OCaml Datatypes

COS 326
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OCaml So Far

- We have seen a number of basic types:
  - int
  - float
  - char
  - string
  - bool
- We have seen a few structured types:
  - pairs
  - tuples
  - options
  - lists
- In this lecture, we will see some more general ways to define our own new types and data structures

Type Abbreviations

- We have already seen some type abbreviations:

```
type point = float * float
```

- These abbreviations can be helpful documentation:

```
let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

- But they add nothing of *substance* to the language
  - they are equal in every way to an existing type

Type Abbreviations

- We have already seen some type abbreviations:

```
type point = float * float
```

- As far as OCaml is concerned, you could have written:

```
let distance (p1:float*float)
             (p2:float*float) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

- Since the types are equal, you can *substitute* the definition for the name wherever you want
  - we have not added any new data structures

Data types

- OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives

```
type my_bool = Tru | Fal
```

a value with type my_bool is one of two things:

- Tru, or
- Fal

read the "|" as "or"

Data types

- OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives

```
type my_bool = Tru | Fal
```

Tru and Fal are called "constructors"

a value with type my_bool is one of two things:

- Tru, or
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read the "|" as "or"

Data types

- OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives

```
type my_bool = Tru | Fal
```

```
type color = Blue | Yellow | Green | Red
```

there's no need to stop at 2 cases; define as many alternatives as you want

Data types

- OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives

```
type my_bool = Tru | Fal
```

```
type color = Blue | Yellow | Green | Red
```

- Creating values:

```
let b1 : my_bool = Tru
let b2 : my_bool = Fal
let c1 : color = Yellow
let c2 : color = Red
```

use constructors to create values

Data types

```
type color = Blue | Yellow | Green | Red

let c1 : color = Yellow
let c2 : color = Red
```

- Using data type values:

```
let print_color (c:color) : unit =
  match c with
  | Blue ->
  | Yellow ->
  | Green ->
  | Red ->
```

use pattern matching to determine which color you have; act accordingly

Data types

```
type color = Blue | Yellow | Green | Red

let c1 : color = Yellow
let c2 : color = Red
```

- Using data type values:

```
let print_color (c:color) : unit =
  match c with
  | Blue -> print_string "blue"
  | Yellow -> print_string "yellow"
  | Green -> print_string "green"
  | Red -> print_string "red"
```

Data types

```
type color = Blue | Yellow | Green | Red

let c1 : color = Yellow
let c2 : color = Red
```

- Using data type values:

```
let print_color (c:color) : unit =
  match c with
  | Blue -> print_string "blue"
  | Yellow -> print_string "yellow"
  | Green -> print_string "green"
  | Red -> print_string "red"
```

Why not just use strings to represent colors instead of defining a new type?

Data types

```
type color = Blue | Yellow | Green | Red
```

oops!:

```
let print_color (c:color) : unit =
  match c with
  | Blue -> print_string "blue"
  | Yellow -> print_string "yellow"
  | Red -> print_string "red"
```

Warning 8: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched:
Green

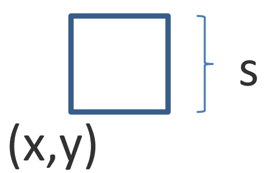
OCaml's datatype mechanism allow you to create types that contain *precisely* the values you want!

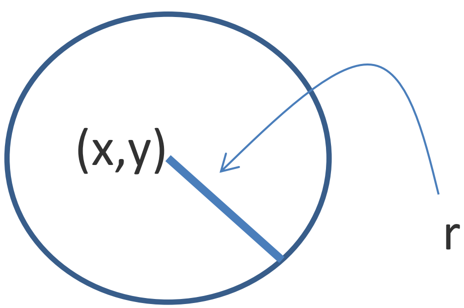
Data Types Can Carry Additional Values

- Data types are more than just enumerations of constants:

```
type point = float * float

type simple_shape =
  Circle of point * float
| Square of point * float
```

- Read as: a simple_shape is either:
  - a Circle, which contains a pair of a point and float, or
  - a Square, which contains a pair of a point and float





Data Types Can Carry Additional Values

- Data types are more than just enumerations of constants:

```
type point = float * float

type simple_shape =
  Circle of point * float
| Square of point * float

let origin : point = (0.0, 0.0)

let circ1  : simple_shape = Circle (origin, 1.0)
let circ2  : simple_shape = Circle ((1.0, 1.0), 5.0)
let square : simple_shape = Square (origin, 2.3)
```

Data Types Can Carry Additional Values

- Data types are more than just enumerations of constants:

```
type point = float * float

type simple_shape =
  Circle of point * float
| Square of point * float

let simple_area (s:simple_shape) : float =
  match s with
  | Circle (_, radius) -> 3.14 *. radius *. radius
  | Square (_, side) -> side *. side
```

Compare

- Data types are more than just enumerations of constants:

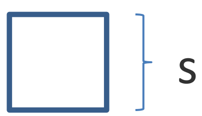
```
type point = float * float

type simple_shape =
  Circle of point * float
| Square of point * float

let simple_area (s:simple_shape) : float =
  match s with
  | Circle (_, radius) -> 3.14 *. radius *. radius
  | Square (_, side) -> side *. side
```

```
type my_shape = point * float

let simple_area (s:my_shape) : float =
  (3.14 *. radius *. radius)  ?? or ??  (side *. side)
```

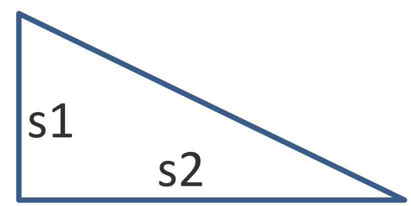
More General Shapes

```
type point = float * float

type shape =
    Square of float
  | Ellipse of float * float
  | RtTriangle of float * float
  | Polygon of point list
```

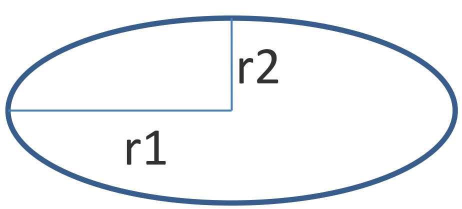
Square s =



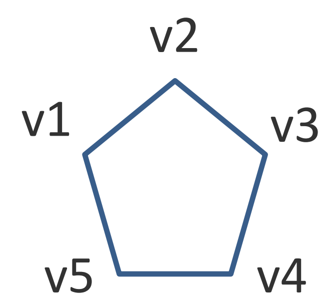
RtTriangle (s1, s2) =



Ellipse (r1, r2) =



Polygon [v1; ...;v5] =

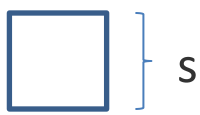


More General Shapes

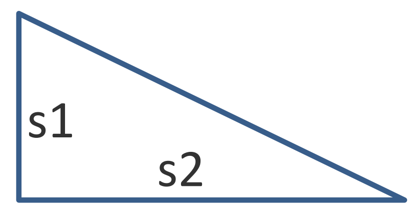
```
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list
```

Type abbreviations can aid readability

Square s =



RtTriangle (s1, s2) =



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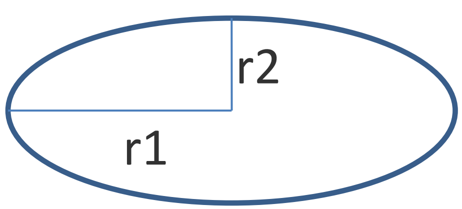

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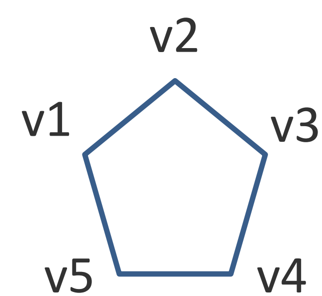

More General Shapes

```
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list
```

Square builds a shape from a single side

```
let sq   : shape = Square 17.0
let ell  : shape = Ellipse (1.0, 2.0)
let rt   : shape = RtTriangle (1.0, 1.0)
let poly : shape = Polygon [(0., 0.); (1., 0.); (0.; 1.)]
```

RtTriangle builds a shape from a pair of sides

they are all shapes; they are constructed in different ways

Polygon builds a shape from a list of points (where each point is itself a pair)

More General Shapes

```
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list
```

a data type also defines a pattern for matching

```
let area (s : shape) : float =
  match s with
  | Square s ->
  | Ellipse (r1, r2)->
  | RtTriangle (s1, s2) ->
  | Polygon ps ->
```

More General Shapes

```
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
  | Ellipse of radius * radius
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  | Polygon of point list
```

a data type also defines a pattern for matching

```
let area (s : shape) : float =
  match s with
  | Square s ->
  | Ellipse (r1, r2)->
  | RtTriangle (s1, s2) ->
  | Polygon ps ->
```

Square carries a value with type float so s is a pattern for float values

RtTriangle carries a value with type float * float so (s1, s2) is a pattern for that type

More General Shapes

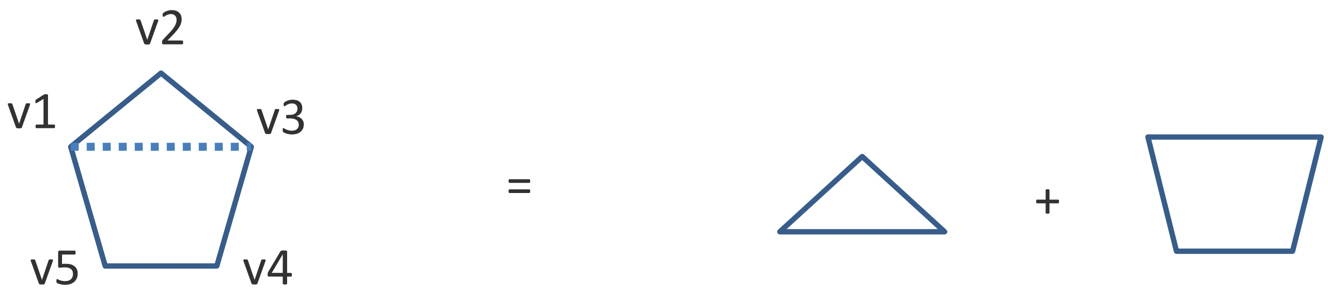
```
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list
```

a data type also defines a pattern for matching

```
let area (s : shape) : float =
  match s with
  | Square s -> s *. s
  | Ellipse (r1, r2)-> pi *. r1 *. r2
  | RtTriangle (s1, s2) -> s1 *. s2 /. 2.
  | Polygon ps -> ???
```

Computing Area

- How do we compute polygon area?
- For convex polygons:
  - Case: the polygon has fewer than 3 points:
    - it has 0 area! (it is a line or a point or nothing at all)
  - Case: the polygon has 3 or more points:
    - Compute the area of the triangle formed by the first 3 vertices
    - Delete the second vertex to form a new polygon
    - Sum the area of the triangle and the new polygon



Computing Area

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- For convex polygons:
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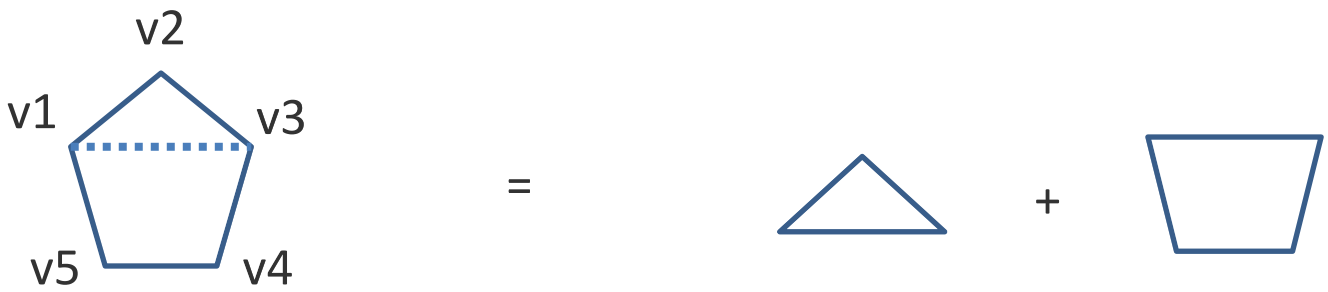
  - Case: the polygon has 3 or more points:
    - Compute the area of the triangle formed by the first 3 vertices
    - Delete the second vertex to form a new polygon
    - Sum the area of the triangle and the new polygon
- Note: This is a beautiful inductive algorithm:
  - the area of a polygon with n points is computed in terms of a smaller polygon with only n-1 points!

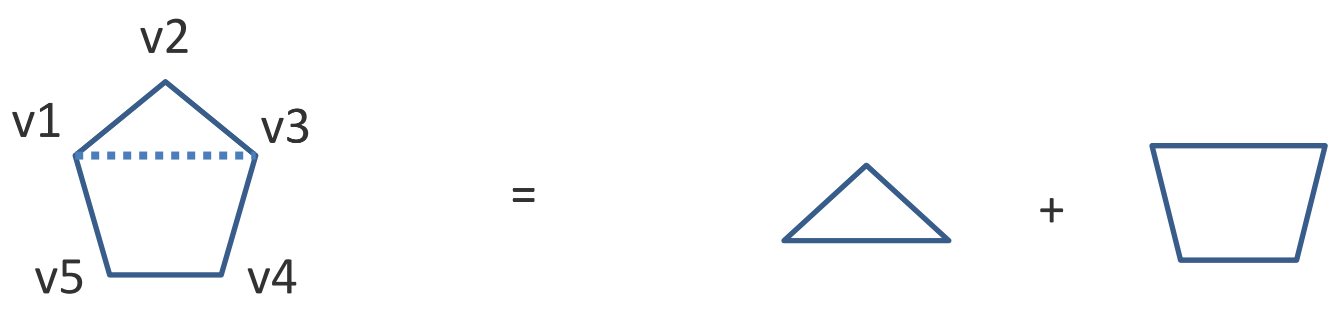


Computing Area

```
let area (s : shape) : float =
  match s with
  | Square s -> s *. s
  | Ellipse (r1, r2)-> r1 *. r2
  | RtTriangle (s1, s2) -> s1 *. s2 /. 2.
  | Polygon ps -> poly_area ps
```

This pattern says the list has at least 3 items

```
let poly_area (ps : point list) : float =
  match ps with
  | p1 :: p2 :: p3 :: tail ->
     tri_area p1 p2 p3 +. poly_area (p1::p3::tail)
  | _ -> 0.
```



Computing Area

```
let tri_area (p1:point) (p2:point) (p3:point) : float =
  let a = distance p1 p2 in
  let b = distance p2 p3 in
  let c = distance p3 p1 in
  let s = 0.5 *. (a +. b +. c) in
  sqrt (s *. (s -. a) *. (s -. b) *. (s -. c))
```

```
let rec poly_area (ps : point list) : float =
  match ps with
  | p1 :: p2 :: p3 :: tail ->
     tri_area p1 p2 p3 +. poly_area (p1::p3::tail)
  | _ -> 0.
```

```
let area (s : shape) : float =
  match s with
  | Square s -> s *. s
  | Ellipse (r1, r2)-> pi *. r1 *. r2
  | RtTriangle (s1, s2) -> s1 *. s2 /. 2.
  | Polygon ps -> poly_area ps
```

INDUCTIVE DATA TYPES

Inductive data types

- We can use data types to define inductive data
- A binary tree is:
  - a Leaf containing no data
  - a Node containing a key, a value, a left subtree and a right subtree

Inductive data types

- We can use data types to define inductive data
- A binary tree is:
  - a Leaf containing no data
  - a Node containing a key, a value, a left subtree and a right subtree

```
type key = string
type value = int

type tree =
  Leaf
| Node of key * value * tree * tree
```

Inductive data types

```
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree
```

```
let rec insert (t:tree) (k:key) (v:value) : tree =
```

Inductive data types

```
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree
```

```
let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf ->
  | Node (k', v', left, right) ->
```

Again, the type definition specifies the cases you must consider

Inductive data types

```
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree
```

```
let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf -> Node (k, v, Leaf, Leaf)
  | Node (k', v', left, right) ->
```

Inductive data types

```
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree
```

```
let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
   | Leaf -> Node (k, v, Leaf, Leaf)
   | Node (k', v', left, right) ->
       if k < k' then
         Node (k', v', insert left k v, right)
       else if k > k' then
         Node (k', v', left, insert right k v)
       else
         Node (k, v, left, right)
```

Inductive data types

```
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree
```

```
let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
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      if k < k' then
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      else if k > k' then
        Node (k', v', left, insert right k v)
      else
        Node (k, v, left, right)
```

Inductive data types

```
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree
```

Note on memory use

```
let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf -> Node (k, v, Leaf, Leaf)
  | Node (k', v', left, right) ->
      if k < k' then
        Node (k', v', insert left k v, right)
      else if k > k' then
        Node (k', v', left, insert right k v)
      else
        Node (k, v, left, right)
```

Inductive data types: Another Example

- Recall, we used the type "int" to represent natural numbers
  - but that was kind of broken: it also contained negative numbers
  - we had to use a dynamic test to guard entry to a function:

```
let double (n : int) : int =
  if n < 0 then
    raise (Failure "negative input!")
  else
    double_nat n
```

  - it would be nice if there was a way to define the natural numbers exactly, and use OCaml's type system to guarantee no client ever attempts to double a negative number

Inductive data types

- Recall, a natural number n is either:
  - zero, or
  - m + 1
- We use a data type to represent this definition exactly:

Inductive data types

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```
type nat = Zero | Succ of nat
```

Inductive data types

- Recall, a natural number n is either:
  - zero, or
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- We use a data type to represent this definition exactly:

```
type nat = Zero | Succ of nat

let rec nat_to_int (n : nat) : int =
 match n with
   Zero -> 0
 | Succ n -> 1 + nat_to_int n
```

Inductive data types

- Recall, a natural number n is either:
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- We use a data type to represent this definition exactly:

```
type nat = Zero | Succ of nat

let rec nat_to_int (n : nat) : int =
 match n with
   Zero -> 0
 | Succ n -> 1 + nat_to_int n

let rec double_nat (n : nat) : nat =
  match n with
   | Zero -> Zero
   | Succ m -> Succ (Succ(double_nat m))
```

Lists!

- Recall, a list is either:
  - nil, or
  - the cons of a *head* value with a *tail* list

- We use a data type to represent this definition exactly:

```
type 'a list =  []  |  :: of 'a * 'a list
```

Summary of Part I

- OCaml data types: a powerful mechanism for defining complex data structures:
  - They are precise
    - contain exactly the elements you want, not more elements
  - They are general
    - recursive, non-recursive (mutually recursive and polymorphic)
  - The type checker helps you detect errors
    - missing cases in your functions

OCaml Datatypes Part II: An Exercise in Type Design

Example Type Design

IBM developed GML (Generalize Markup Language) in 1969

- http://en.wikipedia.org/wiki/IBM_Generalized_Markup_Language
- Precursor to SGML, HTML and XML

```
:h1.Chapter 1: Introduction
:p.GML supported hierarchical containers, such as
:ol
:li.Ordered lists (like this one),
:li.Unordered lists, and
:li.Definition lists
:eol.
as well as simple structures.
:p.Markup Minimization (later generalized and
formalized in SGML), allowed the end-tags to be
omitted for the “h1” and “p” elements.
```

Simplified GML

To process a GML document, an OCaml program would:

- Read a series of characters from a text file & Parse GML structure
- Represent the information content as an OCaml data structure
- Analyze or transform the data structure
- Print/Store/Communicate results

We will focus on how to *represent* and *transform* the information content of a GML document.

Example Type Design

- A GML document consists of:
  - a list of elements
- An element is either:
  - a word or markup applied to an element
- Markup is either:
  - italicize, bold, or a font name

Example Type Design

- A GML document consists of:
  - a list of elements
- An element is either:
  - a word or markup applied to an element
- Markup is either:
  - italicize, bold, or a font name

```
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

Example Data

```
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

```
let d = [ Formatted (Bold,
            Formatted (Font “Arial”,
              Words [“Chapter”;“One”]));

          Words [“It”; ”was”; ”a”; ”dark”;
                 ”&”; ”stormy; ”night.”; "A"];

          Formatted (Ital, Words[“shot”]);

          Words [“rang”; ”out.”] ];;
```

Challenge

- Change all of the “Arial” fonts in a document to “Courier”.
- Of course, when we program functionally, we implement *change* via a function that
  - receives one data structure as input
  - builds a new (different) data structure as an output

Challenge

- Change all of the “Arial” fonts in a document to “Courier”.

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Challenge

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type doc = elt list
```

- Technique: approach the problem top down, work on doc first:

```
let rec chfonts (elts:doc) : doc =
```

Challenge

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type doc = elt list
```

- Technique: approach the problem top down, work on doc first:

```
let rec chfonts (elts:doc) : doc =
  match elts with
  | [] ->
  | hd::tl ->
```

Challenge

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```
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
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type doc = elt list
```

- Technique: approach the problem top down, work on doc first:

```
let rec chfonts (elts:doc) : doc =
  match elts with
  | [] -> []
  | hd::tl -> (chfont hd)::(chfonts tl)
```

Changing fonts in an element

- Change all of the "Arial" fonts in a document to "Courier".

```
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

- Next work on changing the font of an element:

```
let rec chfont (e:elt) : elt =
```

Changing fonts in an element

- Change all of the “Arial” fonts in a document to “Courier”.

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type markup = Ital | Bold | Font of string

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- Next work on changing the font of an element:

```
let rec chfont (e:elt) : elt =
  match e with
   | Words ws ->
   | Formatted(m,e) ->
```

Changing fonts in an element

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- Next work on changing the font of an element:

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let rec chfont (e:elt) : elt =
  match e with
   | Words ws -> Words ws
   | Formatted(m,e) ->
```

Changing fonts in an element

- Change all of the "Arial" fonts in a document to "Courier".

```
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

- Next work on changing the font of an element:

```
let rec chfont (e:elt) : elt =
  match e with
  | Words ws -> Words ws
  | Formatted(m,e) -> Formatted(chmarkup m, chfont e)
```

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- Next work on changing a markup:

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Changing fonts in an element

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```

- Next work on changing a markup:

```
let chmarkup (m:markup) : markup =
  match m with
  | Font “Arial” -> Font “Courier”
  | _ -> m
```

Summary: Changing fonts in an element

- Change all of the "Arial" fonts in a document to "Courier"
- Lesson: function structure follows type structure

```
let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | _ -> m

let rec chfont (e:elt) : elt =
  match e with
  | Words ws -> Words ws
  | Formatted(m,e) -> Formatted(chmarkup m, chfont e)

let rec chfonts (elts:doc) : doc =
  match elts with
  | [] -> []
  | hd::tl -> (chfont hd)::(chfonts tl)
```

Poor Style

- Consider again our definition of markup and markup change:

```
type markup =
  Ital | Bold | Font of string


let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | _ -> m
```

Poor Style

- What if we make a change:

```
type markup =
  Ital | Bold | Font of string | TTFont of string


let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | _ -> m
```

the underscore silently catches all possible alternatives

this may not be what we want -- perhaps there is an Arial TT font

it is better if we are alerted of all functions whose implementation may need to change

Better Style

- Original code:

```
type markup =
  Ital | Bold | Font of string

let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | Ital | Bold -> m
```

Better Style

- Updated code:

```
type markup =
  Ital | Bold | Font of string | TTFont of string

let chmarkup (m:markup) : markup =
  match m with
   | Font “Arial” -> Font “Courier”
   | Ital | Bold -> m
```

```
..match m with
    | Font "Arial" -> Font "Courier"
    | Ital | Bold -> m..
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched:
TTFont _
```

Better Style

- Updated code, fixed:

```
type markup =
  Ital | Bold | Font of string | TTFont of string

let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | TTFont "Arial" -> TTFont "Courier"
  | Font s -> Font s
  | TTFont s -> TTFont s
  | Ital | Bold -> m
```

- Lesson: use the type checker where possible to help you maintain your code

A couple of practice problems

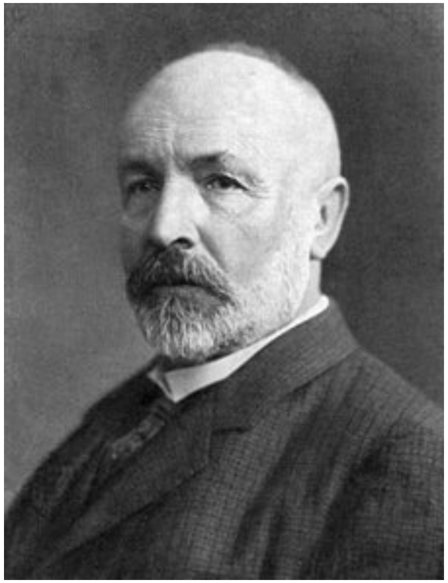
- Write a function that gets rid of immediately redundant markup in a document.
  - Formatted(Ital, Formatted(Ital,e)) can be simplified to Formatted(Ital,e)
  - write maps and folds over markups
- Design a datatype to describe bibliography entries for publications. Some publications are journal articles, others are books, and others are conference papers. Journals have a name, number and issue; books have an ISBN number; All of these entries should have a title and author.
  - design a sorting function
  - design maps and folds over your bibliography entries

To Summarize

- Design recipe for writing OCaml code:
  - write down English specifications
    - try to break problem into obvious sub-problems
  - write down some sample test cases
  - write down the signature (types) for the code
  - use the signature to guide construction of the code:
    - tear apart inputs using pattern matching
      - make sure to cover all of the cases! (OCaml will tell you)
    - handle each case, building results using data constructor
      - this is where human intelligence comes into play
      - the "skeleton" given by types can almost be done automatically!
    - clean up your code
  - use your sample tests (and ideally others) to ensure correctness

WHERE DID TYPE SYSTEMS COME FROM?

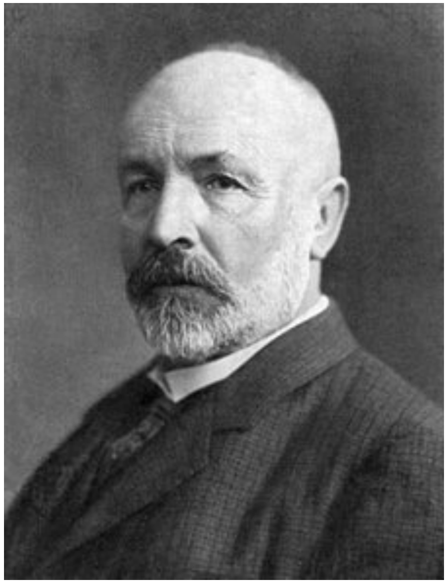
Origins of Type Theory



Georg Cantor

* http://www.math.ups.edu/~bryans/Current/Journal_Spring_1999/JEarly_232_S99.html

Origins of Type Theory



Georg Cantor

Über eine Eigenshaft des Inbegriffes aller reellen algebraischen Zahlen. 1874

(On a Property of the System of all the Real Algebraic Numbers)

"Considered the first purely theoretical paper on set theory." *

* http://www.math.ups.edu/~bryans/Current/Journal_Spring_1999/JEarly_232_S99.html

Origins of Type Theory

Bertrand Russell

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He noticed that Cantor's set theory allows the definition of this set S:

$\{ A \mid A \text{ is a set and } A \notin A \}$

Origins of Type Theory

Bertrand Russell

He noticed that Cantor's set theory allows the definition of this set S:

{ A | A is a set and A ∉ A }

If we assume S is not in the set S, then by definition, it must belong to that set.

If we assume S is in the set S, then it contradicts the definition of S.

Russell's paradox

Origins of Type Theory

Bertrand Russell

He noticed that Cantor's set theory allows the definition of this set S:

$\{ A \mid A \text{ is a set and } A \notin A \}$

Russell's solution:

Each set has a distinct type:
type 1, 2, 3, 4, 5, ...

A set of type i+1 can only have elements of type i so it can't include itself.

Aside

Ernst Zermelo

Abraham Fraenkel

Developers of Zermelo-Fraenkel set theory (1921).
An alternative solution to Russell’s paradox.

Origins of Type Theory

Alonzo Church, 1903-1995
Princeton Professor, 1929-1967

Developed the lambda calculus
(ancestor of ML / OCaml)

and "The simple theory of types"
(ancestor of ML's type system)