116	-32
								-23	61
									84

Profiling the Better Algorithm

```
❶     for (<1>lb = 0; <11>lb < n; <10>lb++) {
          int sum = <10>0;
❷         for (<10>ub = lb; <65>ub < n; <55>ub++) {
              <55>sum += x[ub];
              if (<55>sum > max) {
                  <6>max = sum;
                  <6>l = lb;
                  <6>r = ub;
              }
          }
      }
      <1>printf("x[%d..%d] = %d\n", l, r, max);
```

- **For $N = 10$**

Loop	❶	**is executed**	**$11 \approx 10^1$**	**times**
	❷		**$65 \approx 10^2/2$**	

Execution time $\approx N^2$, but can't solve $N = 1{,}000{,}000$, because 10^{12} microseconds $\approx$ 11 days

- **There is a divide-and-conquer algorithm that takes $\approx N \lg N$, but there's even a better way**

The Optimal Algorithm

- **Keep track of the maximum sum so far *and* the sum of the sublist that ends at `x[i]`**

  Suppose `max` is the maximum sum in `x[0..i-1]`; extend that solution to `x[i]`

31	-41	59	26	-53	58	97	-93	-23	84
			85	*32*					

31	-41	59	26	-53	58	97	-93	-23	84
					90				

```
void sublist(int x[], int n) {
     int i, l, r, max = 0, maxi = 0;

     for (i = 0; i < n; i++) {
          if (maxi + x[i] > 0)
               maxi += x[i];
          } else
               maxi = 0;
          if (maxi > max)
               max = maxi;
     }
     printf("x[%d..%d] = %d\n", l, r, max);
}
```

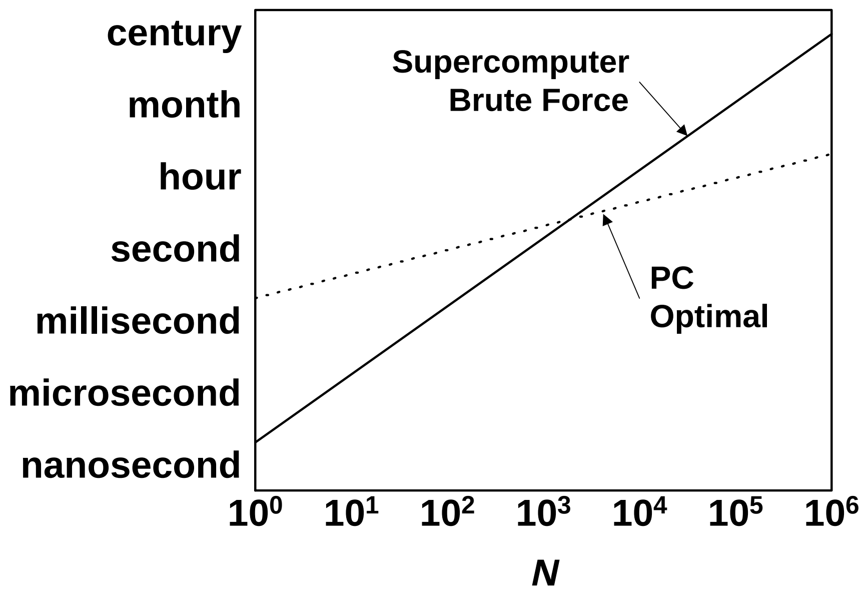
31	-41	59	26	-53	58	97	-93	-23	84
31	*0*	*59*	*85*	*32*	*90*	*187*	*94*	*71*	*155*
31	*31*	*59*	*85*	*85*	*90*	*187*	*187*	*187*	*187*

- **Execution time ≈ *N*, because there's just one loop; *N* = 1,000,000 takes ≈ 1 second**
- **See `sublistn.c` for details of computing `l` and `r`**

Summary

- A good algorithm can be more powerful than a supercomputer

		Thousand	Million
Brute Force	N^3	17 min	300 centuries!
Better	N^2	1 sec	11 days
Divide and Conquer	$N \lg N$	0.01 sec	20 sec
Optimal	N	0.001 sec	1 sec



- For more, see J. Bentley, *Programming Pearls*, Addison Wesley, 1986