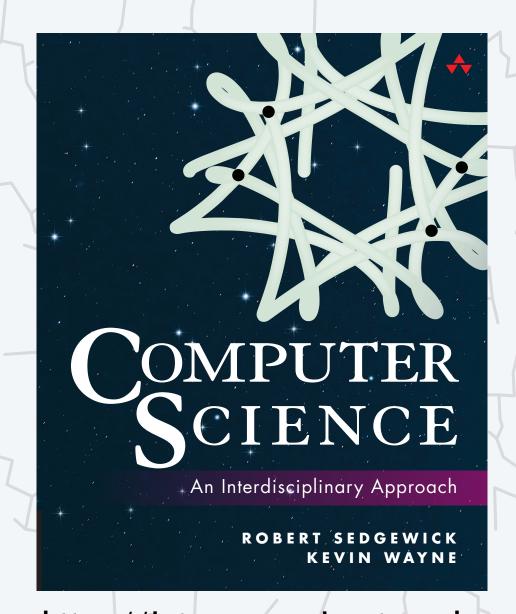
Computer Science

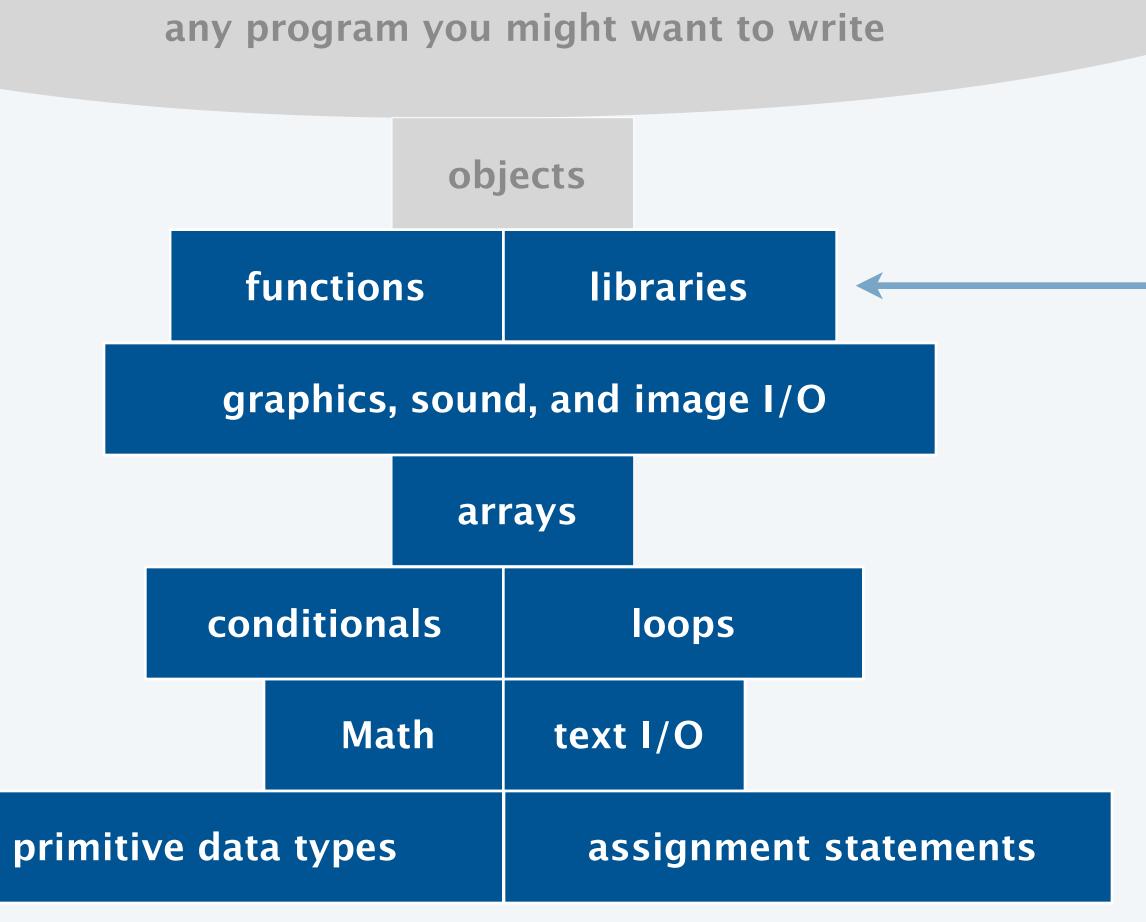


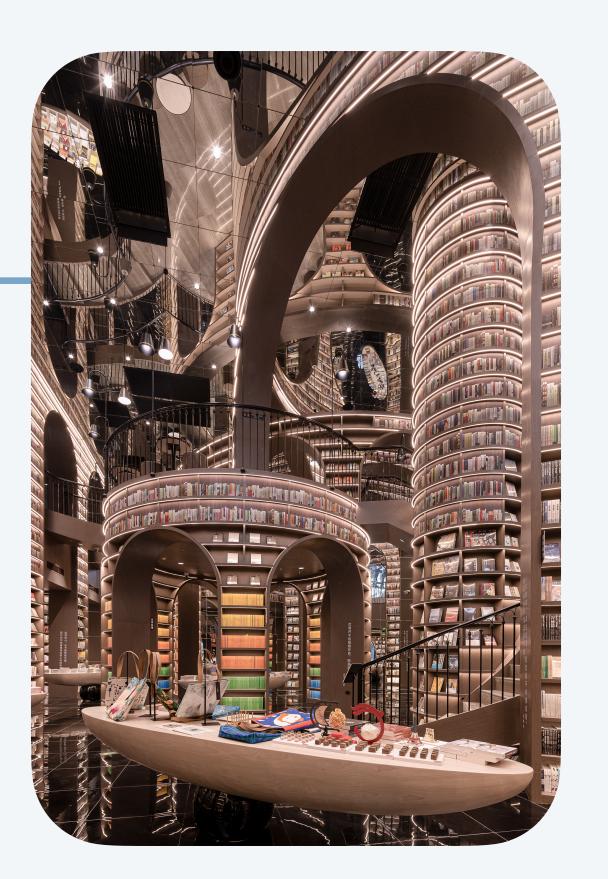
https://introcs.cs.princeton.edu

2.2 LIBRARIES AND CLIENTS

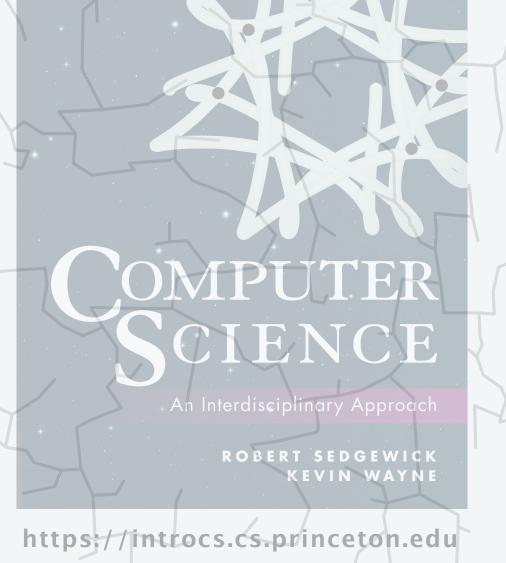
- random number library
- designing libraries
- sound synthesis
- synthesizer library

Basic building blocks for programming





build reusable libraries



2.2 LIBRARIES AND CLIENTS

- random number library
- designing libraries
- sound synthesis
- synthesizer library

```
int getRandomNumber()
{

return 4; // chosen by fair dice roll.

// guaranteed to be random.
}
```

https://xkcd.com/221/

Standard random library



Goal. Design a library to generate pseudo-random numbers.

```
public class StdRandom
static double uniformDouble()
                                                           real number between 0 and 1
static double uniformDouble(double lo, double hi)
                                                           real number between 10 and hi
static boolean bernoulli(double p)
                                                           true with probability p, false otherwise
static int
                uniformInt(int n)
                                                           integer between 0 and n-1
static double gaussian()
                                                           normal with mean 0 and stddev 1
static double gaussian(double mu, double sigma)
                                                           normal with mean mu and stddev sigma
static void
                shuffle(String[] a)
                                                           shuffle the string array a[]
static int
                discrete(int[] freq)
                                                           i with probability proportion to freq[i]
```

Standard random implementation: random numbers from various distributions

```
public class StdRandom {
   public static double uniformDouble() {
                                                       calls a method
      return Math.random(); ←
                                                     (in a different class)
   public static double uniformDouble(double lo, double hi) {
      return lo + (uniformDouble() * (hi - lo));
   public static boolean bernoulli(double p) {
                                                                                      you could re-implement
      return uniformDouble() < p;</pre>
                                                                                   these methods in each program,
                                                                                     but now you don't have to!
   public static int uniformInt(int n) {
      return (int) (uniformDouble() * n);
                       calls a method
                      (in the same class)
```

Standard random implementation: random numbers from a Gaussian distribution

```
public class StdRandom {
   public static double gaussian() {
                                                                                  can call a method without
      double r, x, y;
                                                                                knowing how it is implemented
        x = uniformDouble(-1.0, 1.0);
         y = uniformDouble(-1.0, 1.0);
        r = x*x + y*y;
       while (r >= 1 | | r == 0);
      return x * Math.sqrt(-2 * Math.log(r) / r);
   public static double gaussian(double mu, double sigma) {
      return mu + gaussian() * sigma;
                                                                                           \mu
```

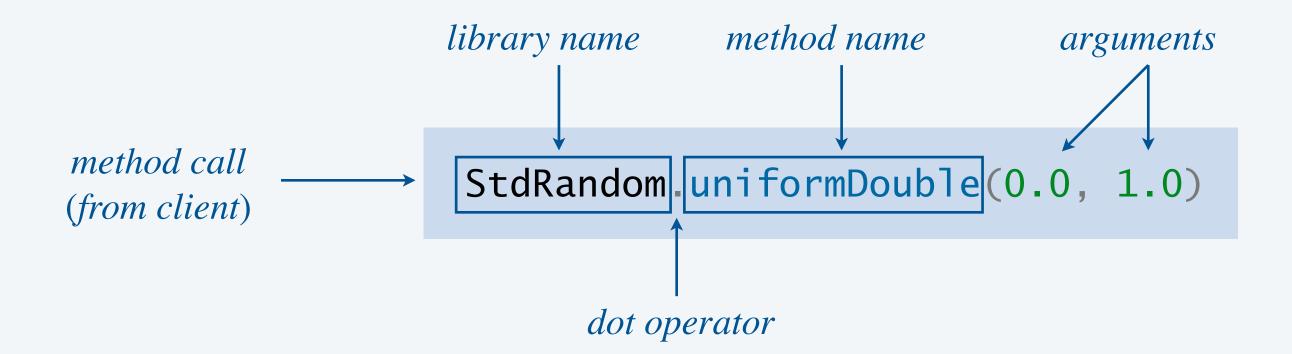
Standard random implementation: shuffling the elements in an array

```
public class StdRandom {
                                                                                   private helper method
   private static void exch(String[] a, int i, int j) {
                                                                            (cannot be called from outside this class)
      String temp = a[i];
      a[i] = a[j];
      a[j] = temp;
   public static void shuffle(String[] a) {
      for (int i = 0; i < a.length; i++) {
         int r = uniformInt(i+1);
         exch(a, i, r);
         calls a private method
           (in the same class)
```

7

Calling a library function

Calling from a client. Specify library name, dot operator, function name, and arguments.



Note. Must use fully qualified name if calling a function from another file.

Standard random clients

StdRandom client 1

```
public class Shuffle {
  public static void main(String[] args) {
    StdRandom.shuffle(args);
    for (int i = 0; i < args.length; i++) {
        StdOut.print(args[i] + " ");
    }
    StdOut.println();
}</pre>
```

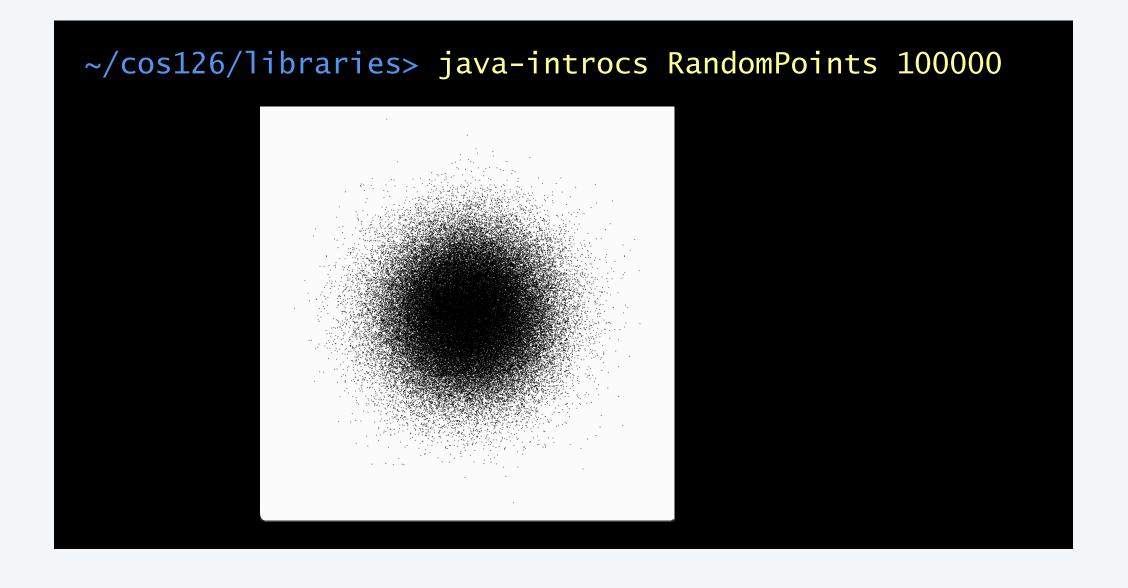
```
~/cos126/libraries> java-introcs Shuffle A B C D E
E A D B C

~/cos126/libraries> java-introcs Shuffle A B C D E
C A E B D

~/cos126/libraries> java-introcs Shuffle 2C 2D 2H ... AS
4S 2D AC 9H QH 8C ... JS 4H 2S
```

StdRandom client 2

```
public class RandomPoints {
   public static void main(String[] args) {
     int n = Integer.parseInt(args[0]);
     for (int i = 0; i < n; i++) {
        double x = StdRandom.gaussian(0.5, 0.1);
        double y = StdRandom.gaussian(0.5, 0.1);
        StdDraw.point(x, y);
     }
}</pre>
```





Libraries

Def. A module is a set of functions stored in a single file.

Def. A library is a module whose primary purpose is for use by other programs.

definitions for this course

library	description	example method call	source	logo	
StdRandom	generate random numbers	StdRandom.uniformInt(6)	4 41 1-	COMPUTER SCIENCE	
StdDraw	draw geometric shapes	StdDraw.circle(0.5, 0.5, 0.25)	textbook	COMPUTER SCIENCE An Interdisciplinary Approach POSITEST SERVINGE ELVIN WAYNE	
Math	compute mathematical functions	Math.sqrt(2.0)	Torro gradado		
java.util.Arrays	manipulate arrays	Arrays.sort(a)	Java system		
Gaussian	compute Gaussian pdf and cdf	Gaussian.pdf(3.0)	usan dafinad		
SayNumber	speak numbers	SayNumber.sayInteger(126)	user-defined		
• •	• • •		•		

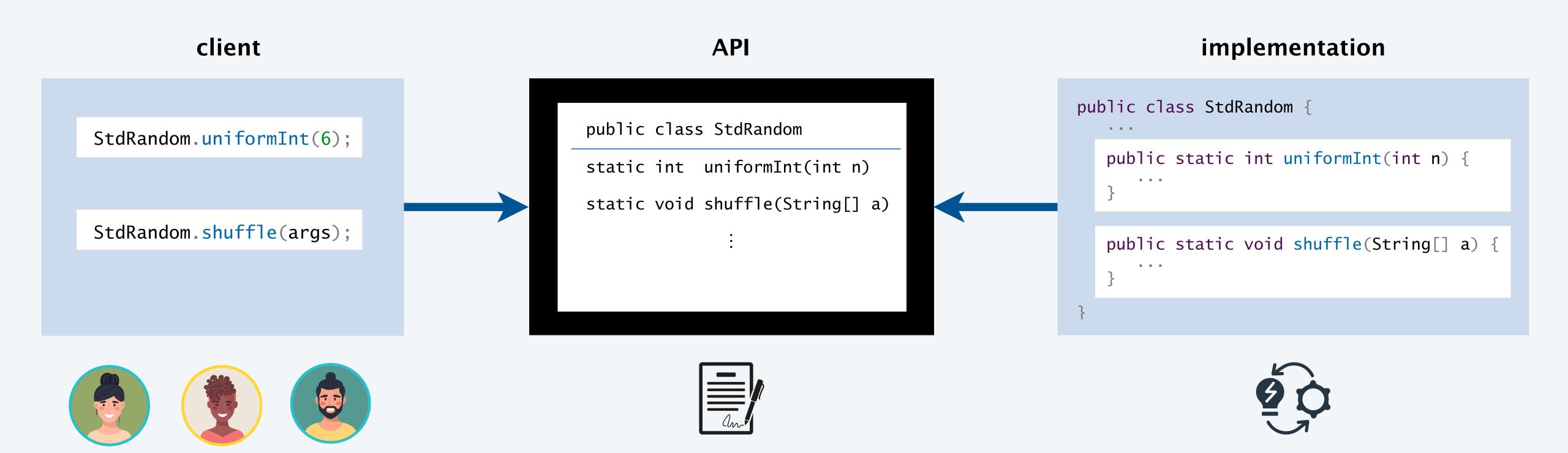
API, client, and implementation

Application programming interface (API). Specifies method headers and behavior for a library.

Implementation. Program that implements the methods in an API.

Client. Program that uses a library through its API.

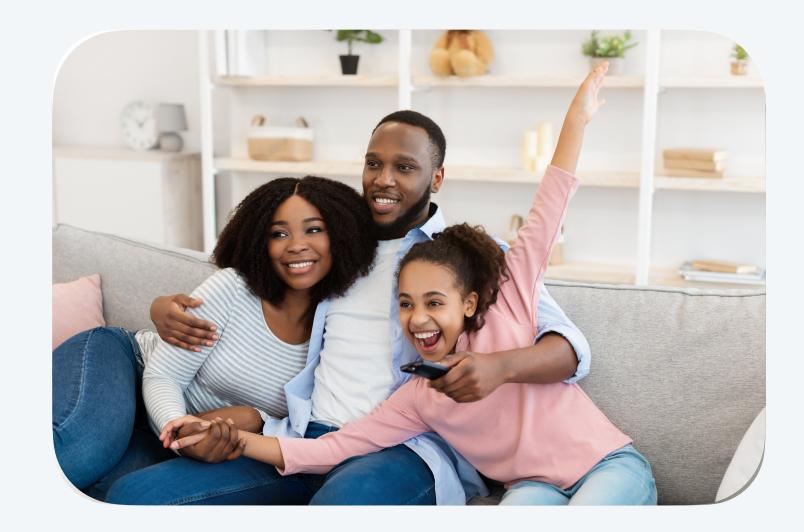
contract between client and implementation



API, client, and implementation

Application programming interface (API). Specifies method headers and behavior for a library. Implementation. Program that implements the methods in an API. Client. Program that uses a library through its API.

client



API



implementation



Encapsulation

Encapsulation. Separating clients from implementation details by hiding information.

Principle. A client does not need to know how a method is implemented in order to use it.



Benefits.

- Can develop client code and implementation code independently.
- Can change implementation details without breaking clients.

Private access modifier. Designates a method as not for use by a client.

- API does not list private methods.
- Compile-time error for client to call a private method.
- Advantage: implementation can add/remove private methods without impacting clients.



Accessing a library

Java classpath. Places where Java looks for user-defined libraries (and other resources).

- Simplest: put library .class file in same directory as client program.
- Best practice: bundle library .class files in a .jar file; add .jar file to Java classpath. ←

stdlib.jar contains:
StdRandom.class
StdIn.class
StdOut.class
StdDraw.class
StdPicture.class
StdAudio.class
:

Unit testing

Best practice. Include a main() method in each class as a test client.

- Call each public method at least once.
- Use result to check behavior.
- Identify failed tests programmatically.

```
minimum requirements
(in this course)
```

```
public class StdRandom {
  public static void main(String[] args) {
      int n = Integer.parseInt(args[0]);
      for (int i = 0; i < n; i++) {
         StdOut.printf("%8.5f", uniformDouble(10.0, 99.0));
         StdOut.printf("%5b " , bernoulli(0.5));
         StdOut.printf("%2d " , uniformInt(100));
         StdOut.printf("%7.5f ", gaussian(9.0, 0.2));
         StdOut.println();
                   unit tests for shuffle()
                      and other methods
```

```
~/cos126/libraries> java-introcs StdRandom 5
85.06009 false 8 8.88418
22.97440 true 40 9.18536
19.46492 false 28 8.89026
53.62835 true 90 8.90420
85.72239 false 5 8.78333
...

looks plausible
(between 0 and 99)
```

Method header comments

Best practice. Every method should include a comment before the method header.

- Describe it purpose.
- Use names of parameter variables in description.
- minimum requirements
 (in this course)

· Identify parameters, return value, and exceptions using Javadoc tags.

```
/**
    * Returns a random integer uniformly in [0, n).
    *
    * @param n number of possible integers
    * @return a random integer uniformly between 0 (inclusive) and n (exclusive)
    */
public static int uniformInt(int n) {
    return (int) (Math.random() * n);
}
```

Javadoc

Javadoc. Automatically generates API and documentation from Javadoc comments.

Class StdRandom

Object

StdRandom

public final class StdRandom
extends Object

Overview. The StdRandom class provides static methods for generating random number from various discrete and continuous distributions, including uniform, Bernoulli, geometric, Gaussian, exponential, Pareto, Poisson, and Cauchy. It also provides method for shuffling an array or subarray and generating random permutations.

Conventions. By convention, all intervals are half open. For example, uniformDouble(-1.0, 1.0) returns a random number between -1.0 (inclusive) and 1.0 (exclusive). Similarly, shuffle(a, lo, hi) shuffles the hi – lo elements in the array a[], starting at index lo (inclusive) and ending at index hi (exclusive).

Performance. The methods all take constant expected time, except those that involve arrays. The *shuffle* method takes time linear in the subarray to be shuffled; the *discrete* methods take time linear in the length of the argument array.

Additional information. For additional documentation, see Section 2.2 of *Computer Science: An Interdisciplinary Approach* by Robert Sedgewick and Kevin Wayne.

Author:

Robert Sedgewick, Kevin Wayne



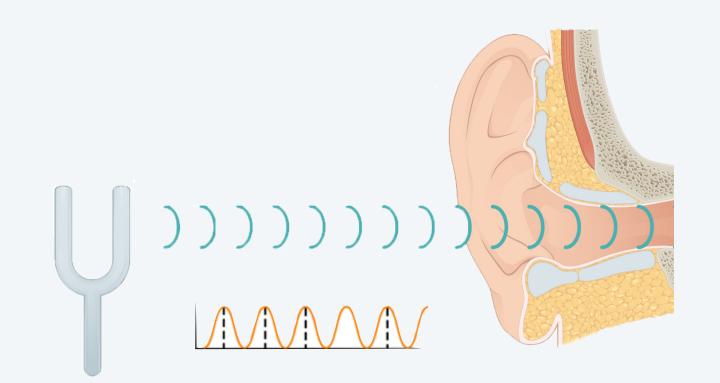
Review: digital audio

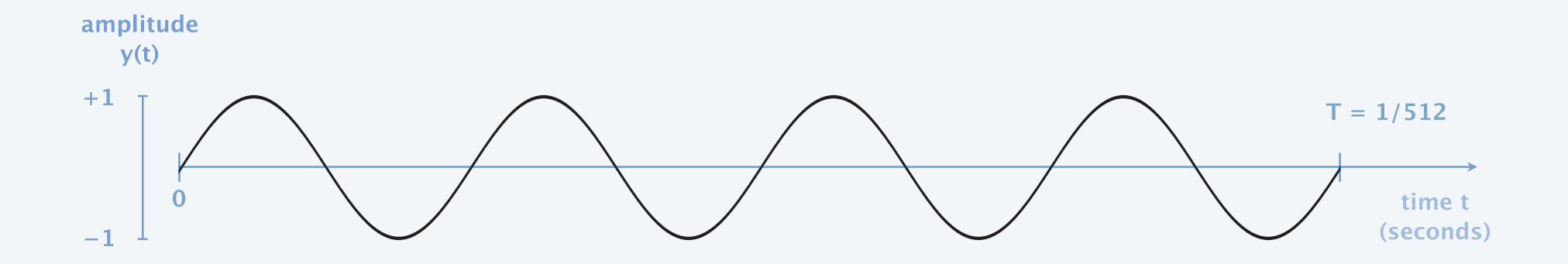


Sound is the perception the vibration of our eardrums.

Audio signal. Real-valued (between -1 and +1) function of time.

Pure tone. Sound wave defined by the sine function of given frequency, amplitude and duration.





$$y(t) = \sin(2\pi \cdot 2048 \cdot t), \quad 0 \le t \le T$$

Review: audio sampling

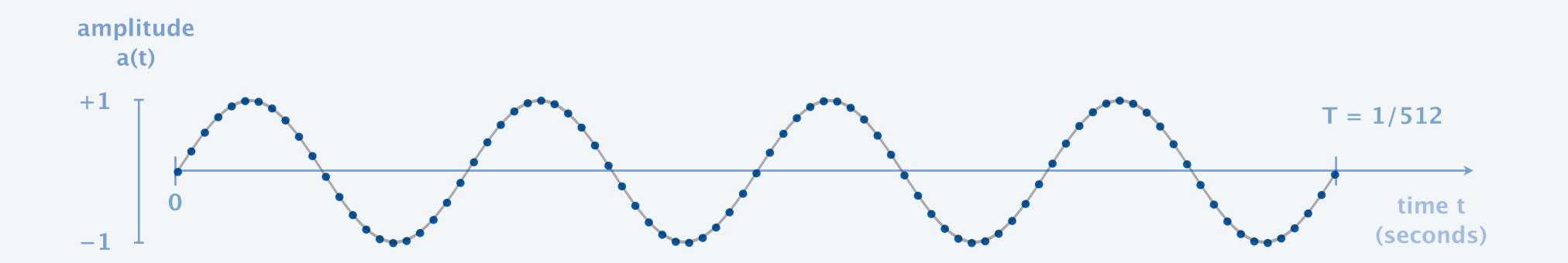
Goal. Convert a continuous-time signal into a discrete-time signal.

- A sample is a signal value at specific point in time.
- Take samples at evenly spaced points.

model sound with an array of

real numbers between −1 and +1

(using 44,100 samples per second)



$$y(t) = \sin(2\pi \cdot 2048 \cdot t), \quad 0 \le t \le T$$

$$a(t) = \sin(2\pi \cdot 2048 \cdot t), \quad t = \frac{0}{44100}, \frac{1}{44100}, \frac{2}{44100}, \dots$$

Review: standard audio API



StdAudio. Our library for playing, reading, and saving digital audio.

public class StdAudio

static int	SAMPLE_RATE	44,100 (CD quality audio)	
static void	play(String filename)	play the audio file	
static void	playInBackground(String filename)	play the audio file in the background	
static void	play(double sample)	play the sample	
static void	play(double[] samples)	play the samples	
	<pre>play(double[] samples) read(String filename)</pre>	play the samples read the samples from an audio file	
		read the samples from an audio file	

Sine wave implementation



```
implementation
public class Synth {
   public static int numberOfSamples(double duration) {
      return (int) (StdAudio SAMPLE_RATE * duration);
                                                                               for internal use only
   private static double sine(double freq, double t) {
                                                                             (private helper methods)
      return Math.sin(2 * Math.PI * freq * t);
   public static double[] sineWave(double freq, double amplitude, double duration) {
      int n = numberOfSamples(duration);
      double[] a = new double[n];
      for (int i = 0; i < n; i++) {
                                                                      sample at n equally
         double t = 1.0 * i / StdAudio.SAMPLE_RATE;
                                                                         spaced points
         a[i] = amplitude * sine(freq, t);
                                                                                                                                  client
      return a;
                                                                                     double[] a = Synth.sineWave(2048.0, 0.5, 3.0);
                                                                                     StdAudio.play(a);
                        a(t) = A \sin(2\pi \cdot f \cdot t), \quad t = \frac{0}{44100}, \frac{1}{44100}, \frac{2}{44100}, \dots
```

Libraries and clients: quiz 1



What sound will the following code fragment produce?

```
double freq = 17400.0;
double amplitude = 0.5;
double duration = 10.0;
double[] a = Synth.sineWave(freq, amplitude, duration);
StdAudio.play(a);
```

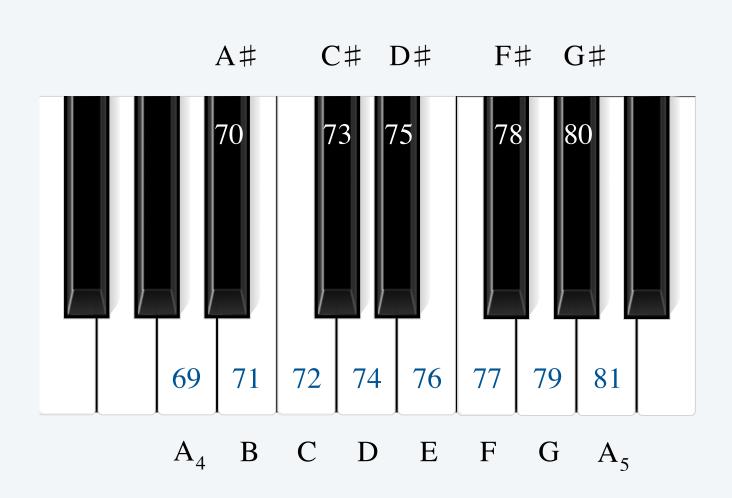


- A. Extremely high-pitched sound.
- B. Inaudible.
- C. Ultrasonic weapon.
- D. All of the above.

Crash course in Western music



- Concert A is 440 Hz.
- An octave is the interval between a note and one with twice its frequency.
- Octave is divided into 12 notes on a logarithmic scale. ← "twelve-tone equal temperament"





note	MIDI (m)	frequency (Hz) $(440 \times 2^{(m-69)/12})$	sine wave
A_4	69	440	
A#/Bb	70	466.16	
В	71	493.88	
C	72	523.25	
C#/Db	73	554.37	
D	74	587.33	
D#/Eb	75	622.25	
E	76	659.26	
F	77	698.46	
F#/Gb	78	739.99	
G	79	783.99	
G#/Ab	80	830.61	
A_5	81	880	

Libraries and clients: quiz 2



Which of the following converts from MIDI note number to frequency?

```
A.
private static double midiToFrequency(int midi) {
   return 440 * Math.pow(2, (midi - 69) / 12);
}
```

```
B.
private static double midiToFrequency(int midi) {
   return 440.0 * 2.0 ^ ((midi - 69.0) / 12.0);
}
```

C. Both A and B.

D. Neither A nor B.

$$frequency = 440 \times 2^{(midi - 69)/12}$$

MIDI-number-to-frequency conversion



Goal. Add methods (and constants) to library that many clients might want to use.

Musical Instrument Digital Interface (MIDI). Digital music standard.

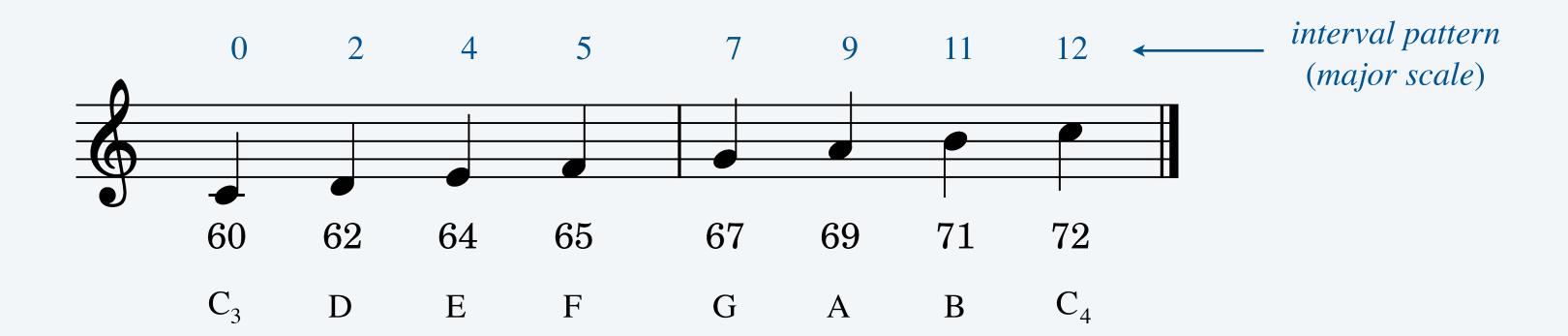
Class constant.

- Declare and initialize "variable" outside of any method, using final and static modifiers.
- Access modifier can be public or private.
- Java naming convention: use SCREAMING_SNAKE_CASE.

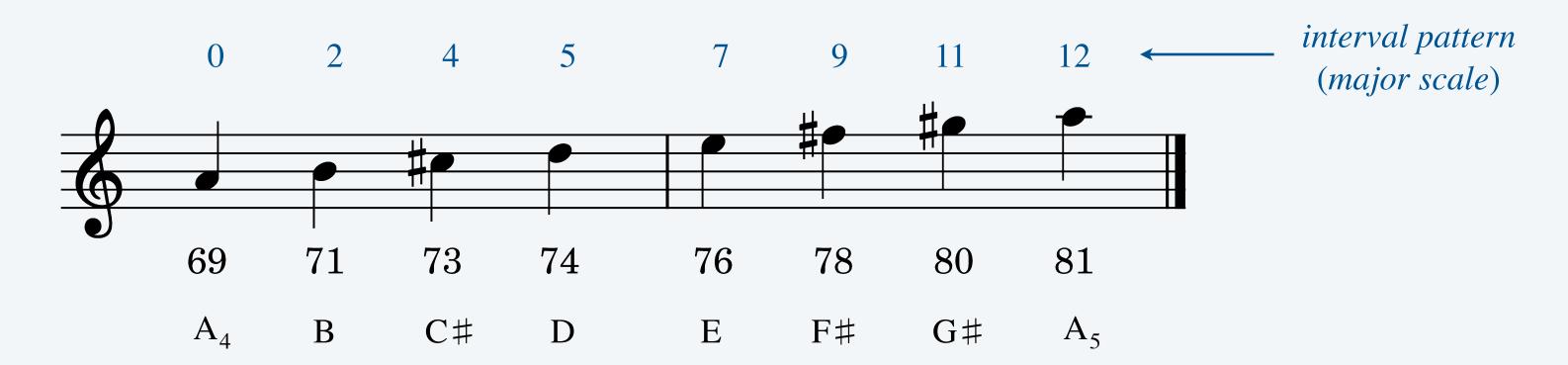
Musical scales

Major scale. Sequence of 8 notes in a specific interval pattern, starting with a root note and ending with the same note one octave higher.

Ex 1. C major scale.



Ex 2. A major scale.



Musical scales



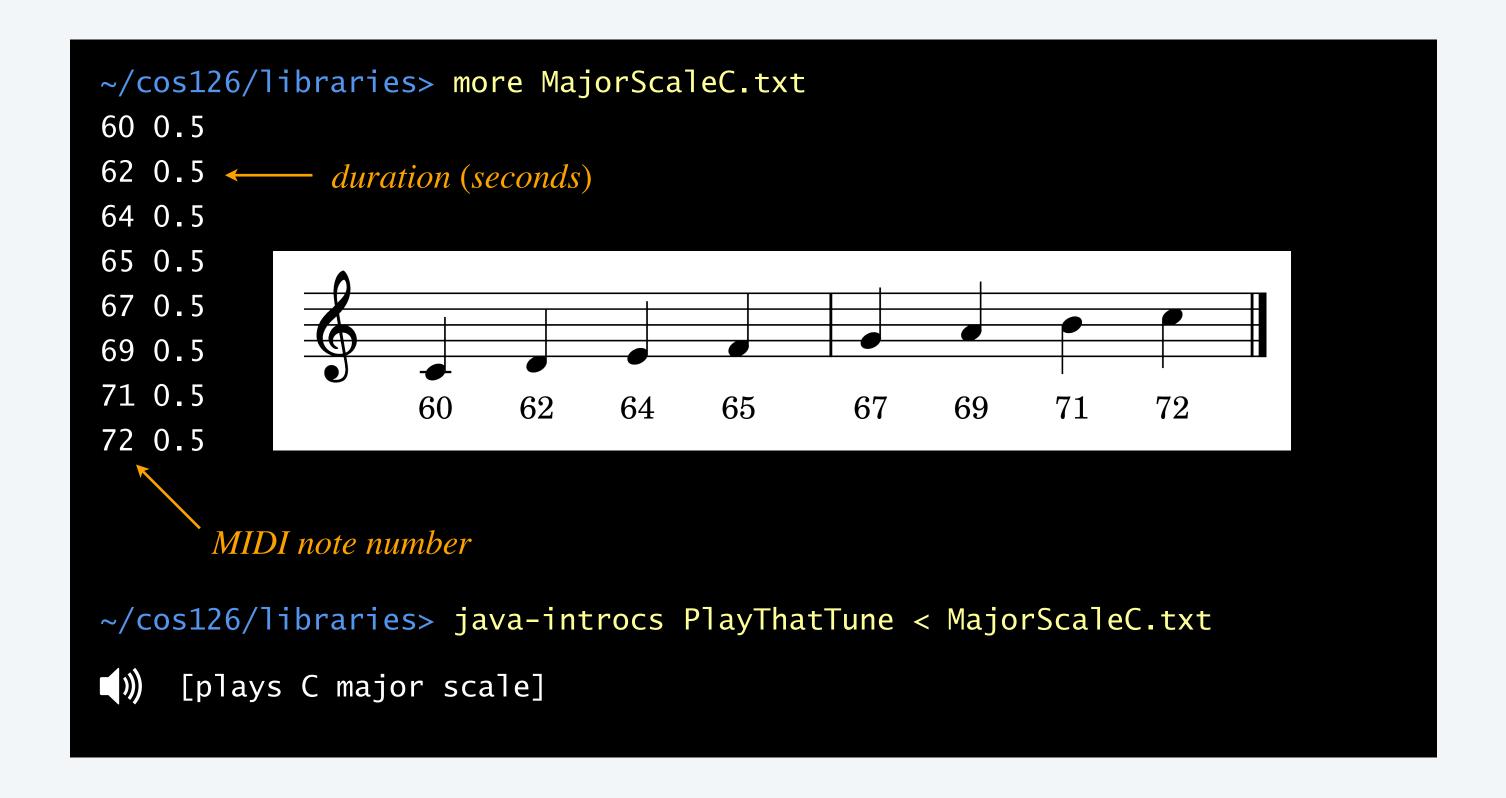
Major scale. Sequence of 8 notes in a specific interval pattern, starting with a root note and ending with the same note one octave higher.

```
public class MajorScale {
                                                            client
   public static void main(String[] args) {
                                                      interval pattern
      int root = Integer.parseInt(args[0]);
                                                       (major scale)
      double duration = 0.5;
      double amplitude = 0.5;
      int[] pattern = { 0, 2, 4, 5, 7, 9, 11, 12 };
      for (int i = 0; i < pattern.length; i++) {
         int midi = root + pattern[i];
         double freq = Synth.midiToFrequency(midi);
         double[] a = Synth.sineWave(freq, amplitude, duration);
         StdAudio.play(a);
                                                                 ~/cos126/libraries> java-introcs MajorScale 60
                                                                     [plays A major scale]
                                                                 ~/cos126/libraries> java-introcs MajorScale 69
                                                                 ()) [plays C major scale]
```

Play that tune



Goal. Read in a sequence of MIDI note numbers and durations from standard input, and play the synthesized results to standard audio.



Play that tune



Goal. Read in a sequence of MIDI note numbers and durations from standard input, and play the synthesized results to standard audio.

client

```
public class PlayThatTune {
   public static void main(String[] args) {
      double amplitude = 0.5;
      while (!StdIn.isEmpty()) {
         int midi = StdIn.readInt();
         double duration = StdIn.readDouble();
         double freq = Synth.midiToFrequency(midi);
         double[] a = Synth.sineWave(freq, amplitude, duration);
         StdAudio.play(a);
                                      ~/cos126/libraries> java-introcs PlayThatTune < Arpeggio.txt
                                          [plays arpeggio]
                                      ~/cos126/libraries> java-introcs PlayThatTune < LooneyTunes.txt
                                          [plays Looney Tunes theme]
                                      ~/cos126/libraries> java-introcs PlayThatTune < FurElise.txt
                                         [plays beginning of Fur Elise]
```



Digital synthesizers



Digital synth. Electronic musical instrument that generates audio signals digitally.

- Sound effects.
- Film and television soundtracks.
- Diverse genres of music (rock, jazz, pop, disco, hip-hop, electronic music, ...).

•



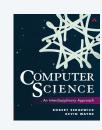


R2-D2 (Star Wars)



Axel F (Harold Faltemeyer)

Synthesizer API



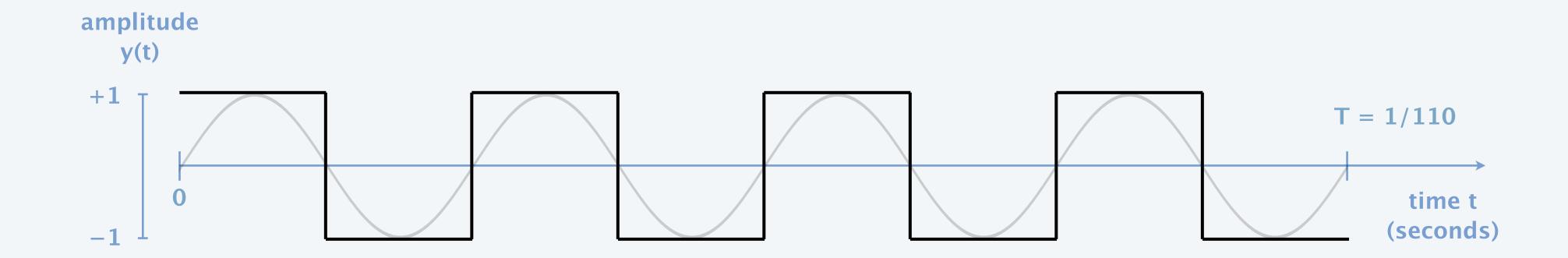
Synth. A library for synthesizing sound.

public class Synth		
static int CONCERT_A	440.0 (concert A pitch in Hz)	
static int numberOfSamples(double duration)	number of audio samples	utility mathods
static double midiToFrequency(int midi)	frequency of MIDI note number	—— utility methods
static double sineWave(double freq, double amplitude, double duration)	sine wave	
static double squareWave(double freq, double amplitude, double duration)	square wave	create sound waves
static double sawWave(double freq, double amplitude, double duration)	saw wave	SOUTHA WAVES
static double supersawWave(double freq, double amplitude, double duration)	supersaw wave	
static double whiteNoise(double amplitude, double duration)	white noise	
<pre>static double[] superpose(double[] a, double[] b)</pre>	add the two waves	manipulate
<pre>static double[] modulate(double[] a, double[] b)</pre>	multiply the two waves	sound waves
static double[] fadeIn(double[] a, double lambda)	exponential fade in	
static double[] fadeOut(double[] a, double lambda)	exponential fade out	

34



Square wave. Alternates between +1 and -1 with frequency f, half the time at each value.



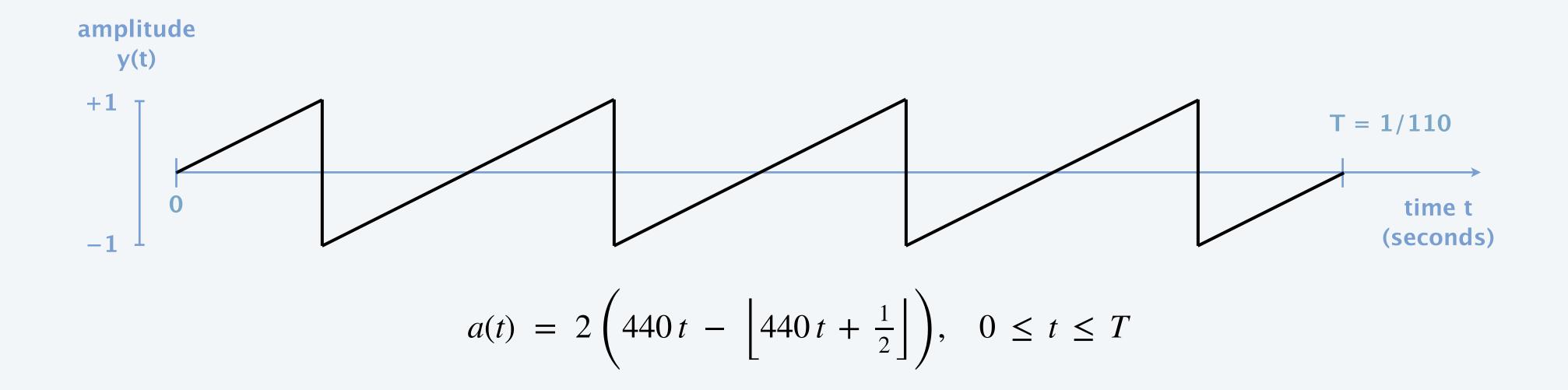
$$a(t) = sgn\left(\sin(2\pi \cdot 440 \cdot t)\right), \quad 0 \le t \le T$$
 $sgn(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ +1 & \text{if } x > 0 \end{cases}$

```
private static double square(double freq, double t) {
    return Math.signum(sine(freq, t));
}

public static double[] squareWave(double freq, double amplitude, double duration) {
    /* similar to sineWave() */
}
```



Sawtooth wave. Rises from -1 to +1 linearly, then drops back to -1, and repeats with frequency f.



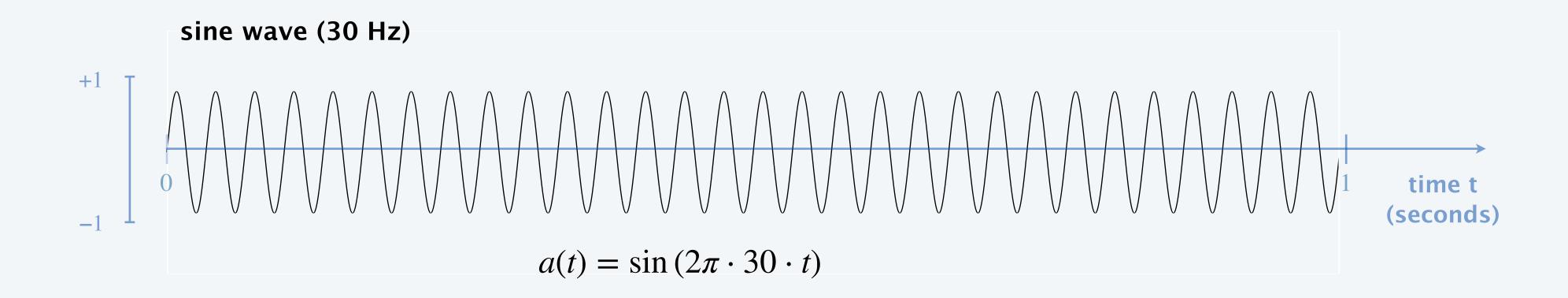
```
private static double saw(double freq, double t) {
    return 2 * (freq*t - Math.floor(freq*t + 0.5));
}

public static double[] sawWave(double freq, double amplitude, double duration) {
    /* similar to sineWave() */
}
```

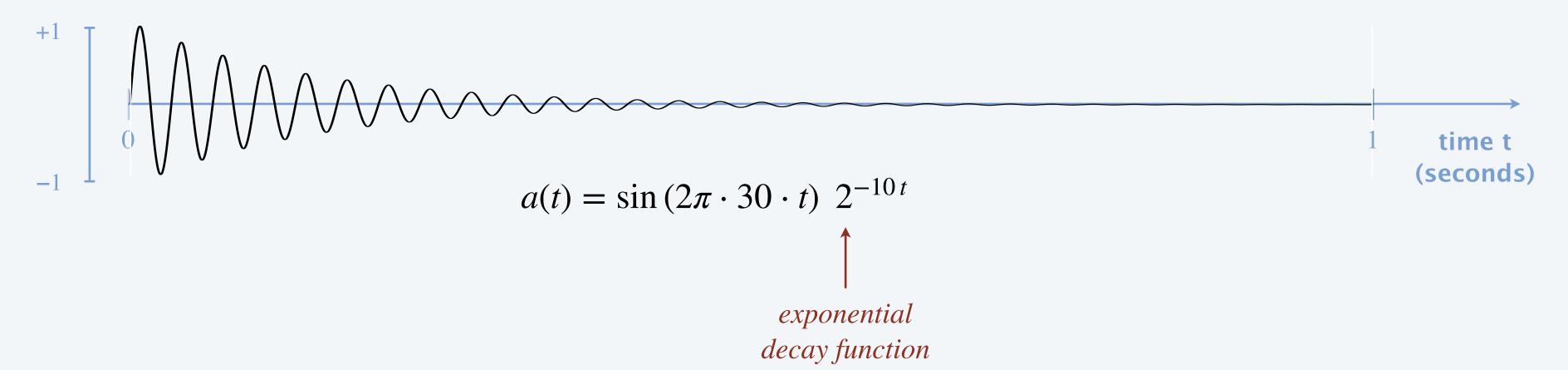
Exponential fade

Sound envelope. Defines how a sound changes over time.

Exponential fade. A sound envelope whose amplitude decays according to exponential function.



sine wave (30 Hz) with exponential fade ($\lambda = 10$)



Exponential fade



```
public class Synth {

public static double[] fadeOut(double[] a, double lambda) {
   int n = a.length;
   double[] result = new double[n];
   for (int i = 0; i < n; i++) {
      double t = 1.0 * i / StdAudio.SAMPLE_RATE;
      result[i] = a[i] * Math.pow(2.0, -lambda * t);
   }
   return result;
}</pre>
```



client



client

```
double[] a = Synth.sineWave(440.0, 0.5, 1.0);
double[] b = Synth.fadeOut(a, 10.0);
StdAudio.play(b);
```

```
double[] a = Synth.squareWave(55.0, 0.25, 1.0);
double[] b = Synth.fadeOut(a, 5.0);
StdAudio.play(b);
```

Libraries and clients: quiz 3



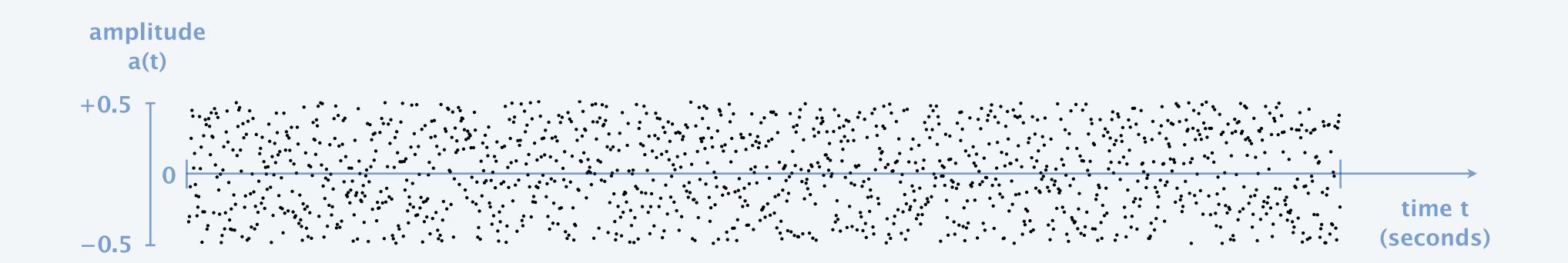
What sound does StdAudio.play(mystery(5.0)) produce?

- A. 5 seconds of concert A (440 Hz).
- **B.** 5 seconds of a random frequency.
- C. 5 seconds of silence.
- D. 5 seconds of static.

```
public static double[] mystery(double duration) {
   int n = numberOfSamples(duration);
   double[] a = new double[n];
   for (int i = 0; i < n; i++) {
       a[i] = StdRandom.uniformDouble(-0.5, 0.5);
   }
   return a;
}</pre>
```



White noise. Samples are uniformly random values.



implementation

```
public static double[] whiteNoise(double amplitude, double duration) {
   int n = numberOfSamples(duration);
   double[] a = new double[n];
   for (int i = 0; i < n; i++) {
        a[i] = StdRandom.uniformDouble(-amplitude, +amplitude);
   }
   return a;
}

while (true) {
        double[] a = Synth.whiteNoise(0.5, 1.0);
        double[] b = Synth.fadeOut(a, 20.0);
        StdAudio.play(b);
}</pre>
```

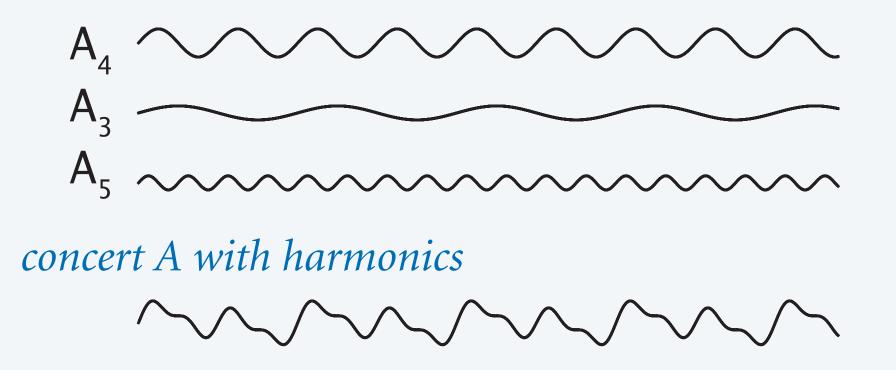
Superposition



Superposition. To combine two (or more) audio signals, add the corresponding samples.

Ex 1. Harmonics.

```
double duration = 5.0;
double[] a4 = Synth.sineWave(440.0, 0.50, duration);
double[] a3 = Synth.sineWave(220.0, 0.25, duration);
double[] a5 = Synth.sineWave(880.0, 0.25, duration);
double[] harmonics = Synth.superpose(a4, a3, a5);
StdAudio.play(harmonics);
```



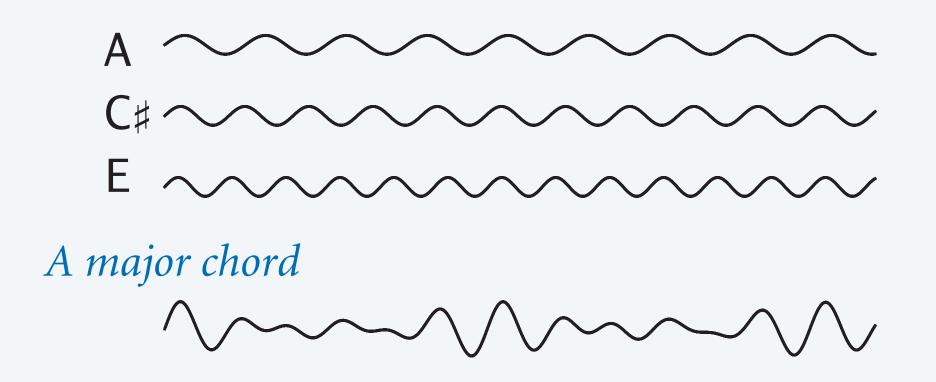
Superposition



Superposition. To combine two (or more) audio signals, add the corresponding samples.

- Ex 1. Harmonics.
- Ex 2. Chord.

```
double duration = 5.0;
double[] a = Synth.sineWave(440.00, 0.33, duration);
double[] c = Synth.sineWave(554.37, 0.33, duration);
double[] e = Synth.sineWave(659.26, 0.33, duration);
double[] chord = Synth.superpose(a, c, e);
StdAudio.play(chord);
```



Superposition



Superposition. To combine two (or more) audio signals, add the corresponding samples.

- Ex 1. Harmonics.
- Ex 2. Chord.
- Ex 3. Supersaw.

```
double freq = 220.0;
double amplitude = 0.05;  "detuned" frequencies

double duration = 10.0;
double[] a0 = Synth.sawWave(freq, amplitude, duration);
double[] a1 = Synth.sawWave(freq - 0.191, amplitude, duration);
double[] a2 = Synth.sawWave(freq - 0.109, amplitude, duration);
double[] a3 = Synth.sawWave(freq - 0.037, amplitude, duration);
double[] a4 = Synth.sawWave(freq + 0.031, amplitude, duration);
double[] a5 = Synth.sawWave(freq + 0.107, amplitude, duration);
double[] a6 = Synth.sawWave(freq + 0.181, amplitude, duration);
double[] supersaw = Synth.superpose(a0, a1, a2, a3, a4, a5, a6);
StdAudio.play(supersaw);
```



Goal. Play that tune, but with a supersaw.

public class SlayThatTune { public static void main(String[] args) { double amplitude = 0.5; transpose one while (!StdIn.isEmpty()) { octave lower int midi = StdIn.readInt(); double duration = StdIn.readDouble(); double freq = Synth.midiToFrequency(midi - 12); double[] a = Synth.supersawWave(freq, amplitude, duration); StdAudio.play(a); ~/cos126/libraries> java-introcs SlayThatTune < Arpeggio.txt [plays arpeggio] ~/cos126/libraries> java-introcs SlayThatTune < AxelF.txt ()) [plays beginning of Axel F]

client

Synth library

```
public class Synth {
                                                                                          implementation
  public static final double CONCERT_A = 440.0;
                    numberOfSamples(double duration) { ... }
  public static int
                                                                 utility methods
  public static double midiToFrequency(int midi)
  private static double sine(double freq, double t) { ... }
                                                                       private helper methods
  private static double square(double freq, double t) { ... }
  public static double[]
                       sineWave(double freq, double amplitude, double duration) { ... }
  public static double[]
                        squareWave(double freq, double amplitude, double duration) { ... }
                                                                                              create
  public static double[]
                           sawWave(double freq, double amplitude, double duration) { ... } 
                                                                                            sound waves
  public static double[] supersawWave(double freq, double amplitude, double duration) { ... }
  public static double[]
                        public static double[] superpose(double[] a, double[] b)
                                                                { ... }
                                                                { ... }
  public static double[] modulate(double[] a, double[] b)
                                                                                   manipulate
                                                                { ... }
  public static double[] fadeIn(double[] a, double lambda)
                                                                                   sound waves
  public static double[] fadeOut(double[] a, double lambda)
                                                                { ... }
  public static void main(String[] args) { ... }
```

Summary

API. Defines method headers and behavior for a library.

Client. Program that calls a library's methods.

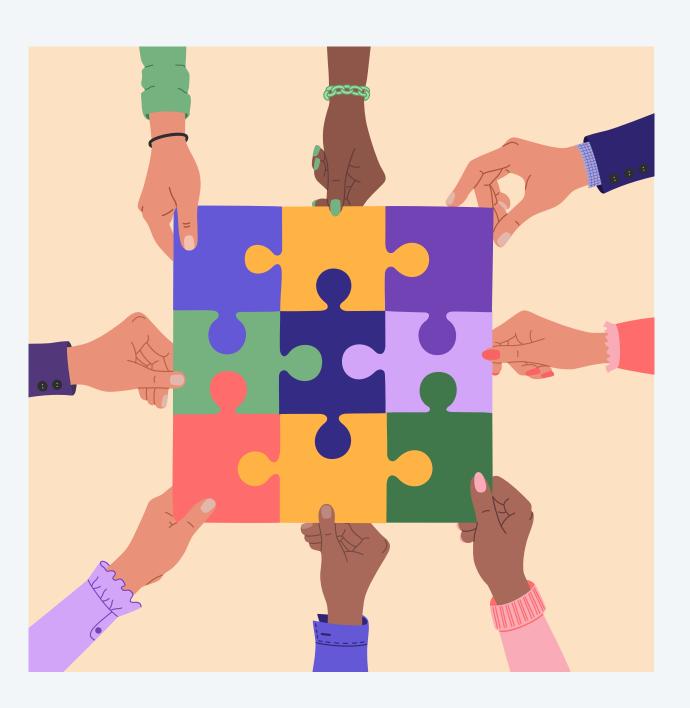
Implementation. Program that implements the library's functionality.

Encapsulation. Separating clients from implementation details by hiding information.

Benefits.

- Reusable libraries.
- Independent development of small programs.
- Collaboration with a team of programmers.

Sound synthesis. You can write programs to synthesize sound.



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