Proving the Equivalence of Two Modules

COS 326
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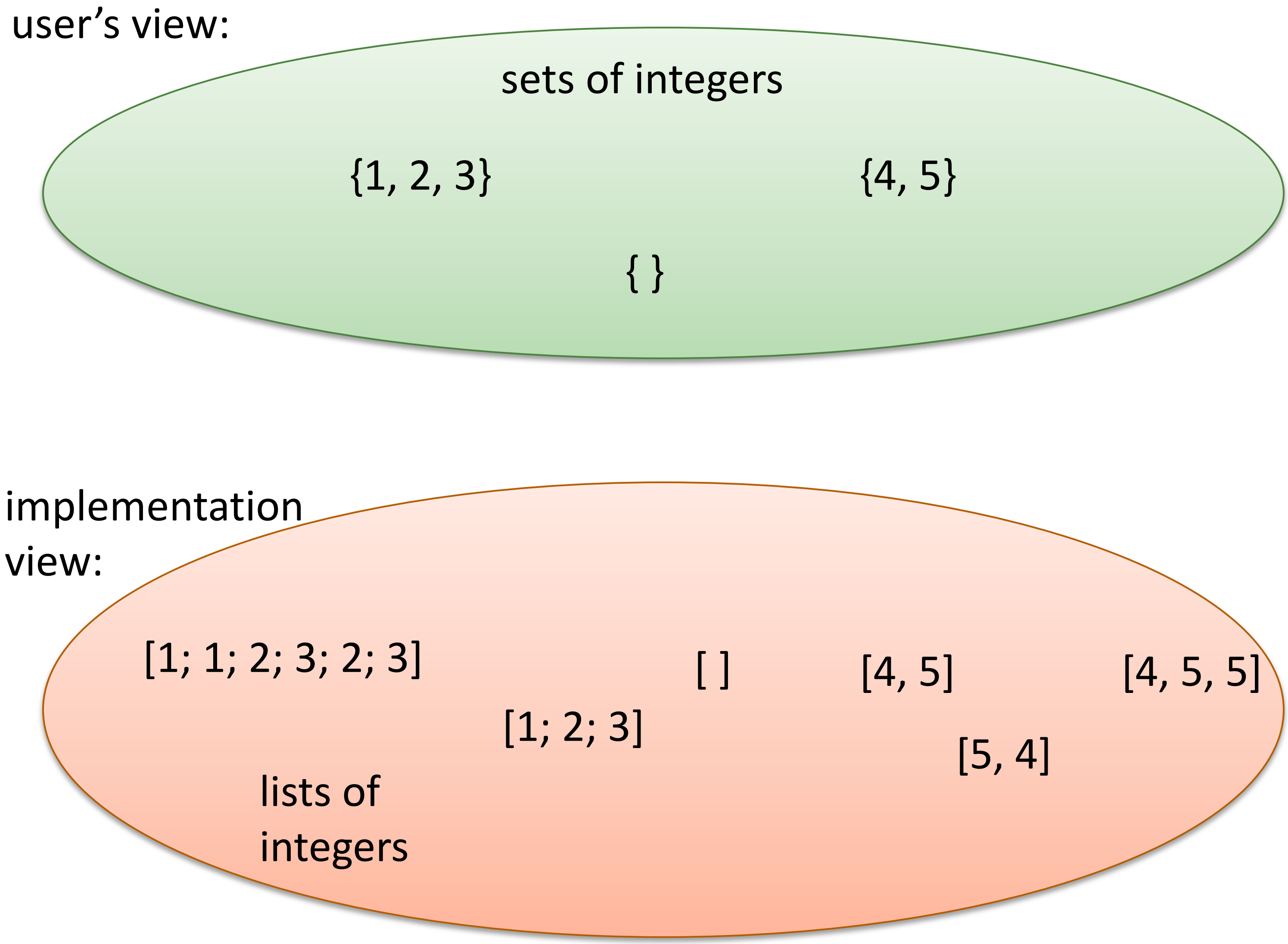
Abstraction

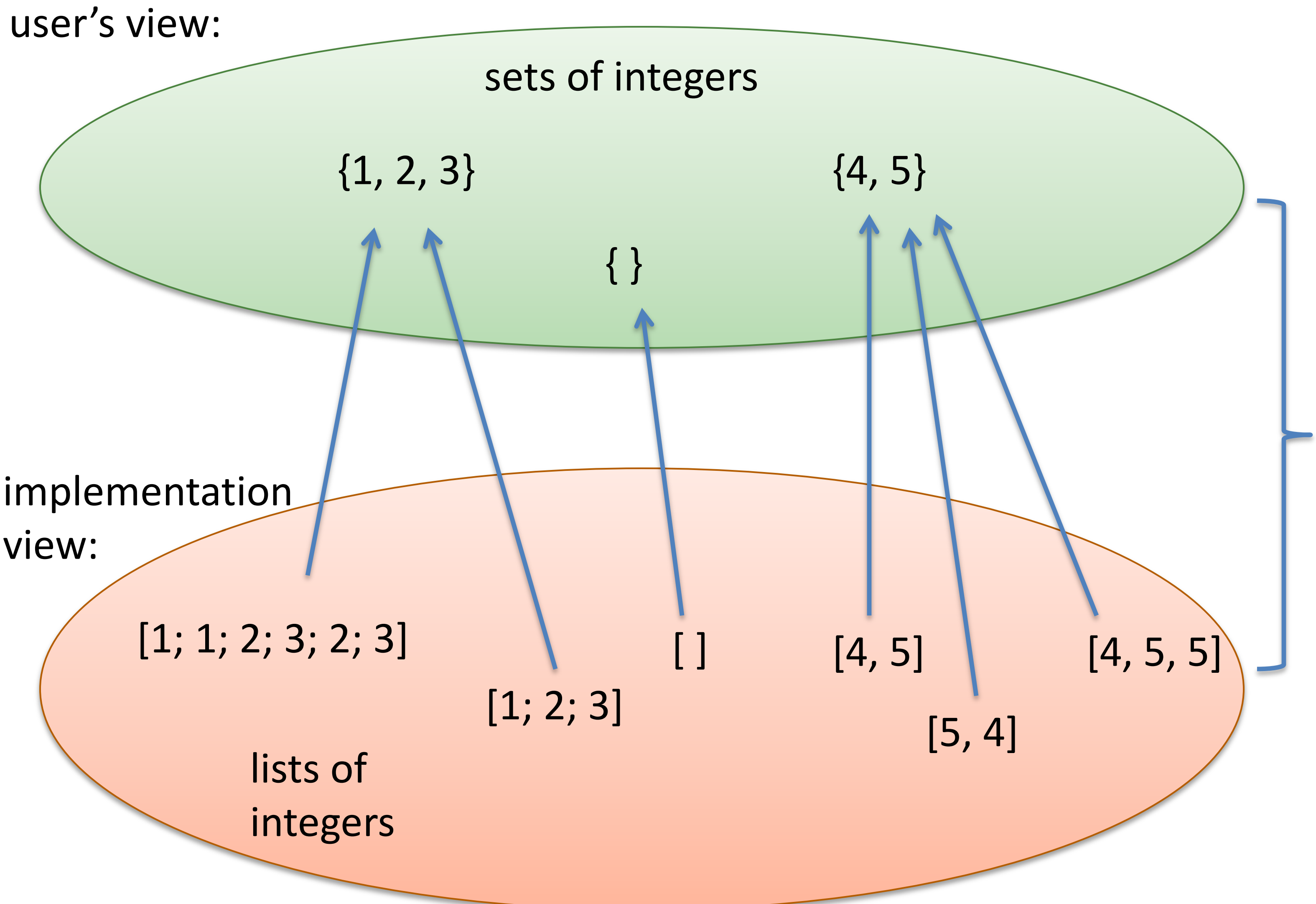
```
module type SET =
  sig
    type 'a set
    val empty : 'a set
    val mem : 'a -> 'a set -> bool
    ...
end
```

- When explaining our modules to clients, we would like to explain them in terms of *abstract values*
  - sets, not the lists (or maybe trees) that implement them
- From a client's perspective, operations act on abstract values
- Signature comments, specifications, preconditions and post-conditions should be defined in terms of those abstract values
- *How are these abstract values connected to the implementation?*

Abstraction



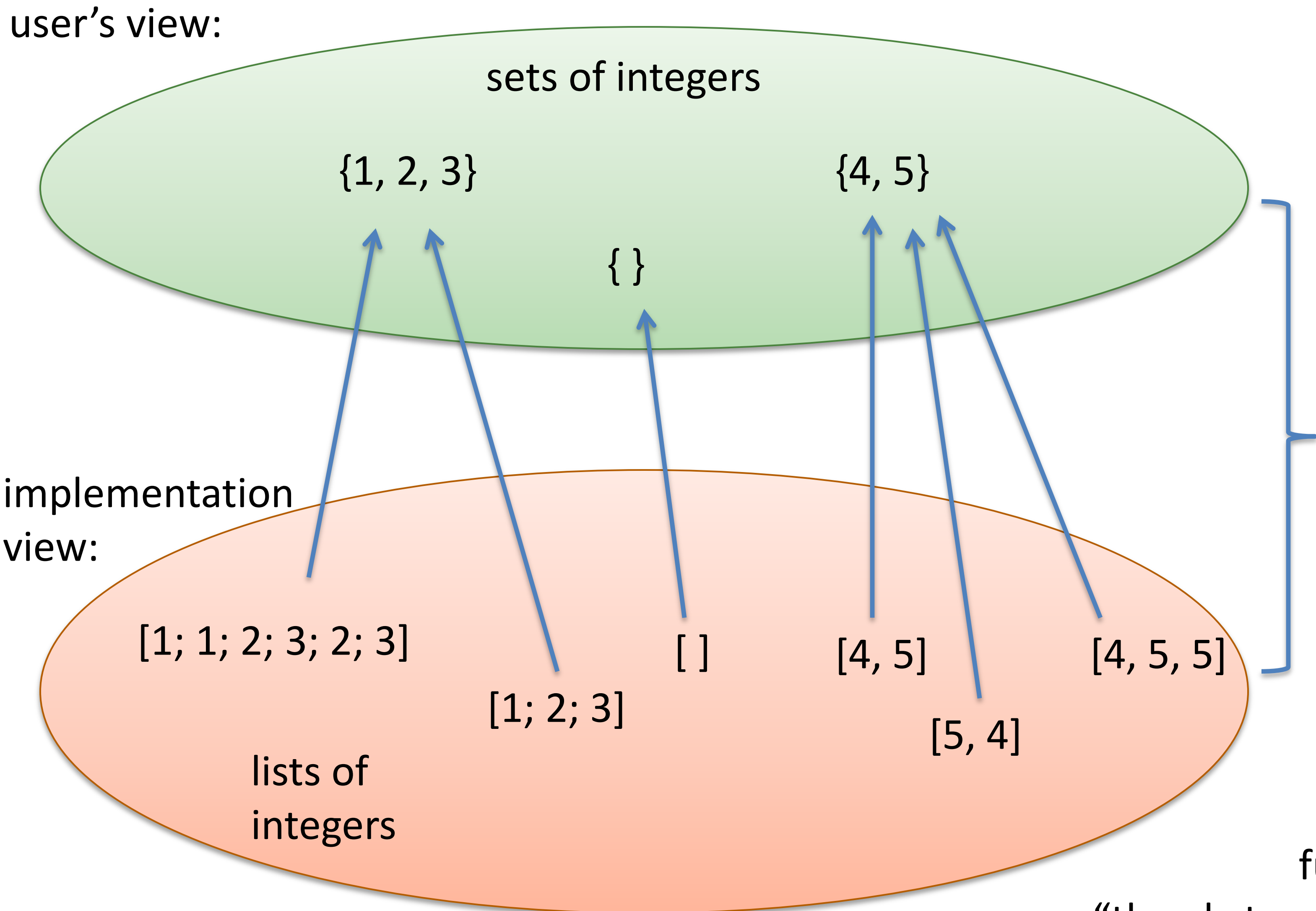
Abstraction



there's a relationship here, of course!

we are trying to *implement* the *abstraction*

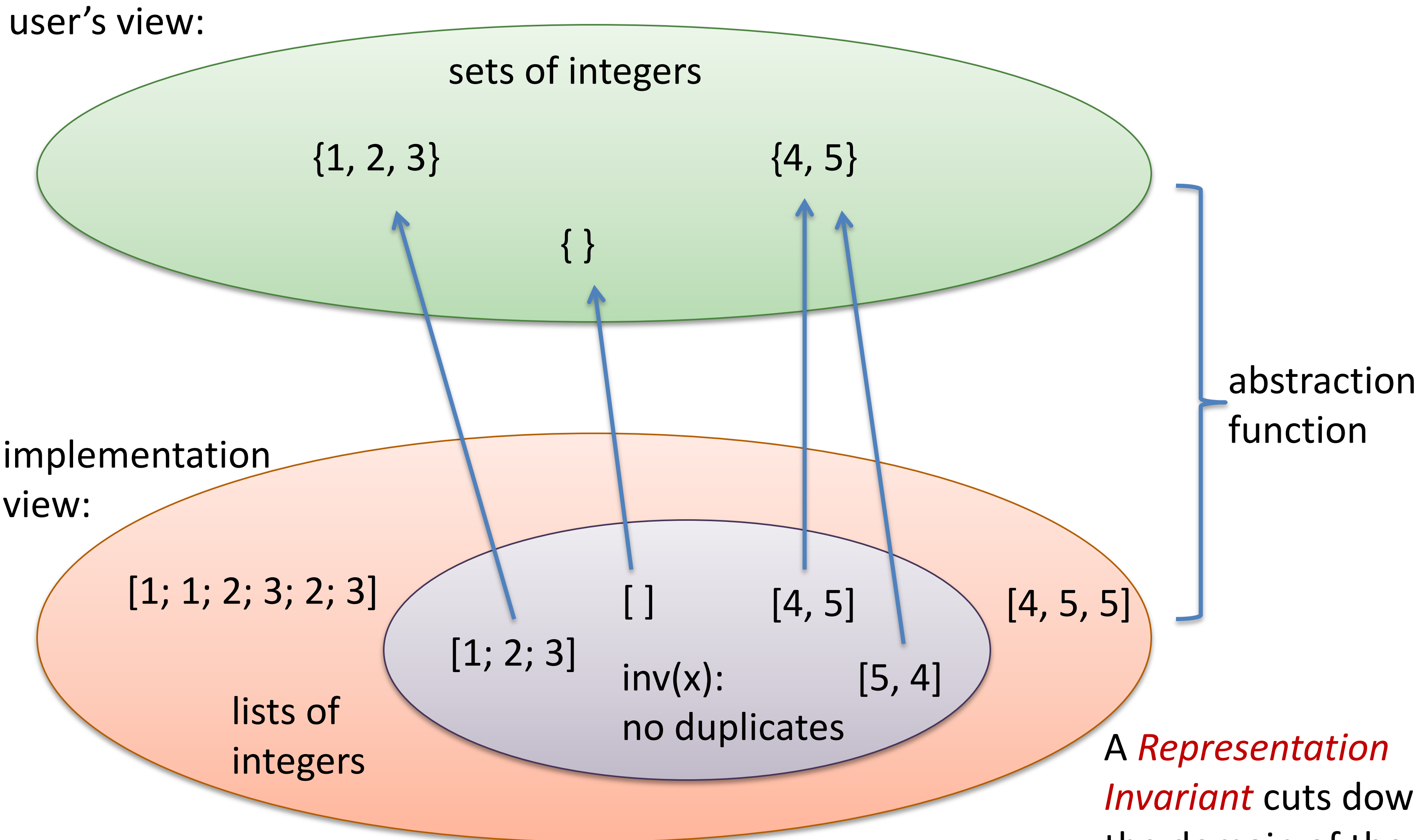
Abstraction



this relationship is a function: *it converts concrete values to abstract ones*

function called "the abstraction function"

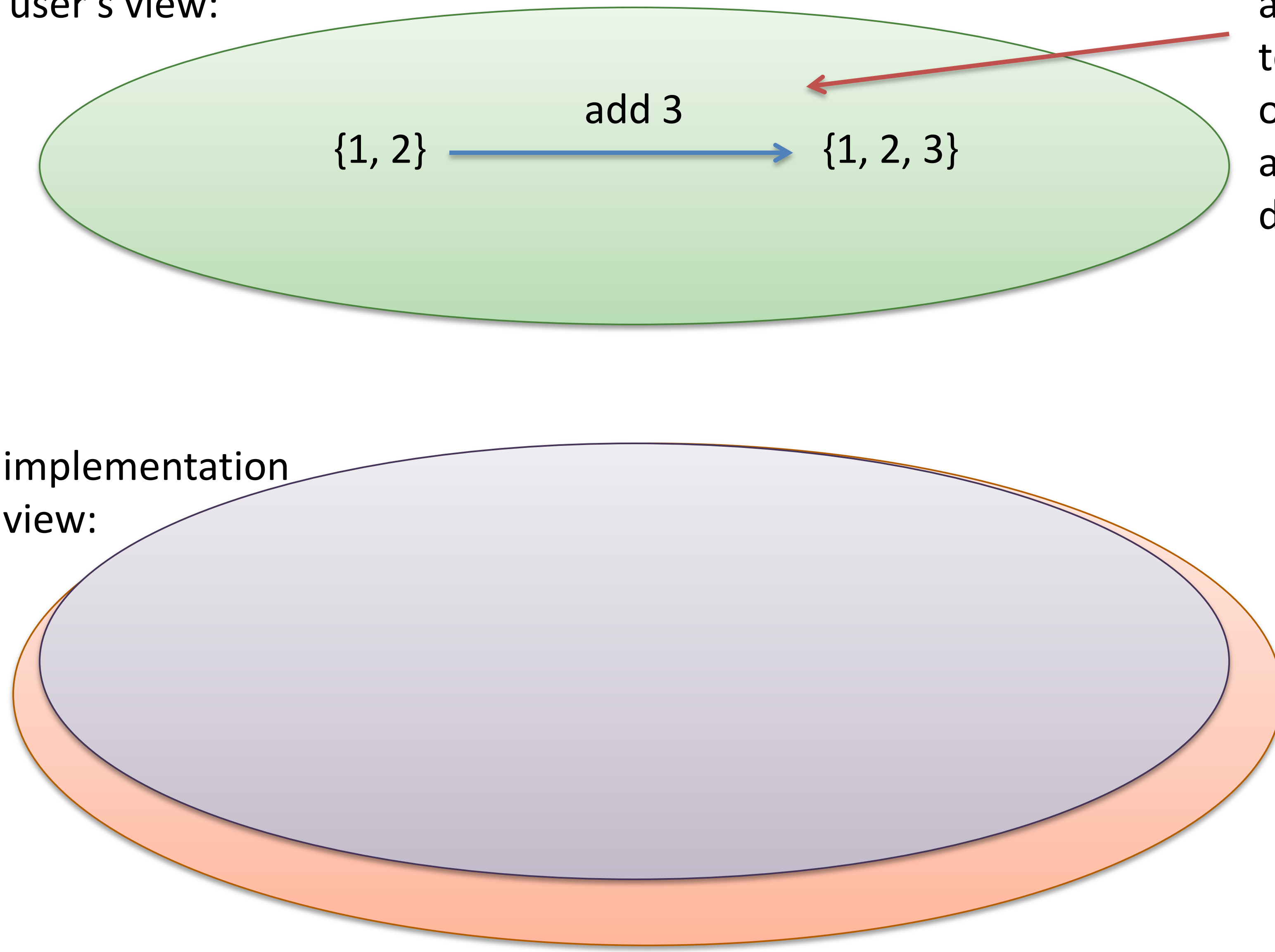
Abstraction



A *Representation Invariant* cuts down the domain of the abstraction function

Specifications

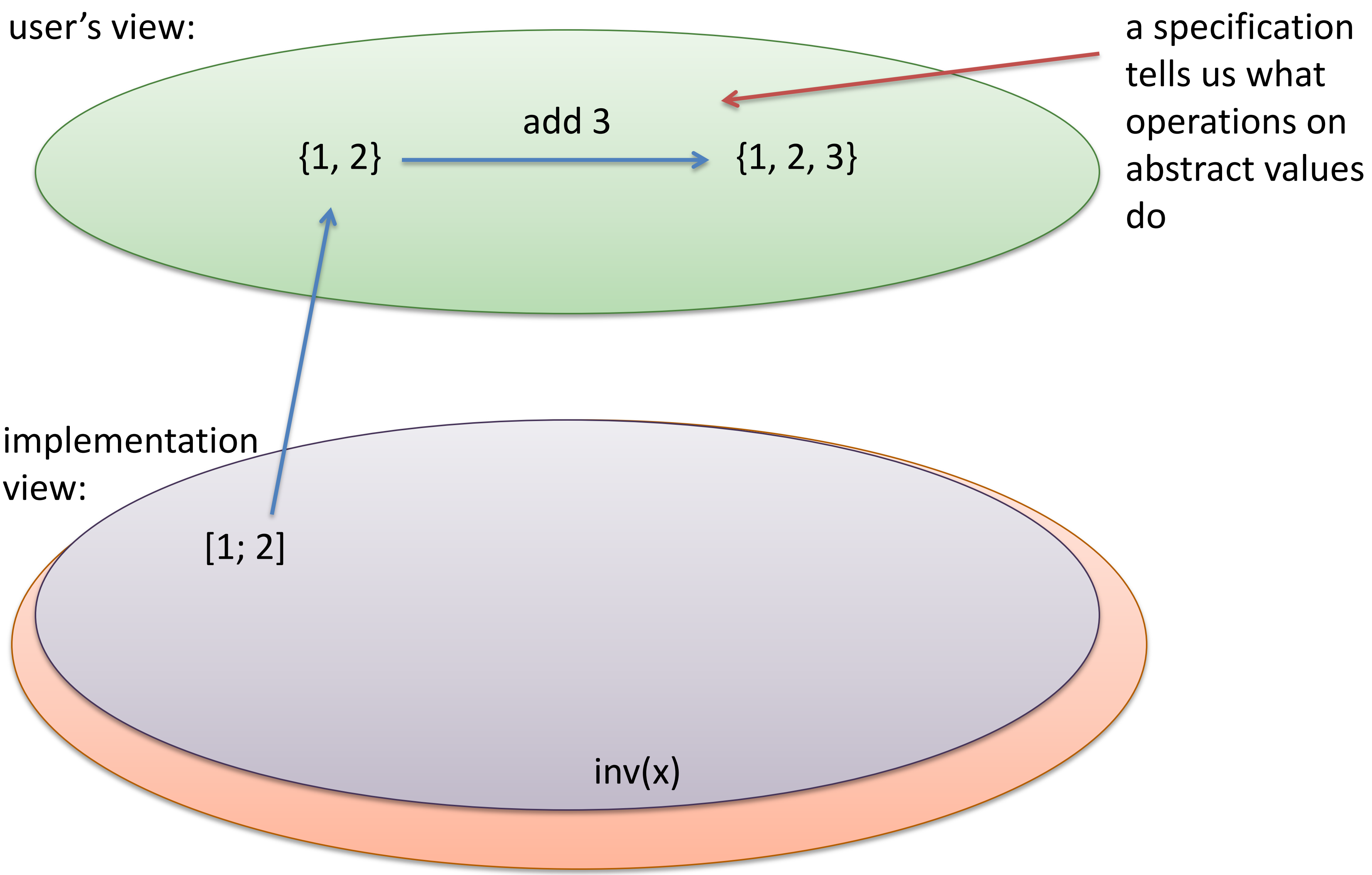
user's view:



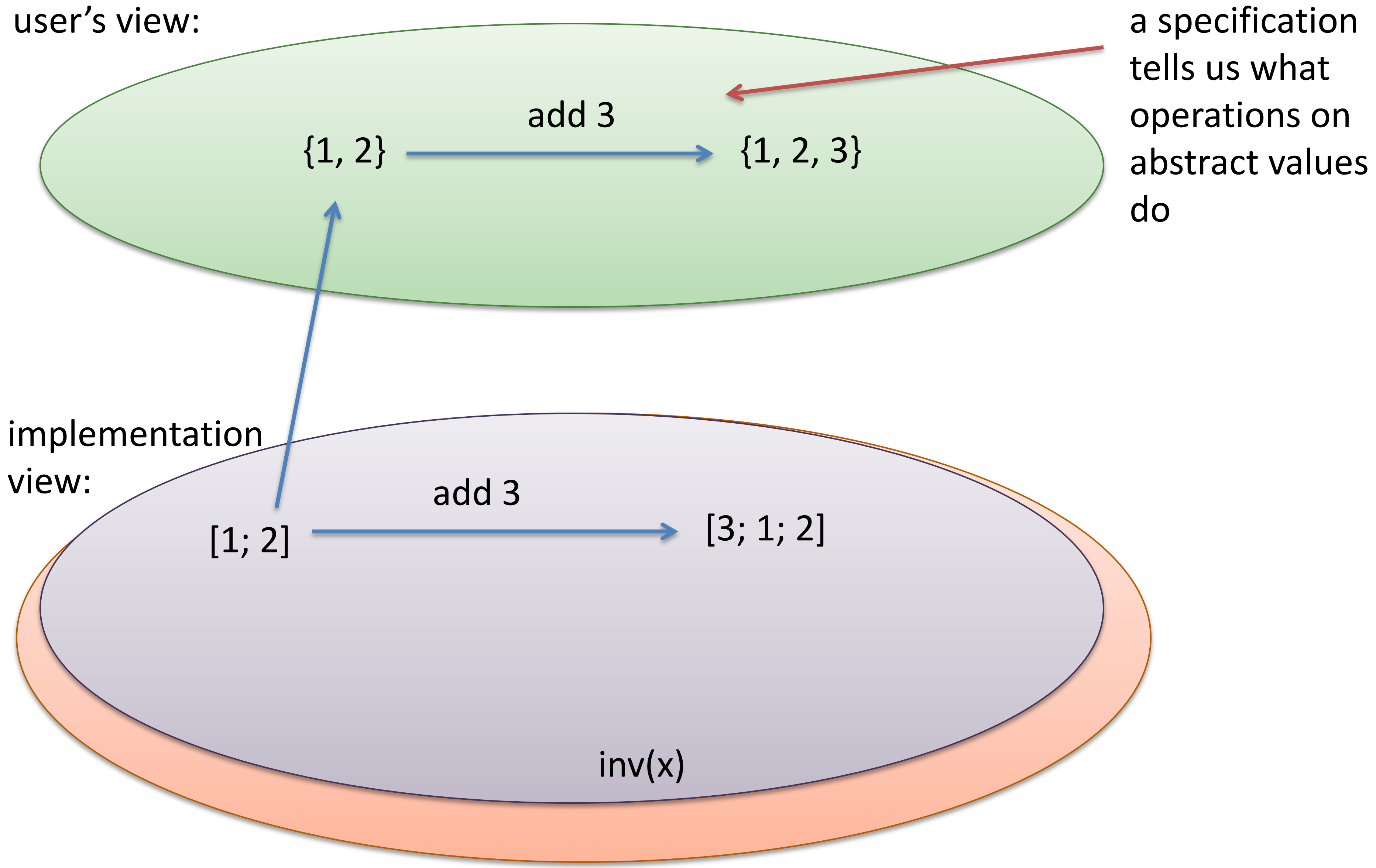
a specification tells us what operations on abstract values do

implementation view:

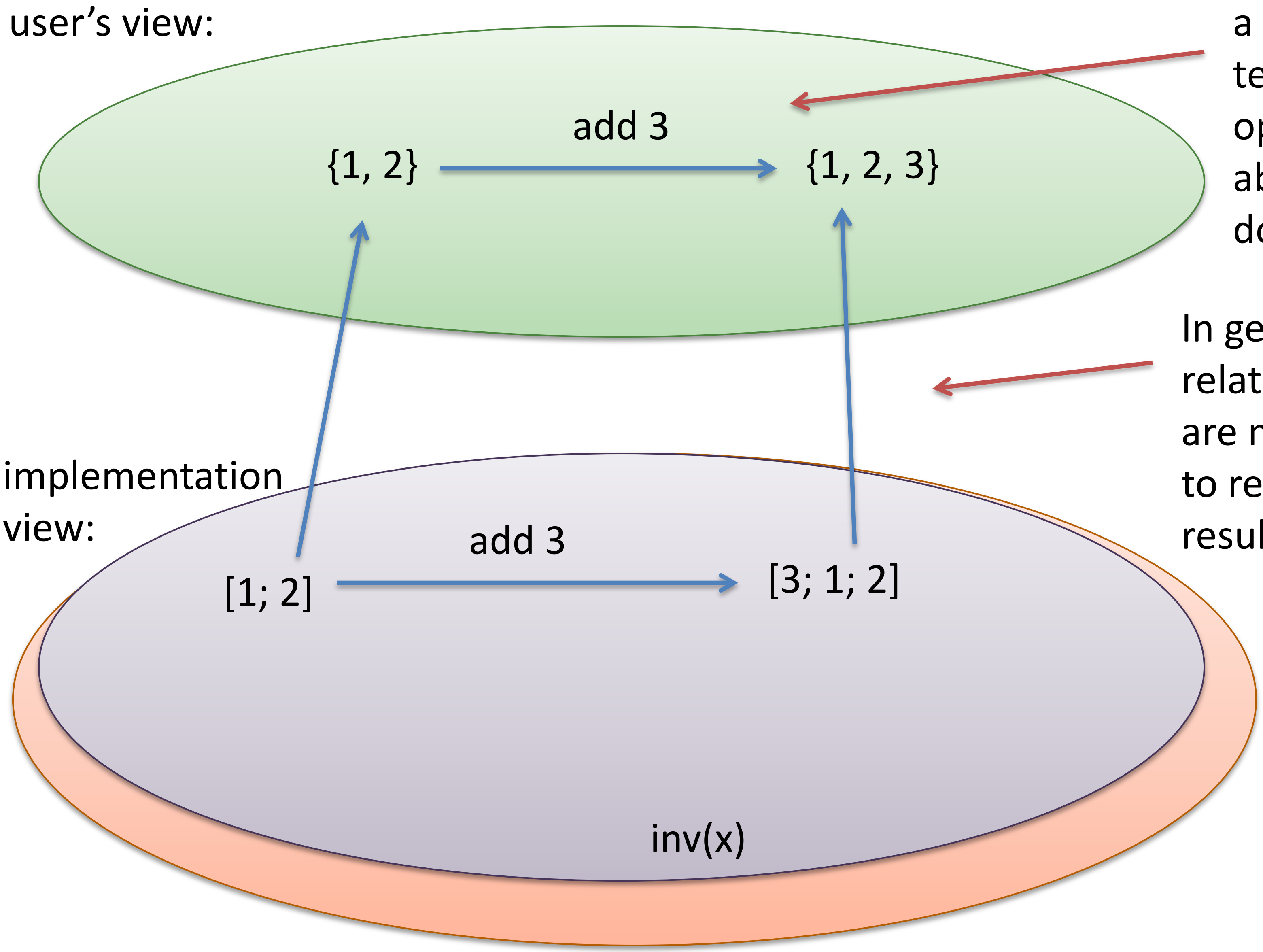
Specifications



Specifications



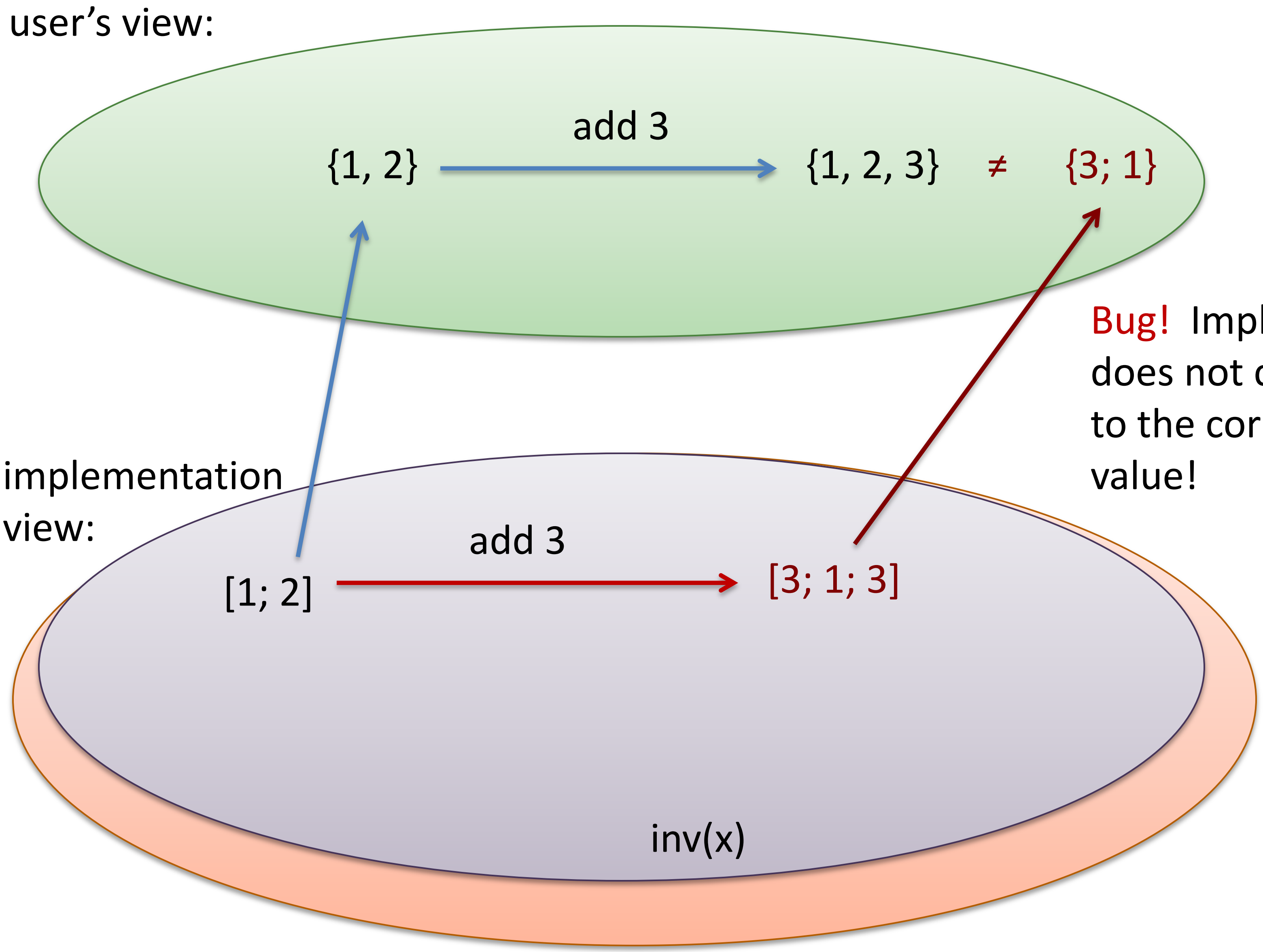
Specifications



a specification tells us what operations on abstract values do

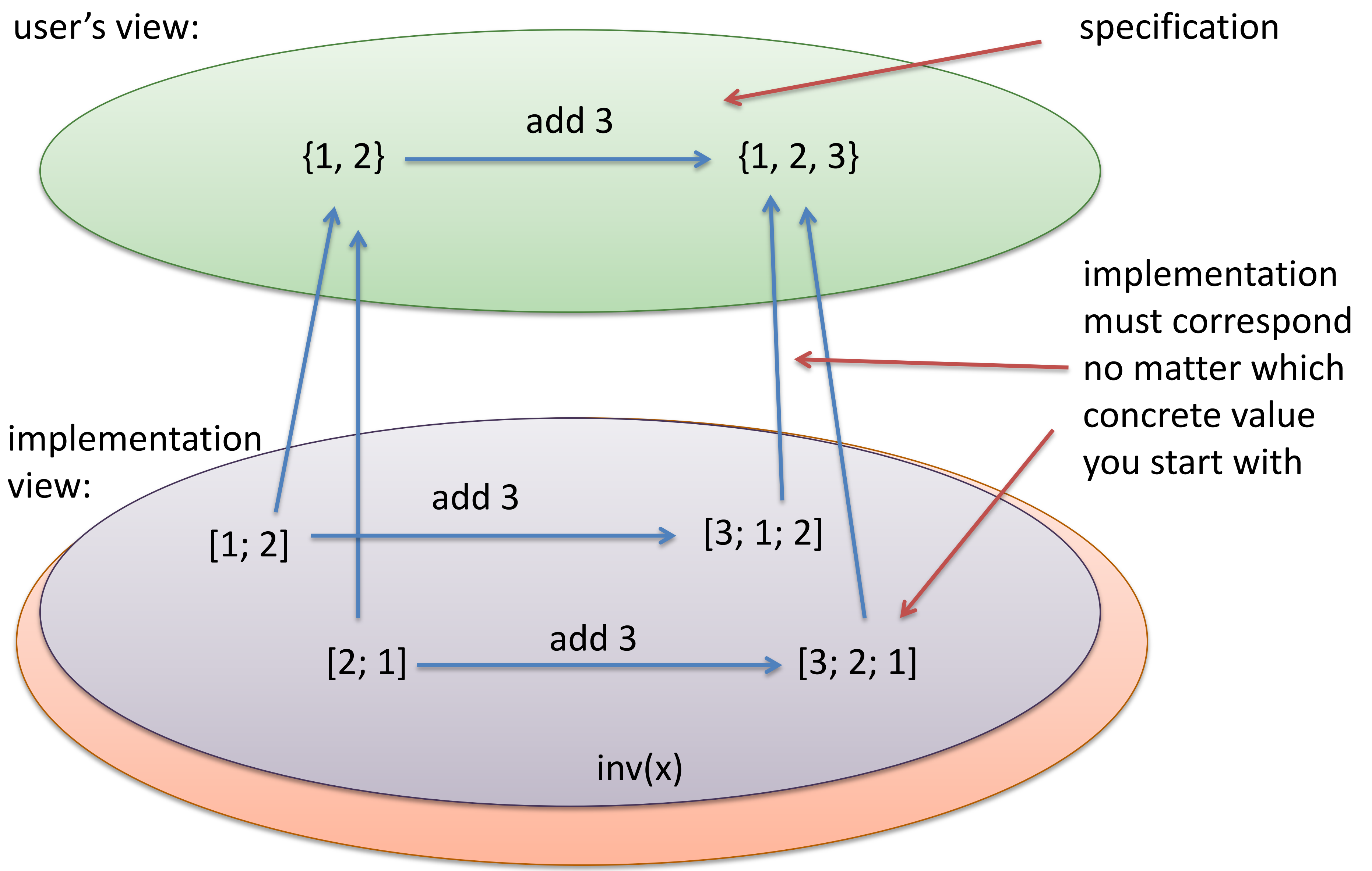
In general: related arguments are mapped to related results

Specifications

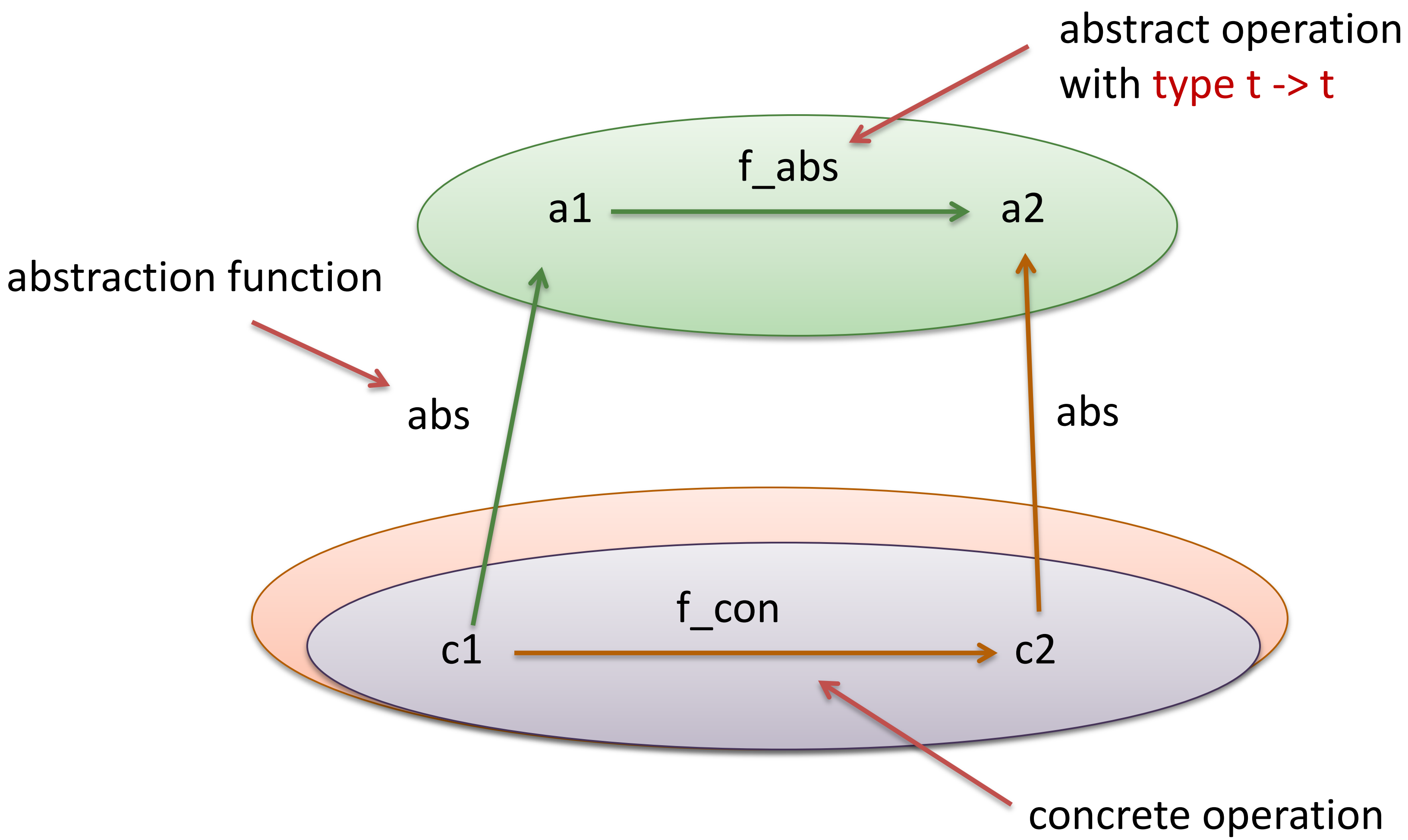


Bug! Implementation does not correspond to the correct abstract value!

Specifications



A more general view



to prove:

for all c1:t, if inv(c1) then f_abs (abs c1) == abs (f_con c1)

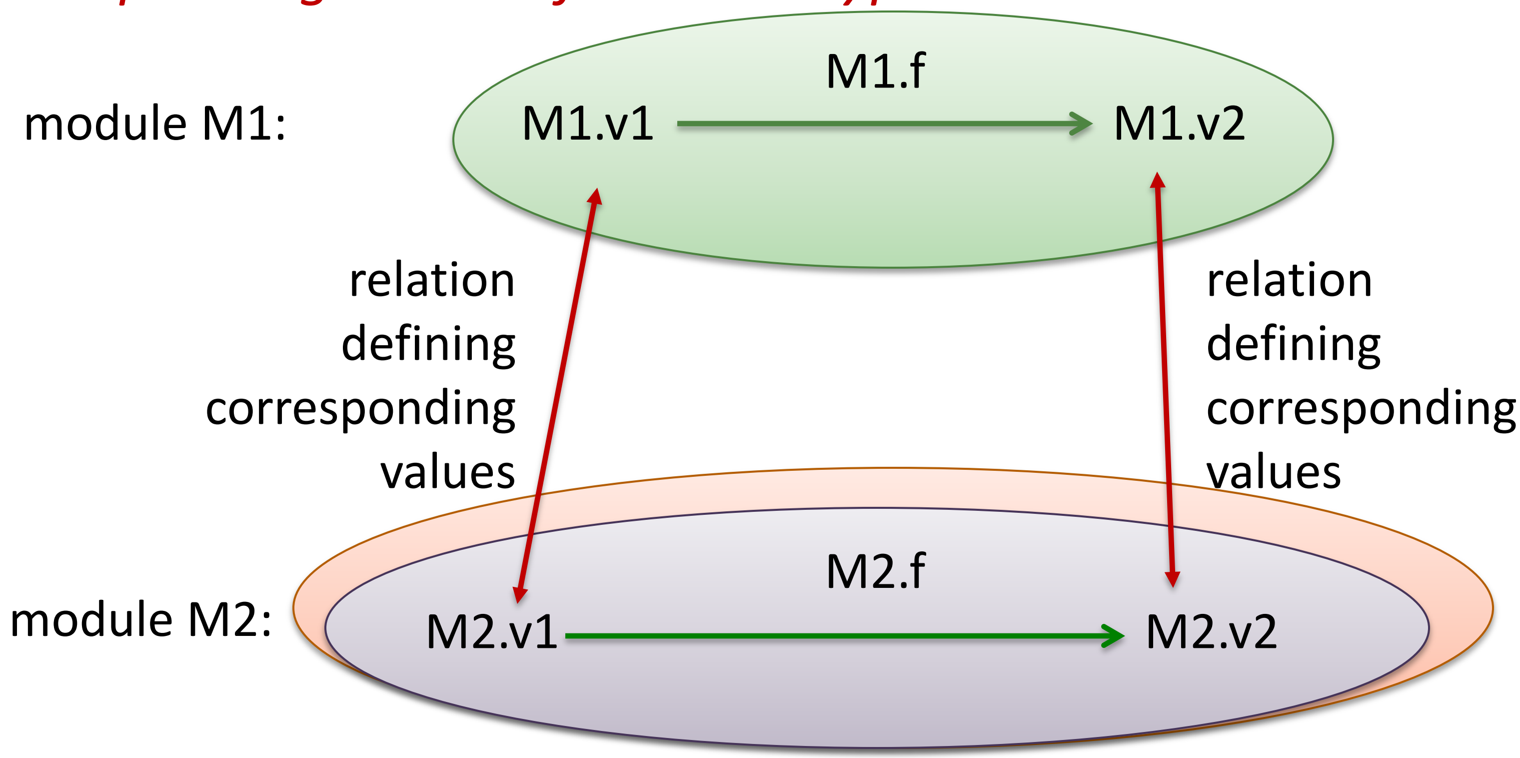
abstract then apply the abstract op == *apply concrete op then abstract*

Another Viewpoint

A specification is really just another implementation (in this viewpoint)

– but it's often simpler ("more abstract")

We can use similar ideas to compare *any two implementations of the same signature. Just come up with a relation between corresponding values of abstract type.*



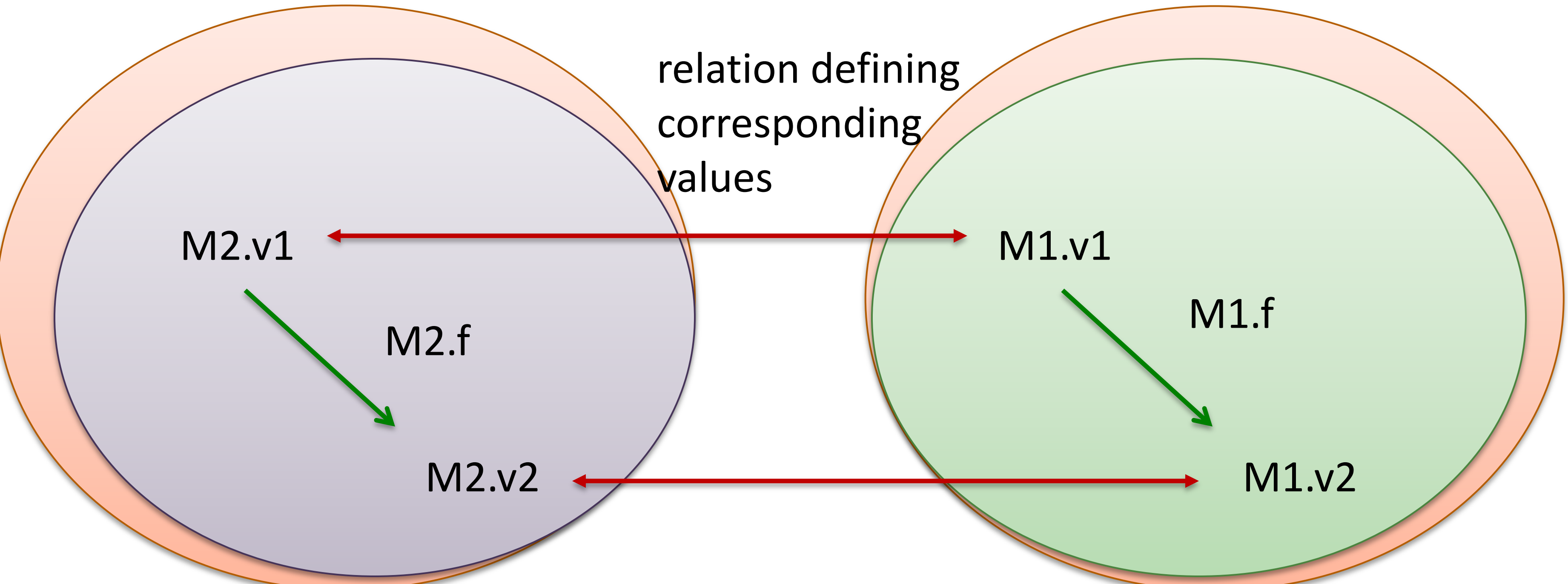
We ask: Do operations like f take related arguments to related results?

What is a specification?

It is really just another implementation

- but it's often simpler ("more abstract")

We can use similar ideas to compare *any two implementations of the same signature. Just come up with a relation between corresponding values of abstract type.*



One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
 end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
 end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
end
```

Consider a client that might use the module:

```
let x1 = M1.bump (M1.bump (M1.zero)
```

```
let x2 = M2.bump (M2.bump (M2.zero)
```

What is the relationship?

```
is_related (x1, x2) =
  x1   ==   x2/2 - 1
```

And it persists: Any sequence of operations produces related results from M1 and M2!

One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
  end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
  end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
end
```

Recall: A representation invariant is a property that holds for all values of abs. type:

- if M.v has abstract type t,
  - we want inv(M.v) to be true

Inter-module relations are a lot like representation invariants!

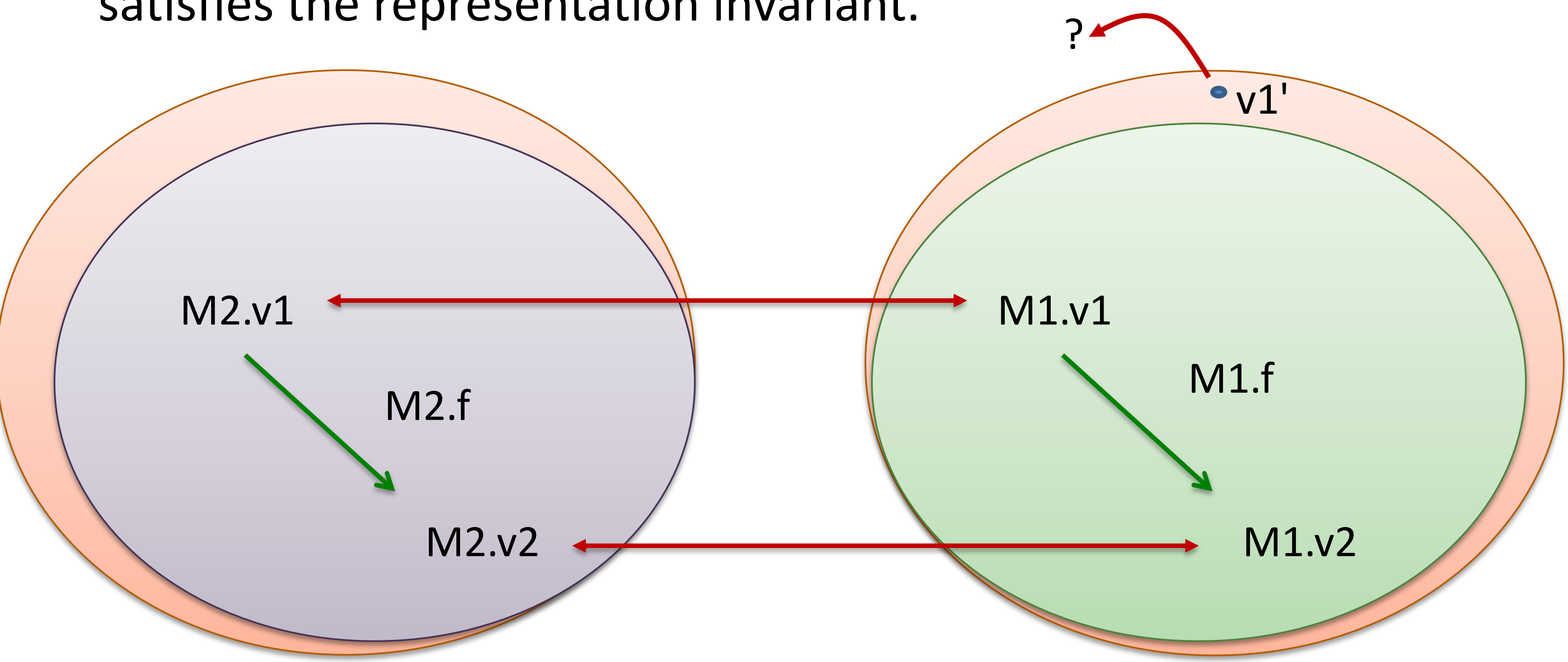
- if M1.v and M2.v have abstract type t,
  - we want is_related(M1.v, M2.v) to be true

It's just a relation between two modules instead of one

Relations may imply the Rep Inv

When defining our relation, we will often do so in a way that implies the representation invariant.

ie: a value in M1 will not be related to any value in M2 unless it satisfies the representation invariant.



One Signature, Two Implementations

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module type S =
  sig
   type t
   val zero : t
   val bump : t -> t
   val reveal : t -> int
 end
```

```
module M1 : S =
  struct
   type t = int
   let zero = 0
   let bump n = n + 1
   let reveal n = n
 end
```

```
module M2 : S =
  struct
   type t = int
   let zero = 2
   let bump n = n + 2
   let reveal n = n/2 - 1
 end
```

is_related (x1, x2) =
  (x1   ==   x2/2 – 1) *&& x1 >= 0 && even x2*

is_related (x1, x2) implies *x1 >= 0*

rep inv for M1

is_related (x1, x2) implies *even x2 && x2 > 0*

rep inv for M2

One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
 end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
 end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
end
```

But For Now:

```
is_related (x1, x2) =
  (x1   ==   x2/2 – 1)
```

One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
  end
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module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
  end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
  end
```

Consider zero, which has abstract type t.

```
is_related (x1, x2) =
  x1   ==   x2/2 - 1
```

Must prove: is_related (M1.zero, M2.zero)

Equivalent to proving: M1.zero == M2.zero/2 – 1

Proof:

```
   M1.zero
== 0                    (substitution)
== 2/2 – 1              (math)
== M2.zero/2 – 1        (substitution)
```

One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
  end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
  end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
  end
```

```
is_related (x1, x2) =
  x1   ==   x2/2 - 1
```

Consider bump, which has abstract type t -> t.

Must prove for all v1:int, v2:int
if is_related(v1,v2) then is_related (M1.bump v1, M2.bump v2)

Proof:
(1) Assume is_related(v1, v2).
(2) v1 == v2/2 – 1 (by def)

Next, prove:
(M2.bump v2)/2 – 1 == M1.bump v1

```
   (M2.bump v2)/2 - 1
== (v2 + 2)/2 – 1          (eval)
== (v2/2 – 1) + 1          (math)
== v1 + 1                  (by 2)
== M1.bump v1              (eval, reverse)
```

One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
  end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
  end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
  end
```

```
is_related (x1, x2) =
  x1   ==   x2/2 - 1
```

Consider reveal, which has abstract type t -> int.

Must prove for all v1:int, v2:int
if is_related(v1,v2) then M1.reveal v1 == M2.reveal v2

Proof:
(1) Assume is_related(v1, v2).
(2) v1 == v2/2 – 1 (by def)

Next, prove:
M2.reveal v2 == M1.reveal v1

M2.reveal v2	
== v2/2 – 1	(eval)
== v1	(by 2)
== M1.reveal v1	(eval, reverse)

Summary of Proof Technique

To prove M1 == M2 relative to signature S,

- Start by defining a relation "is_related":
  - is_related (v1, v2) should hold for values with abstract type t when v1 comes from module M1 and v2 comes from module M2

- Extend "is_related" to types other than just abstract t. For example:
  - if v1, v2 have type int, then they must be exactly the same
    - ie, we must prove: v1 == v2
  - if v1, v2 have type s1 -> s2 then we consider arg1, arg2 such that:
    - if is_related(arg1, arg2) at type s1 then we prove
    - is_related(v1 arg1, v2 arg2) at type s2
  - if v1, v2 have type s option then we must prove:
    - v1 == None and v2 == None, or
    - v1 == Some u1 and v2 == Some u2 and is_related(u1, u2) at type s

- For each val v:s in S, prove is_related(M1.v, M2.v) at type s

MODULES WITH DIFFERENT IMPLEMENTATION TYPES

One Signature, Two Implementations

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
 end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump n = n + 1
    let reveal n = n
 end
```

```
module M2 : S =
  struct
    type t = int
    let zero = 2
    let bump n = n + 2
    let reveal n = n/2 - 1
 end
```

Different representation types

```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump x = x + 1
    let reveal x = x
  end
```

```
module M2 : S =
  struct
    type t = Zero | S of t
    let zero = Zero
    let bump x = S x
    let rec reveal x =
      match x with
      | Zero -> 0
      | S x -> 1 + reveal x
  end
```

The Same Principle Applies!

Two modules with abstract type t will be declared equivalent if:

- one can *define a relation between corresponding values of type t*
- one can show that *the relation is preserved by all operations*

If we do indeed show the relation is "preserved" by operations of the module (an idea that depends crucially on the *signature* of the module) then *no client will ever be able to tell the difference between the two modules even though their data structures are implemented by completely different types*!

Different Representation Types

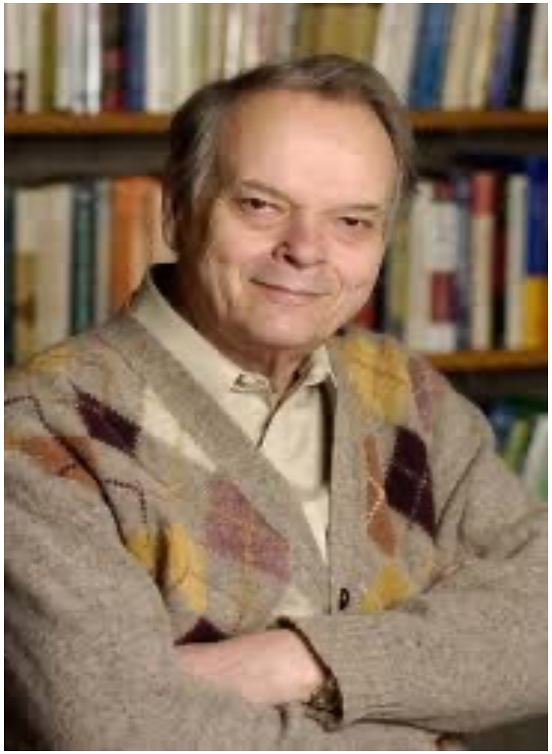
```
module type S =
  sig
    type t
    val zero : t
    val bump : t -> t
    val reveal : t -> int
end
```

```
module M1 : S =
  struct
    type t = int
    let zero = 0
    let bump x = x + 1
    let reveal x = x
  end
```

```
module M2 : S =
  struct
    type t = Zero | S of t
    let zero = Zero
    let bump x = S x
    let rec reveal x =
      match x with
      | Zero -> 0
      | S x -> 1 + reveal x
  end
```

```
is_related (x1, x2) =
  x1   ==   M2.reveal x2
```

Module Abstraction



John Reynolds,   1935-2013

Discovered the polymorphic lambda calculus (first polymorphic type system).

Developed ***Relational Parametricity****:* A technique for proving the equivalence of modules.

Summary: Abstraction and Equivalence

Abstraction functions define the relationship between a concrete implementation and the abstract view of the client

- We should prove concrete operations implement abstract ones described to our customers/clients

We prove any two modules are equivalent by

- Defining a relation between values of the modules with abstract type
- We get to assume the relation holds on inputs; prove it on outputs

Rep invariants and "is_related" predicates are called logical relations

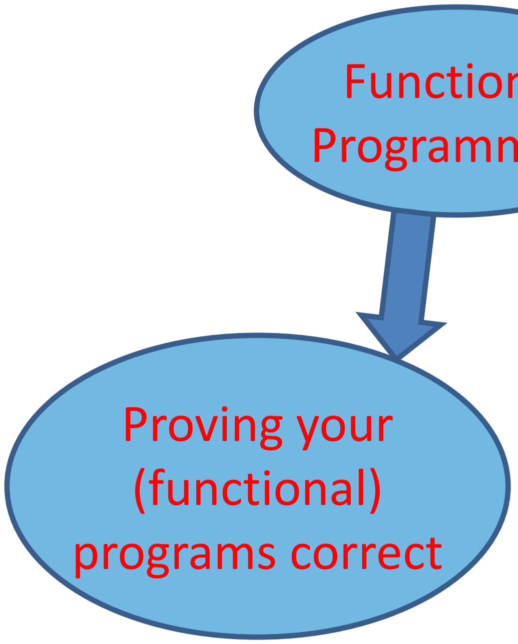
Software Verification

(preview of COS 510 “Programming Languages”)

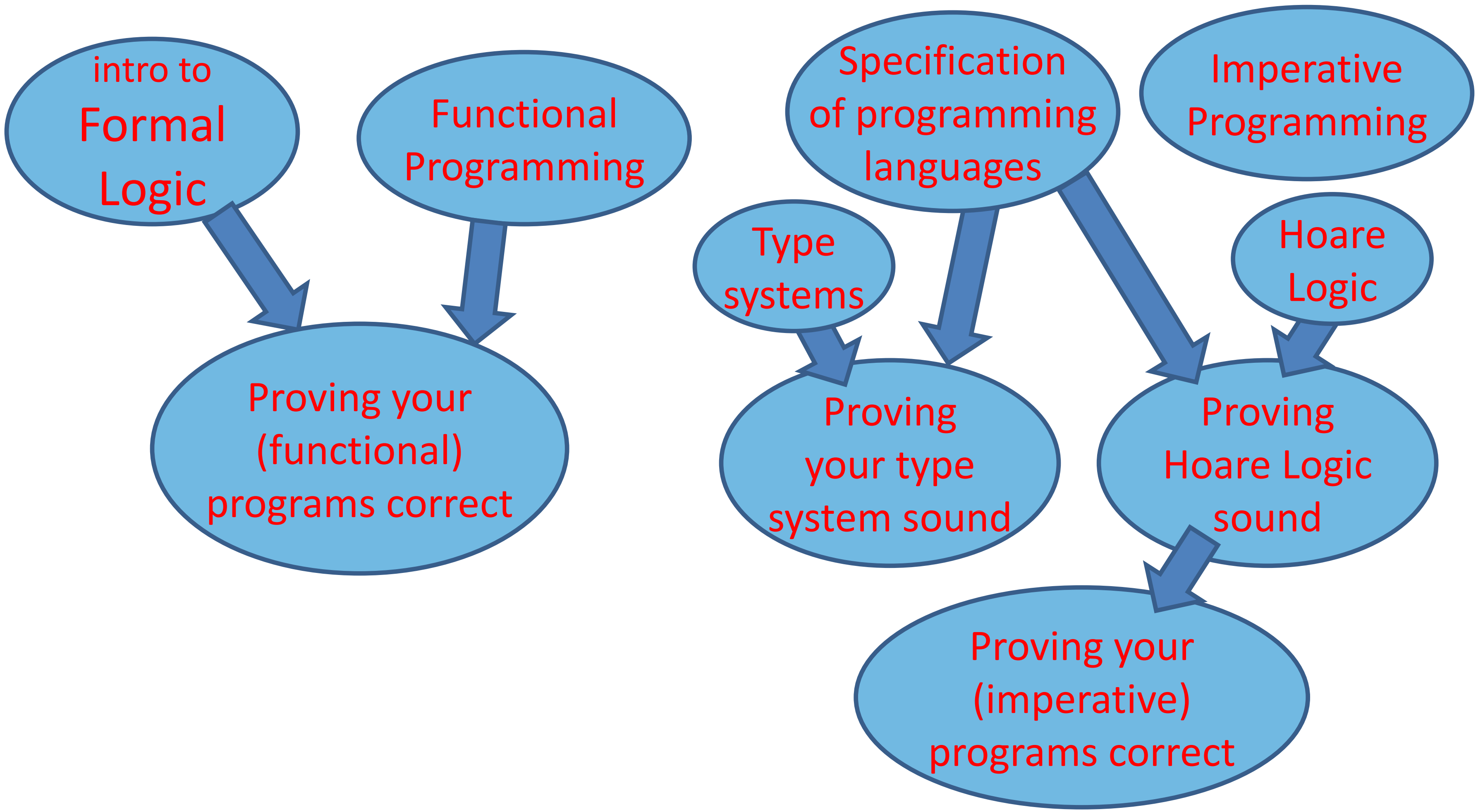
Andrew W. Appel

Princeton
University

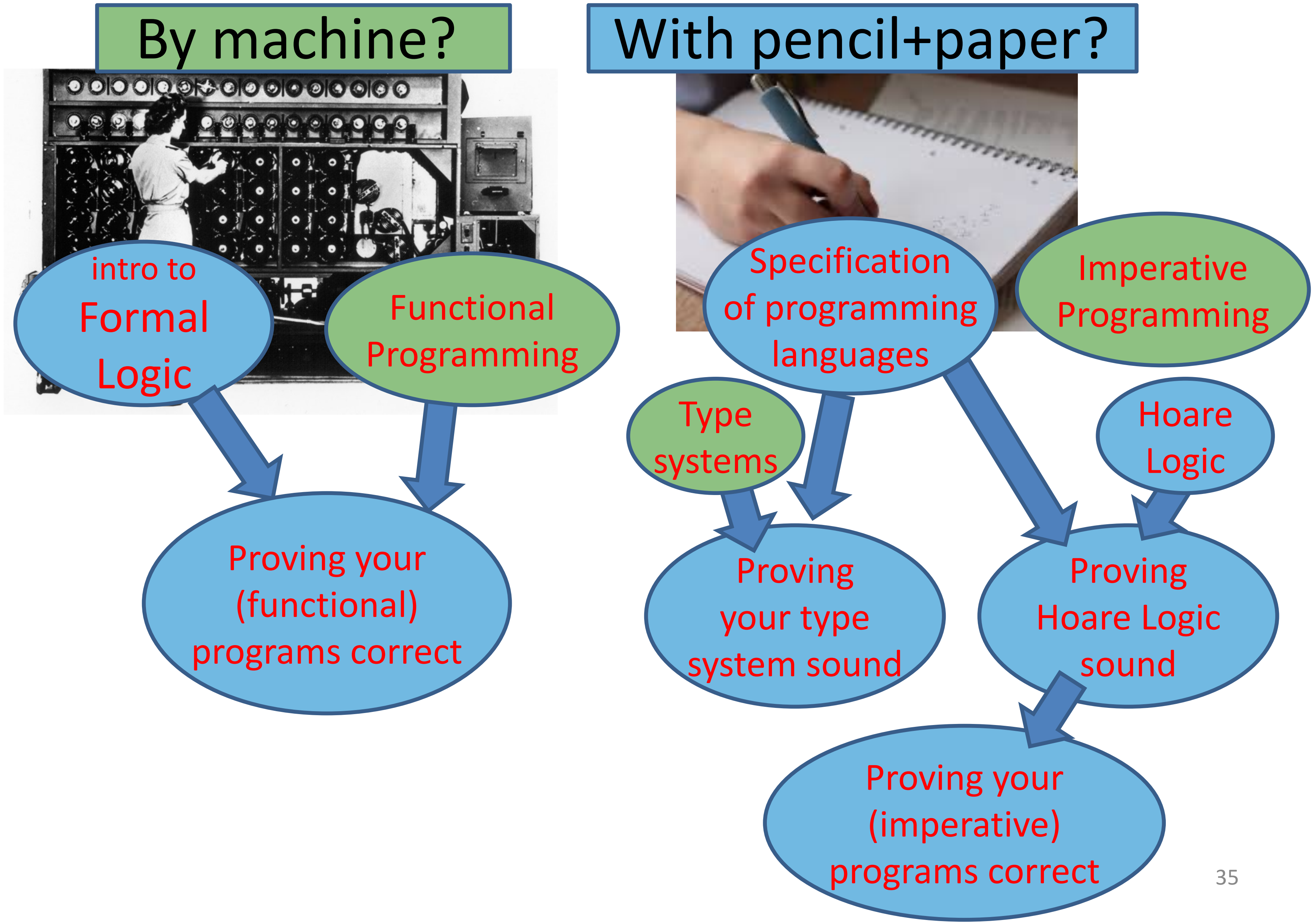
Formal reasoning about programs



Formal reasoning about programs and programming languages

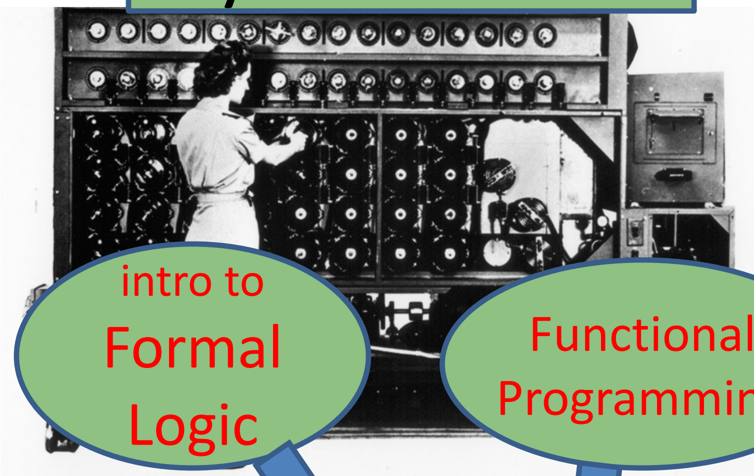


Which of these things do we do



We can do all of these

By machine!

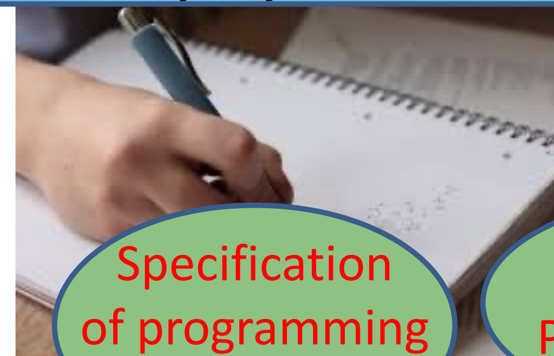


intro to Formal Logic

Functional Programming

Proving your (functional) programs correct

pencil+paper? Really?



Specification of programming languages

Imperative Programming

Type systems

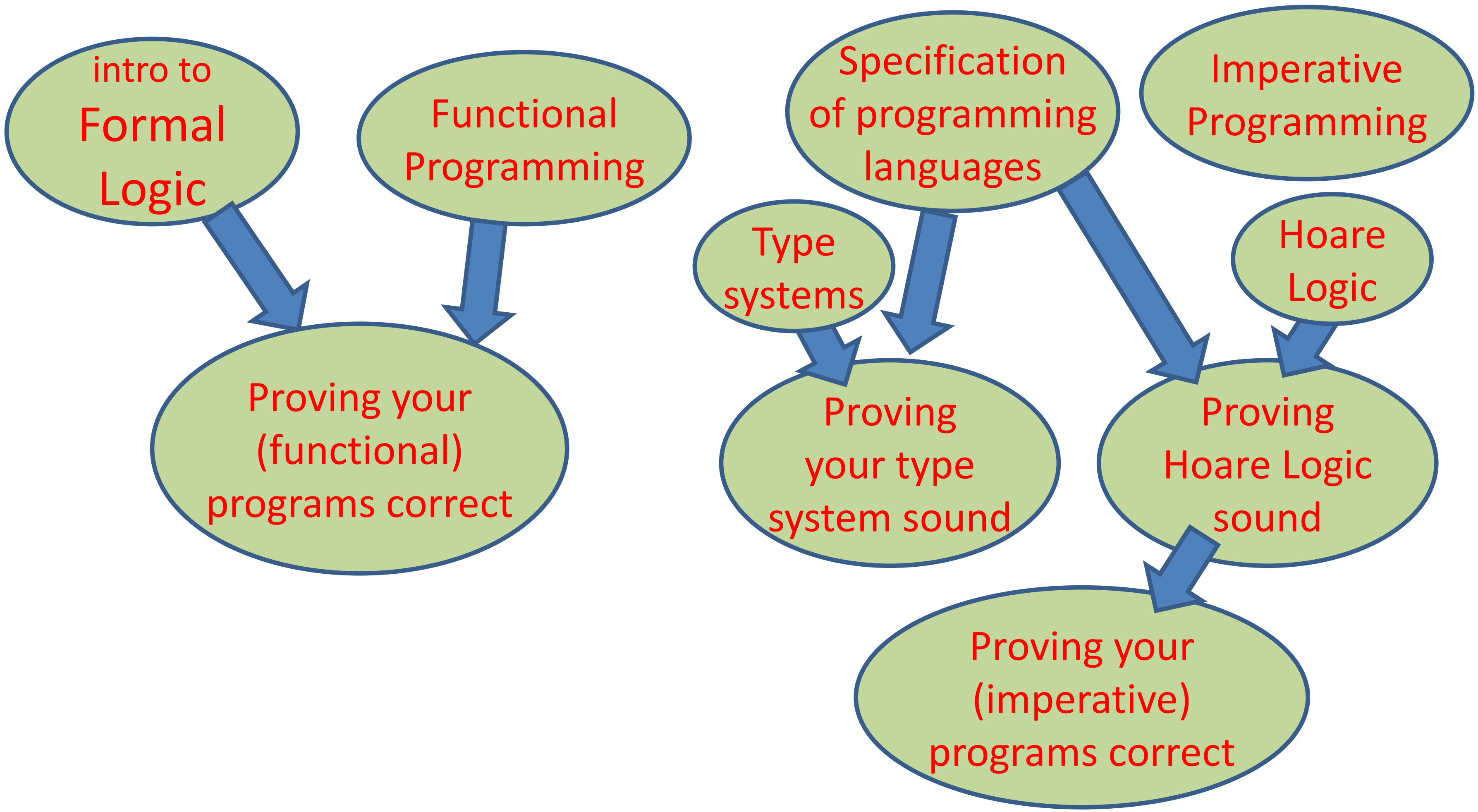
Hoare Logic

Proving your type system sound

Proving Hoare Logic sound

Proving your (imperative) programs correct

COS 510: Machine-checked, formal reasoning about programs and programming languages



EXAMPLE: LENGTH, APP

```
Require Import List.

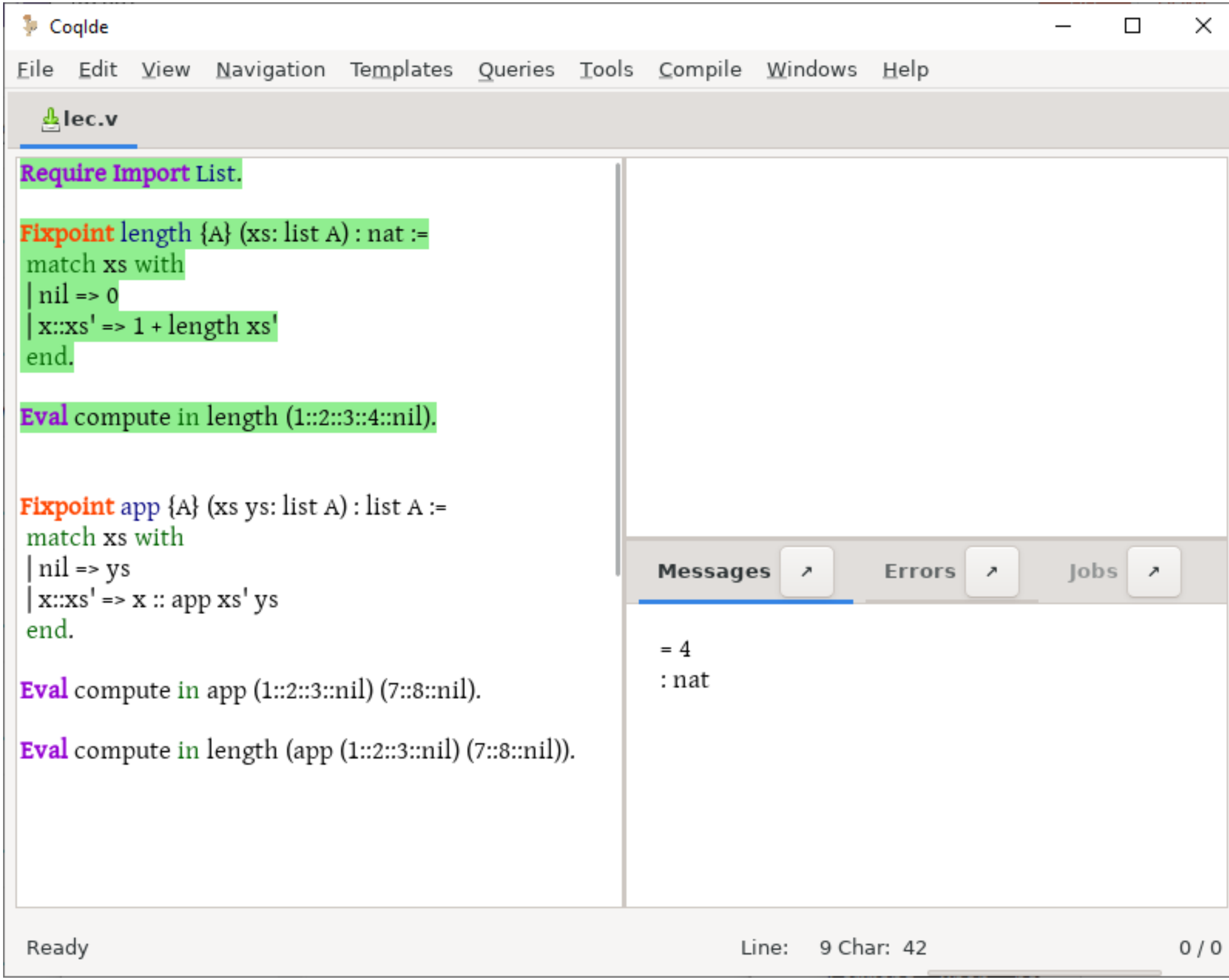
Fixpoint length {A} (xs: list A) : nat :=
 match xs with
 | nil => 0
 | x::xs' => 1 + length xs'
 end.

Eval compute in length (1::2::3::4::nil).


Fixpoint app {A} (xs ys: list A) : list A :=
 match xs with
 | nil => ys
 | x::xs' => x :: app xs' ys
 end.

Eval compute in app (1::2::3::nil) (7::8::nil).

Eval compute in length (app (1::2::3::nil) (7::8::nil)).
```


CoqIde
File Edit View Navigation Templates Queries Tools Compile Windows Help
lec.v
Require Import List.
Fixpoint length {A} (xs: list A) : nat :=
match xs with
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end.
Eval compute in length (1::2::3::4::nil).
Fixpoint app {A} (xs ys: list A) : list A :=
match xs with
| nil => ys
| x::xs' => x :: app xs' ys
end.
Eval compute in app (1::2::3::nil) (7::8::nil).
Eval compute in length (app (1::2::3::nil) (7::8::nil)).
Messages
Errors
Jobs
= 4
: nat
Ready
Line: 9 Char: 42
0 / 0

CoqIde

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lec.v

```
Require Import List.

Fixpoint length {A} (xs: list A) : nat :=
 match xs with
 | nil => 0
 | x::xs' => 1 + length xs'
 end.

Eval compute in length (1::2::3::4::nil).


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 match xs with
 | nil => ys
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 end.

Eval compute in app (1::2::3::nil) (7::8::nil).

Eval compute in length (app (1::2::3::nil) (7::8::nil)).
```

Messages ↗ Errors ↗ Jobs ↗

Ready Line: 17 Char: 1 0 / 0

CoqIde

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lec.v

```
Require Import List.

Fixpoint length {A} (xs: list A) : nat :=
 match xs with
 | nil => 0
 | x::xs' => 1 + length xs'
 end.

Eval compute in length (1::2::3::4::nil).


Fixpoint app {A} (xs ys: list A) : list A :=
 match xs with
 | nil => ys
 | x::xs' => x :: app xs' ys
 end.

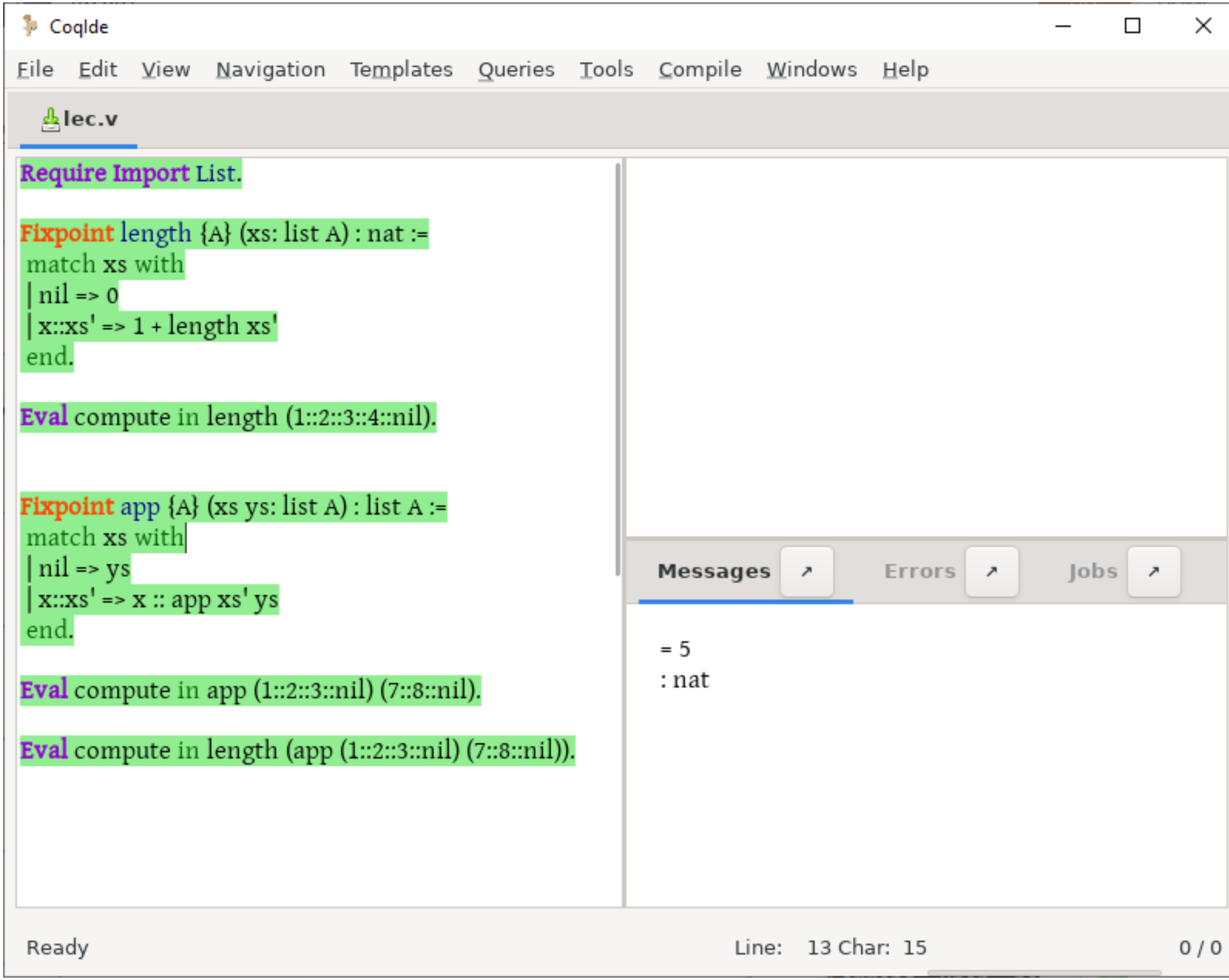
Eval compute in app (1::2::3::nil) (7::8::nil).

Eval compute in length (app (1::2::3::nil) (7::8::nil)).
```

Messages ↗ Errors ↗ Jobs ↗

```
= 1 :: 2 :: 3 :: 7 :: 8 :: nil
: list nat
```

Ready Line: 18 Char: 48 0 / 0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lec.v
Require Import List.
Fixpoint length {A} (xs: list A) : nat :=
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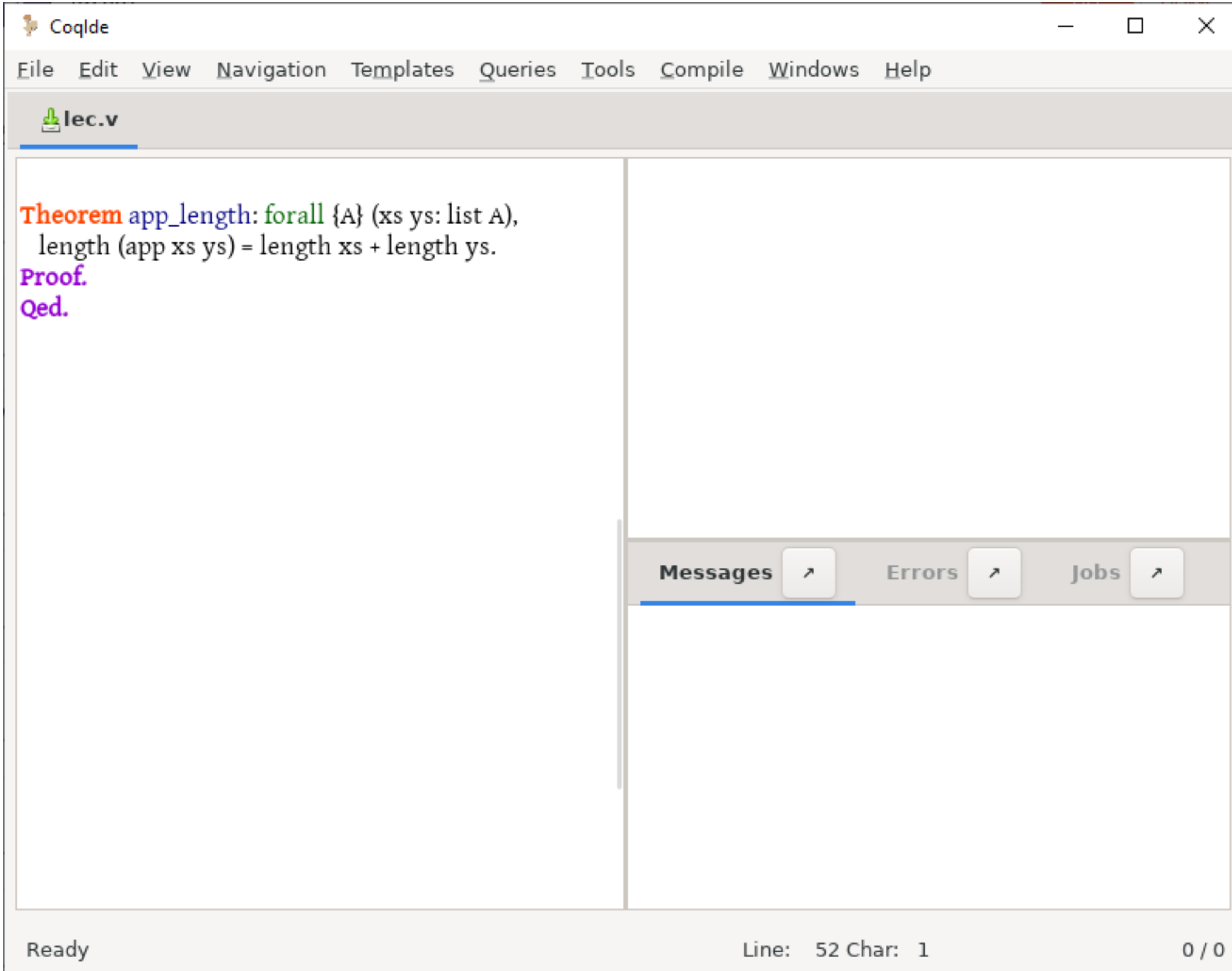
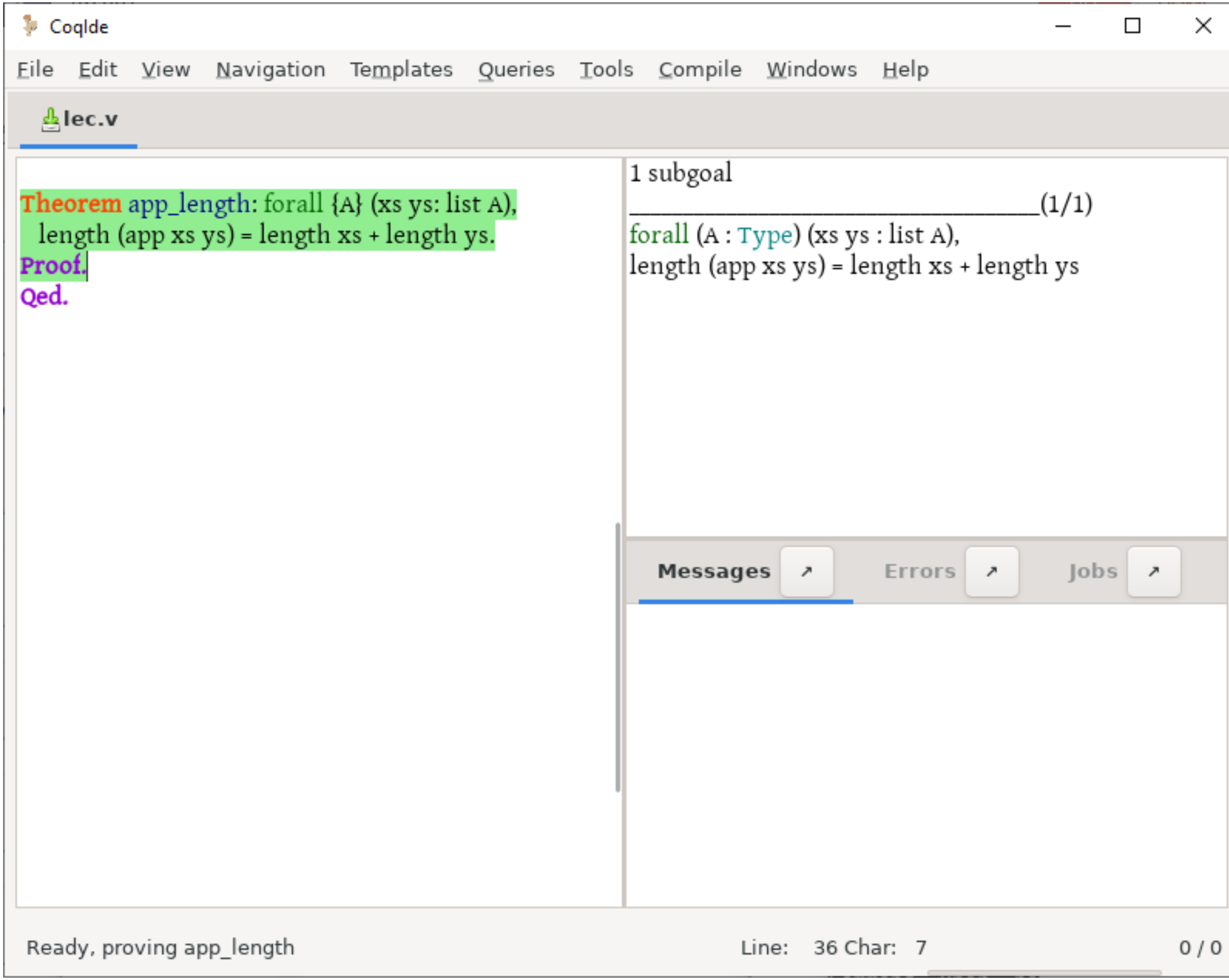
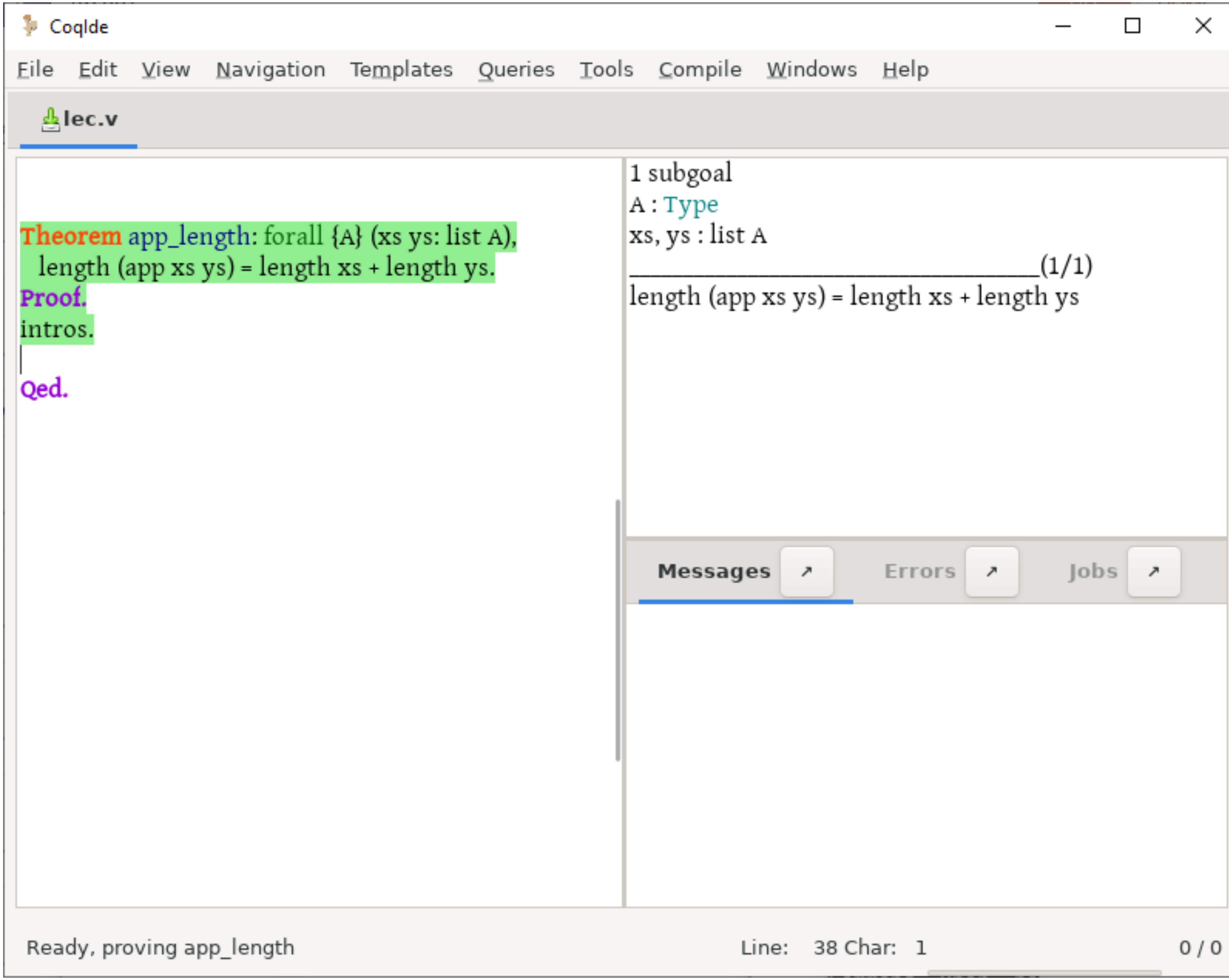
| nil => 0
| x::xs' => 1 + length xs'
end.
Eval compute in length (1::2::3::4::nil).
Fixpoint app {A} (xs ys: list A) : list A :=
match xs with
| nil => ys
| x::xs' => x :: app xs' ys
end.
Eval compute in app (1::2::3::nil) (7::8::nil).
Eval compute in length (app (1::2::3::nil) (7::8::nil)).
Messages Errors Jobs
= 5
: nat
Ready
Line: 13 Char: 15
0 / 0


CoqIde
File Edit View Navigation Templates Queries Tools Compile Windows Help
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
Qed.
Messages
Errors
Jobs
Ready
Line: 52 Char: 1
0 / 0


CoqIde
File Edit View Navigation Templates Queries Tools Compile Windows Help
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
Qed.
1 subgoal
(1/1)
forall (A : Type) (xs ys : list A),
length (app xs ys) = length xs + length ys
Messages
Errors
Jobs
Ready, proving app_length
Line: 36 Char: 7
0 / 0


CoqIde
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lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
Qed.
1 subgoal
A : Type
xs, ys : list A
(1/1)
length (app xs ys) = length xs + length ys
Messages
Errors
Jobs
Ready, proving app_length
Line: 38 Char: 1
0 / 0

CoqIde

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lec.v

```
Theorem app_length: forall {A} (xs ys: list A),
  length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
  simpl.
  reflexivity.
- (* inductive case *)
 simpl.
 reflexivity.
Qed.
```

```
2 subgoals
A : Type
ys : list A
______________________________________(1/2)
length (app nil ys) = length nil + length ys
______________________________________(2/2)
length (app (a :: xs) ys) =
length (a :: xs) + length ys
```

Messages Errors Jobs

Ready, proving app_length   Line: 38 Char: 14   0 / 0

CoqIde

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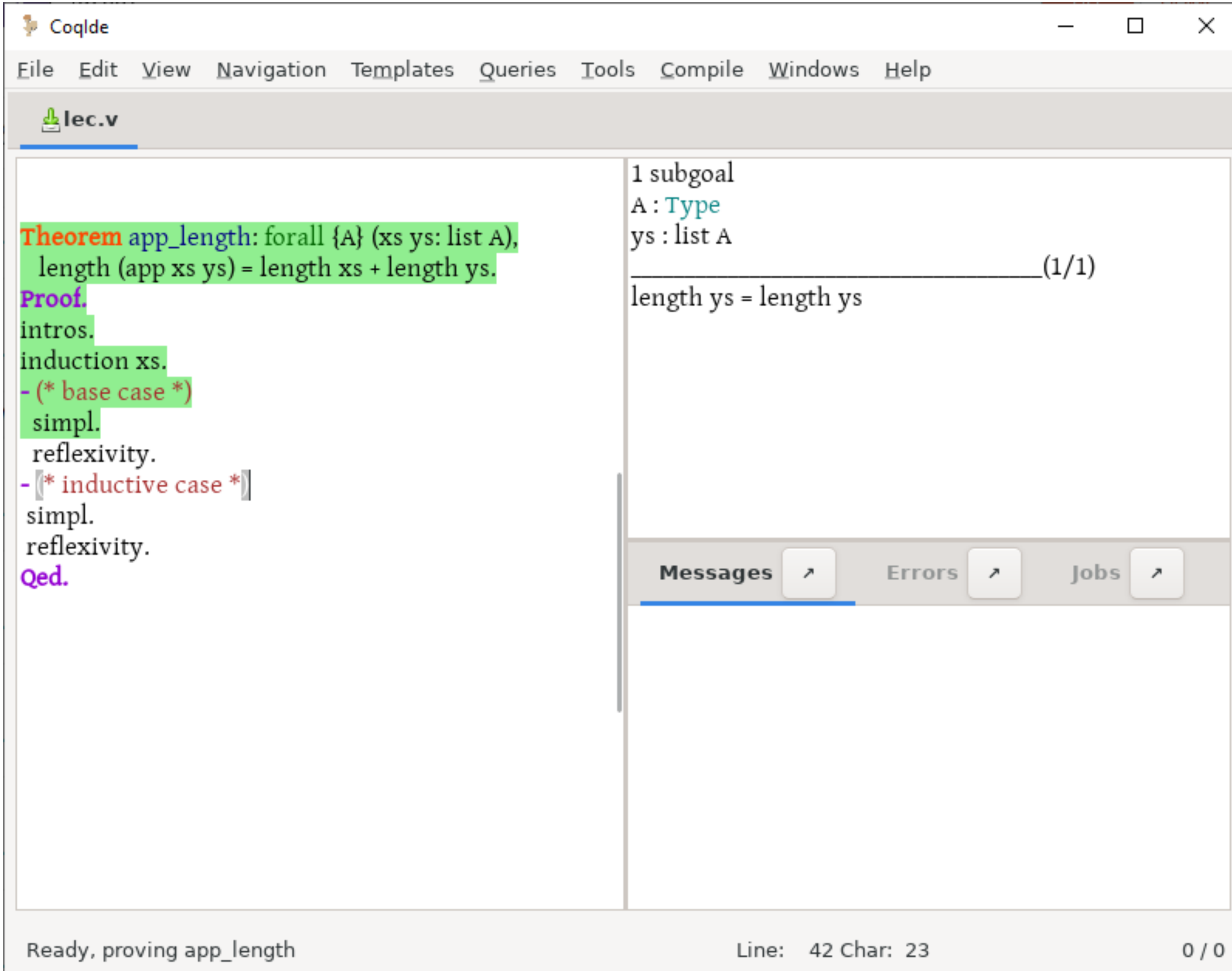
lec.v

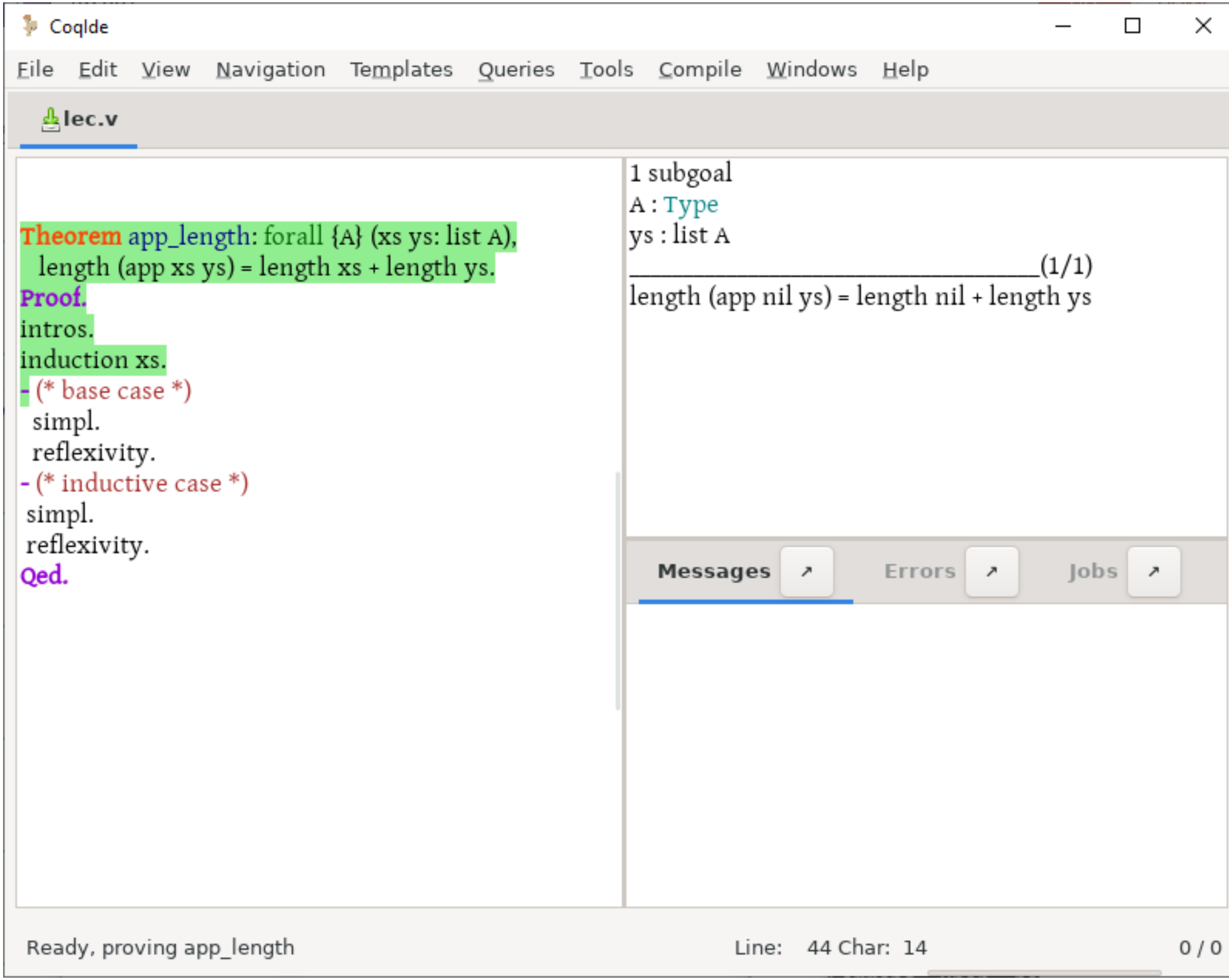
```
Theorem app_length: forall {A} (xs ys: list A),
  length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
  simpl.
  reflexivity.
- (* inductive case *)
 simpl.
 reflexivity.
Qed.
```

```
1 subgoal
A : Type
ys : list A
______________________________________(1/1)
length (app nil ys) = length nil + length ys
```

Messages | Errors | Jobs

Ready, proving app_length   Line: 44 Char: 14   0 / 0


CoqIde
File Edit View Navigation Templates Queries Tools Compile Windows Help
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
reflexivity.
Qed.
1 subgoal
A : Type
ys : list A
(1/1)
length ys = length ys
Messages
Errors
Jobs
Ready, proving app_length
Line: 42 Char: 23
0 / 0


CoqIde
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lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
reflexivity.
Qed.
1 subgoal
A : Type
ys : list A
(1/1)
length (app nil ys) = length nil + length ys
Messages
Errors
Jobs
Ready, proving app_length
Line: 44 Char: 14
0 / 0

CoqIde

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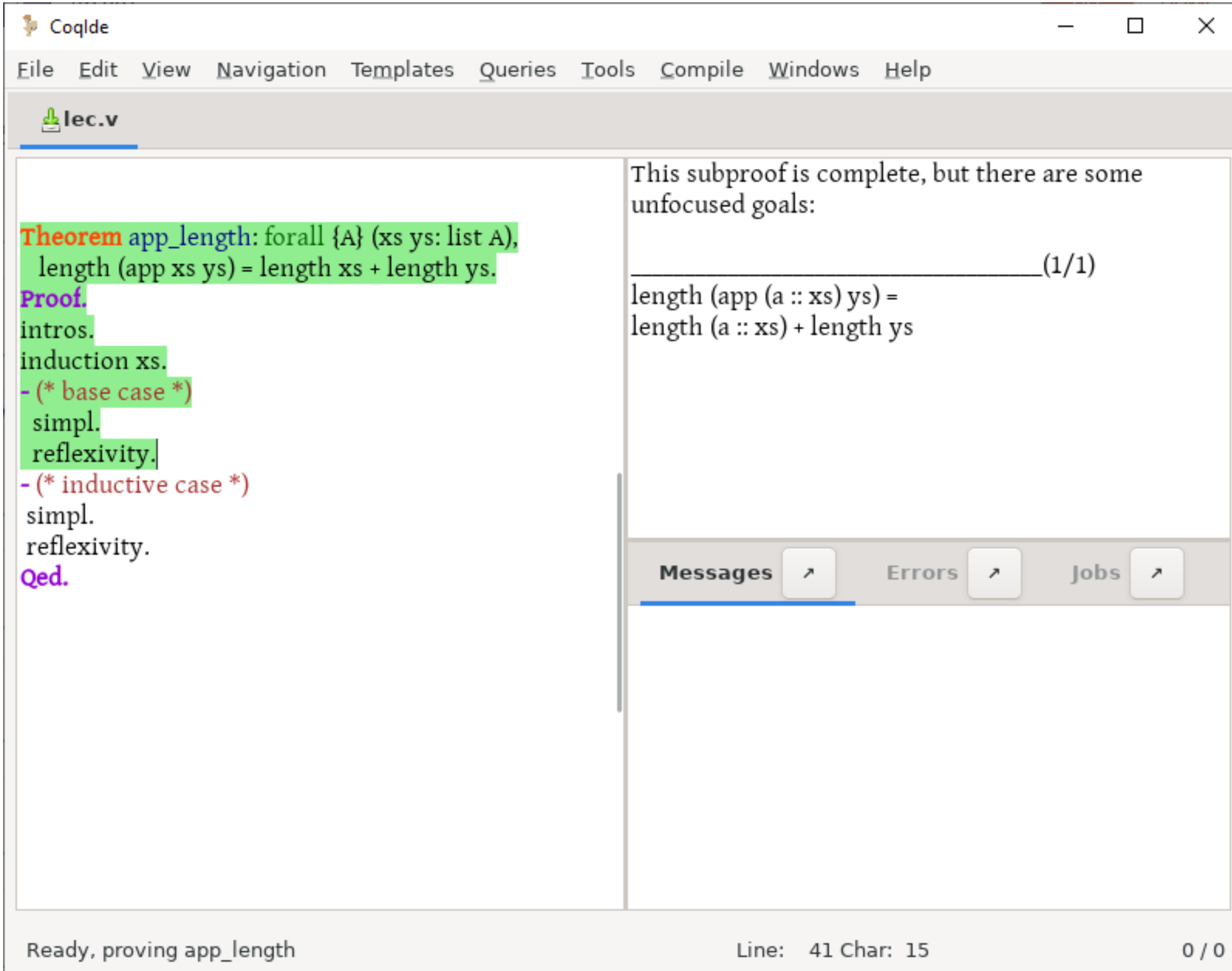
lec.v

```
Theorem app_length: forall {A} (xs ys: list A),
  length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
  simpl.
  reflexivity.
- (* inductive case *)
 simpl.
 reflexivity.
Qed.
```

```
1 subgoal
A : Type
ys : list A
________________________________________(1/1)
length ys = length ys
```

Messages Errors Jobs

Ready, proving app_length Line: 42 Char: 23 0 / 0


CoqIde
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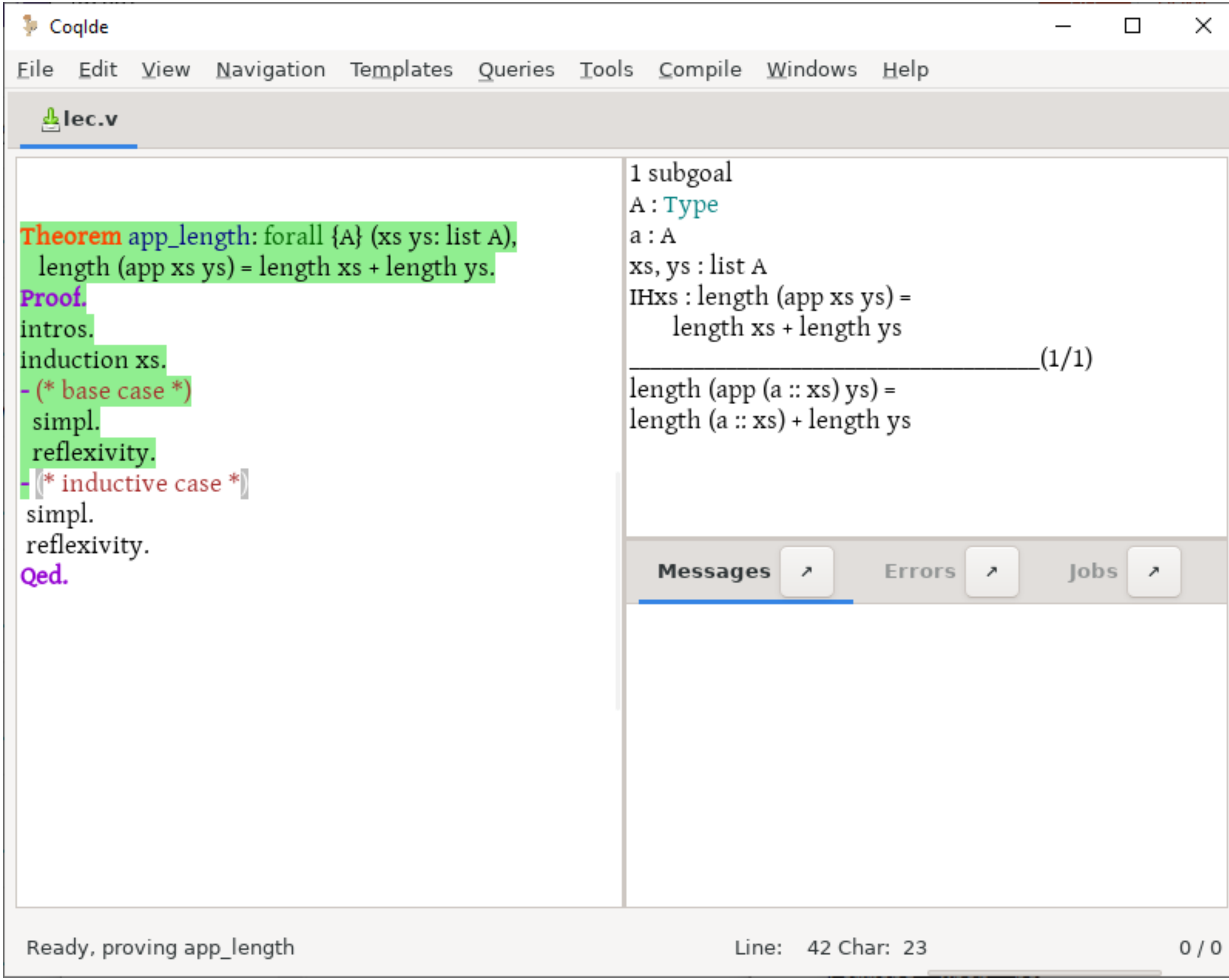
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
reflexivity.
Qed.
This subproof is complete, but there are some unfocused goals:
(1/1)
length (app (a :: xs) ys) =
length (a :: xs) + length ys
Messages
Errors
Jobs
Ready, proving app_length
Line: 41 Char: 15
0 / 0


CoqIde
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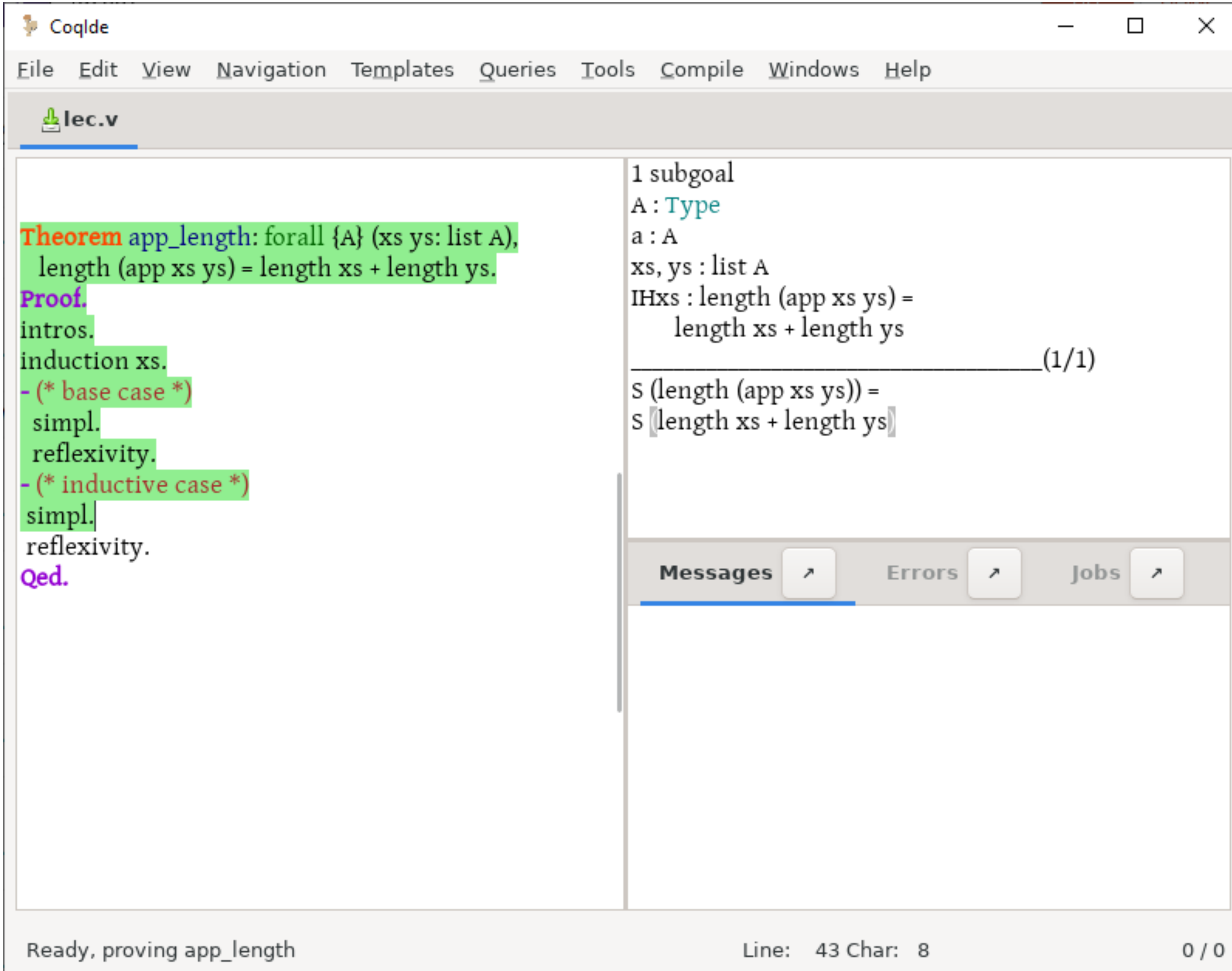
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
reflexivity.
Qed.
1 subgoal
A : Type
a : A
xs, ys : list A
IHxs : length (app xs ys) =
length xs + length ys
(1/1)
length (app (a :: xs) ys) =
length (a :: xs) + length ys
Messages
Errors
Jobs
Ready, proving app_length
Line: 42 Char: 23
0 / 0


CoqIde
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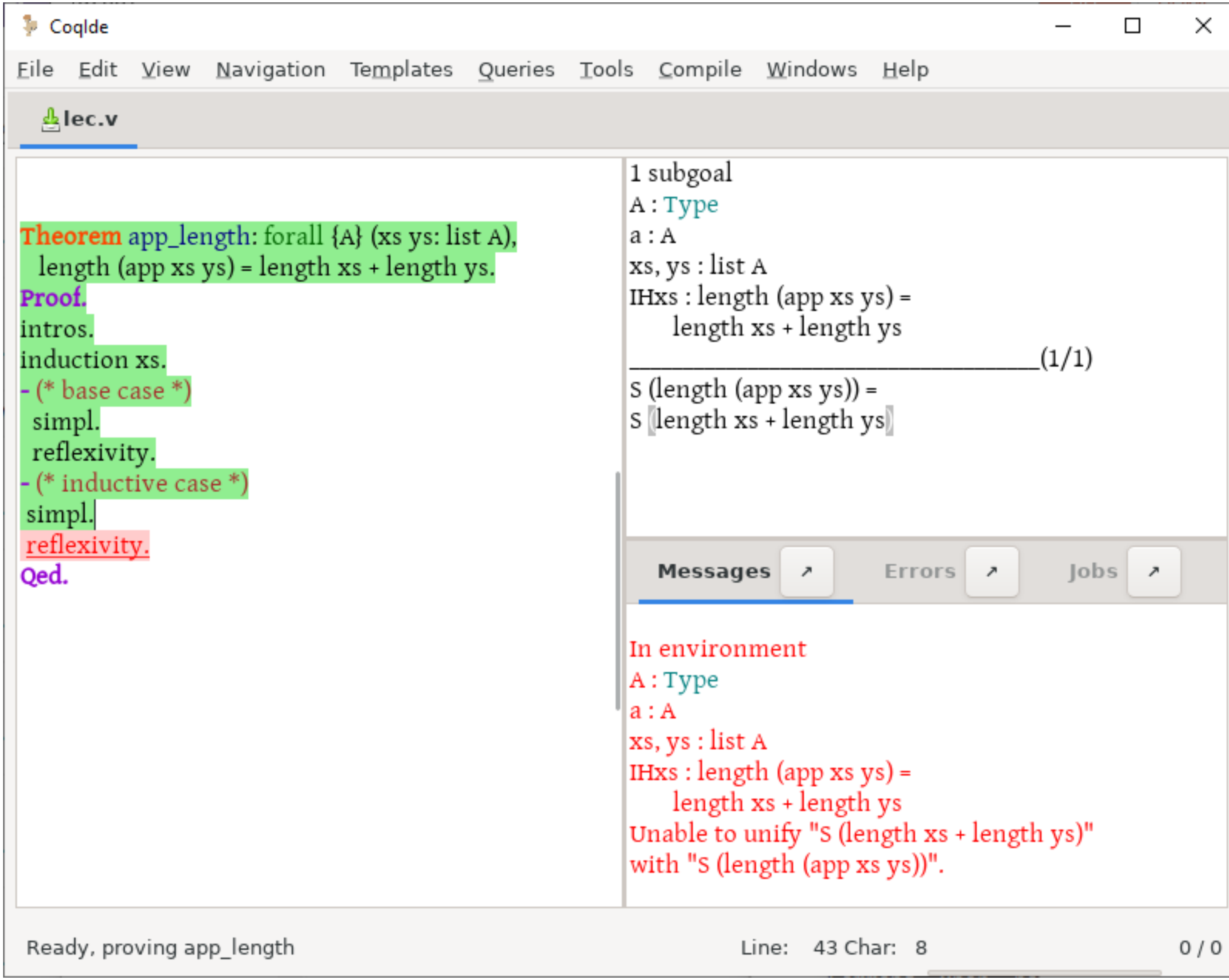
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
reflexivity.
Qed.
1 subgoal
A : Type
a : A
xs, ys : list A
IHxs : length (app xs ys) =
length xs + length ys
(1/1)
S (length (app xs ys)) =
S (length xs + length ys)
Messages
Errors
Jobs
Ready, proving app_length
Line: 43 Char: 8
0 / 0


CoqIde
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lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
reflexivity.
Qed.
1 subgoal
A : Type
a : A
xs, ys : list A
IHxs : length (app xs ys) =
length xs + length ys
(1/1)
S (length (app xs ys)) =
S (length xs + length ys)
Messages
Errors
Jobs
In environment
A : Type
a : A
xs, ys : list A
IHxs : length (app xs ys) =
length xs + length ys
Unable to unify "S (length xs + length ys)"
with "S (length (app xs ys))".
Ready, proving app_length
Line: 43 Char: 8
0 / 0

CoqIde

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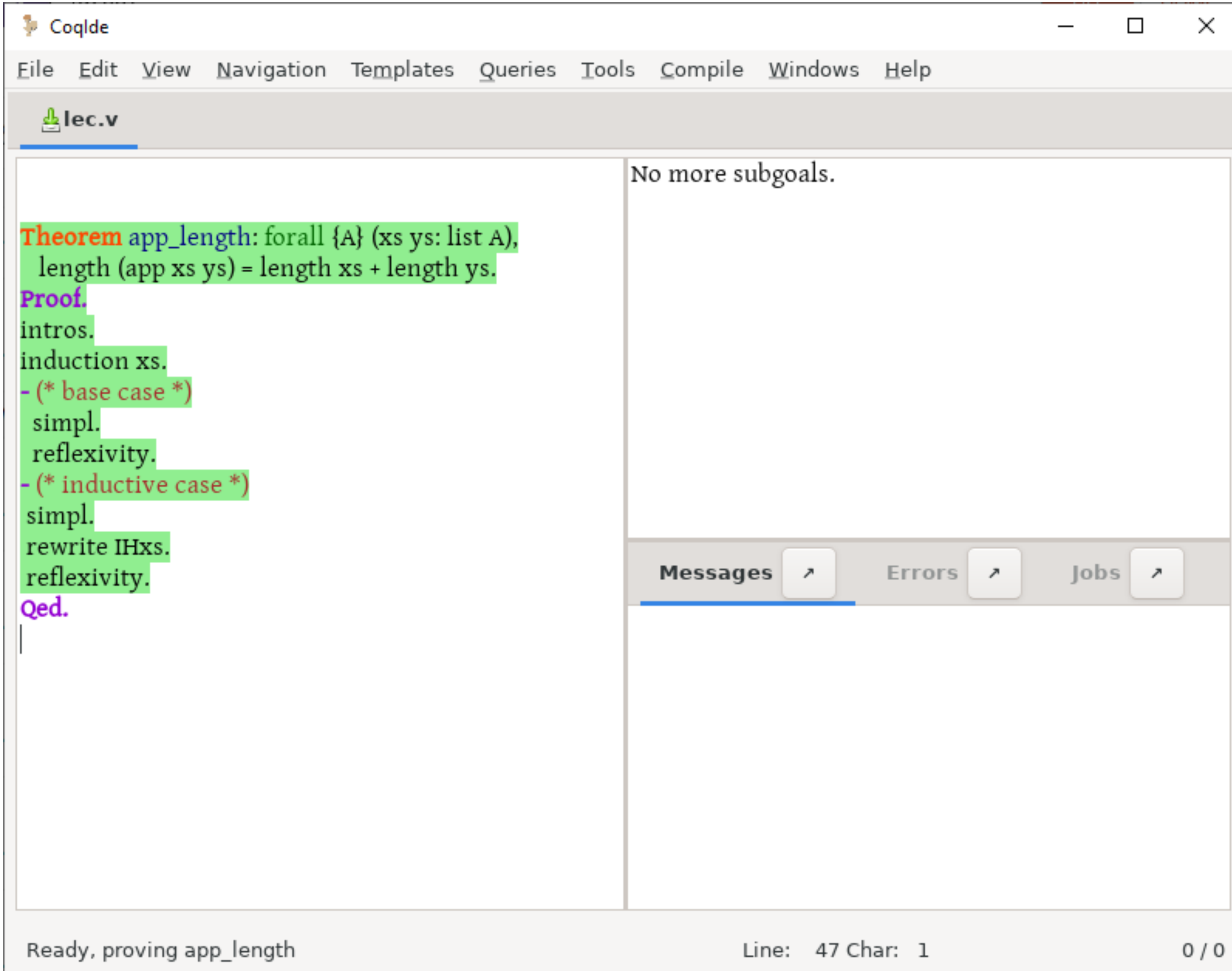
lec.v

```
Theorem app_length: forall {A} (xs ys: list A),
  length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
  simpl.
  reflexivity.
- (* inductive case *)
 simpl.
 rewrite IHxs.
 reflexivity.
Qed.
```

```
1 subgoal
A : Type
a : A
xs, ys : list A
IHxs : length (app xs ys) =
       length xs + length ys
______________________________________(1/1)
S (length xs + length ys) =
S (length xs + length ys)
```

Messages Errors Jobs

Ready, proving app_length   Line: 45 Char: 14   0 / 0


CoqIde
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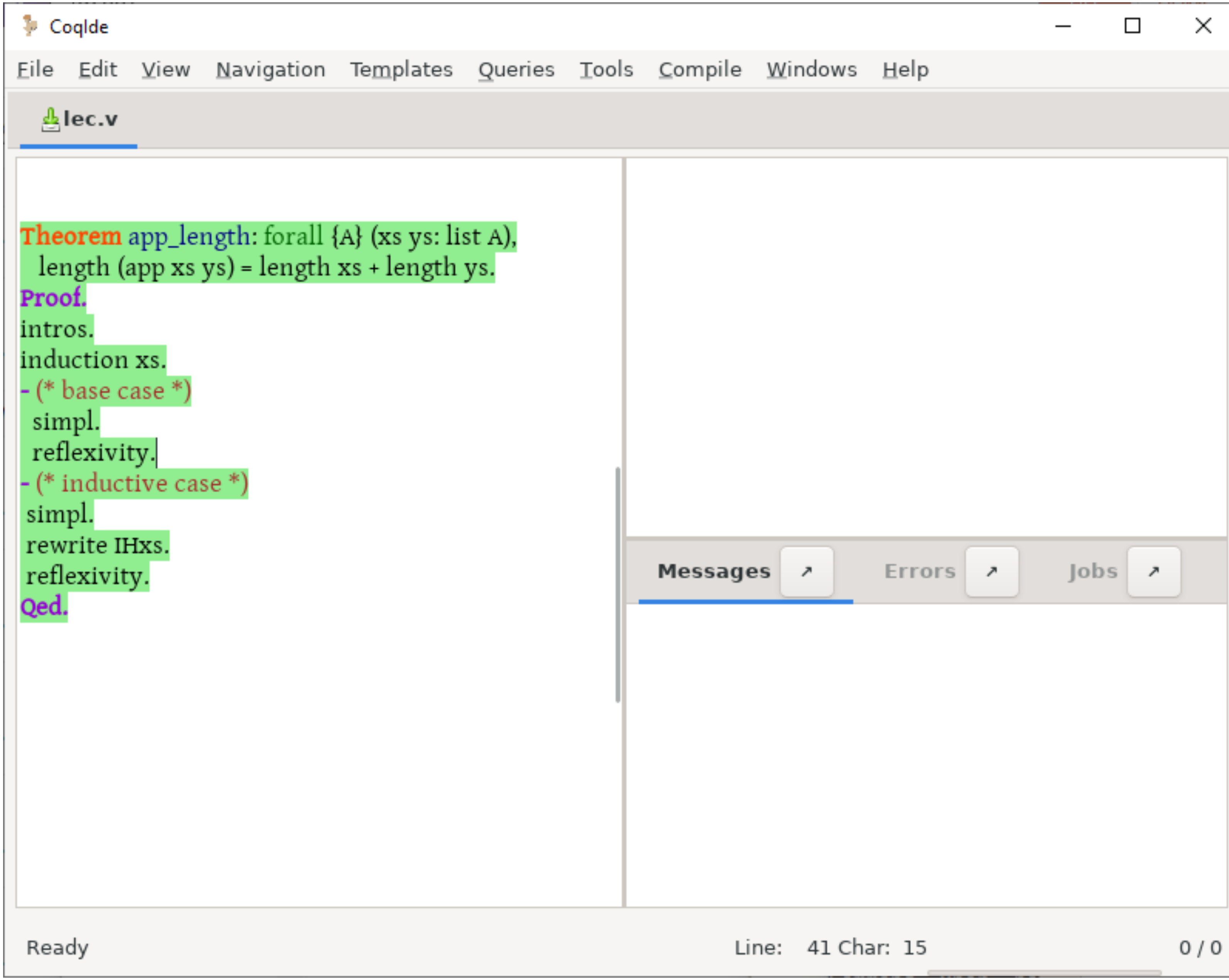
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
rewrite IHxs.
reflexivity.
Qed.
No more subgoals.
Messages
Errors
Jobs
Ready, proving app_length
Line: 47 Char: 1
0 / 0


CoqIde
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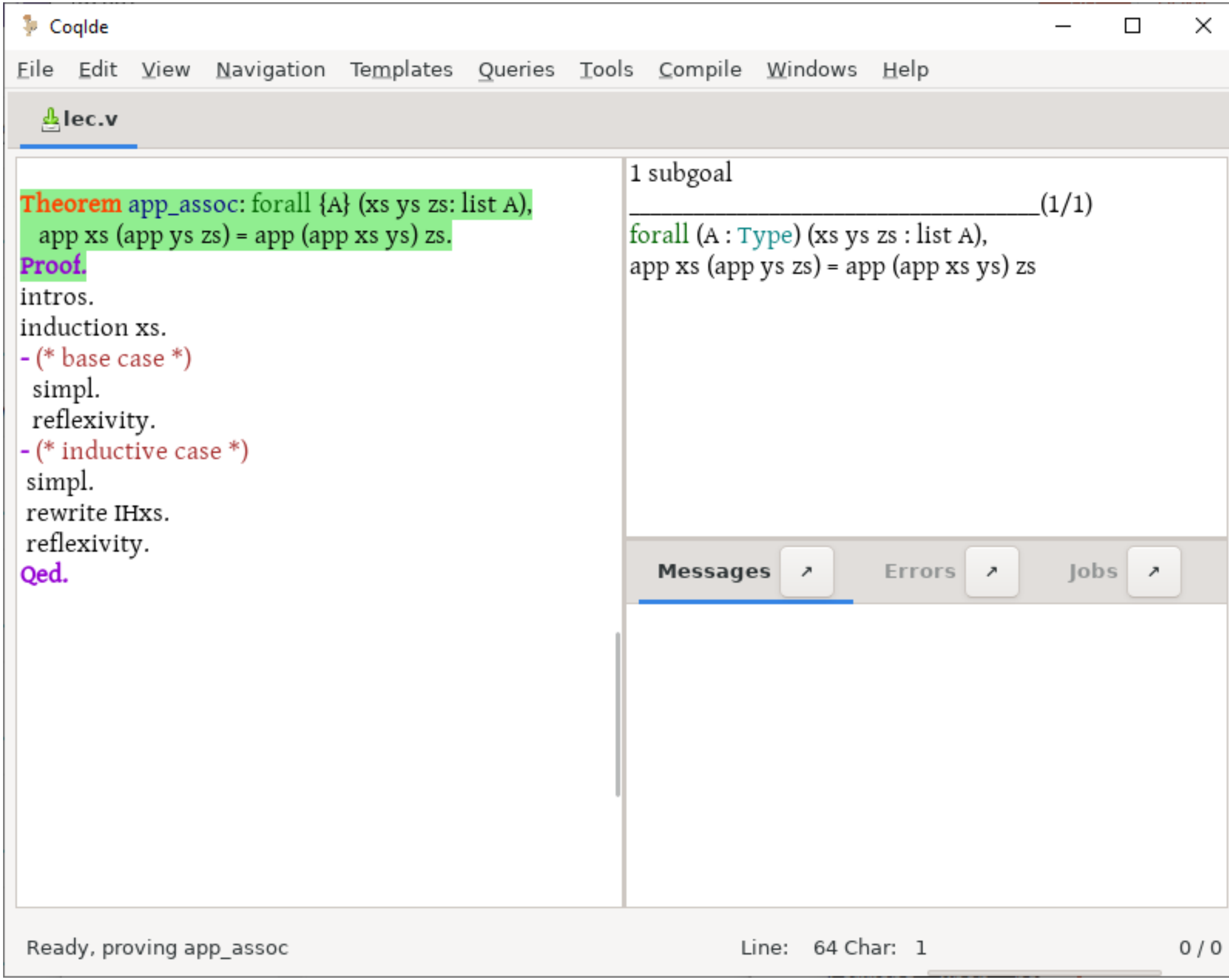
lec.v
Theorem app_length: forall {A} (xs ys: list A),
length (app xs ys) = length xs + length ys.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
rewrite IHxs.
reflexivity.
Qed.
Messages
Errors
Jobs
Ready
Line: 41 Char: 15
0 / 0


CoqIde
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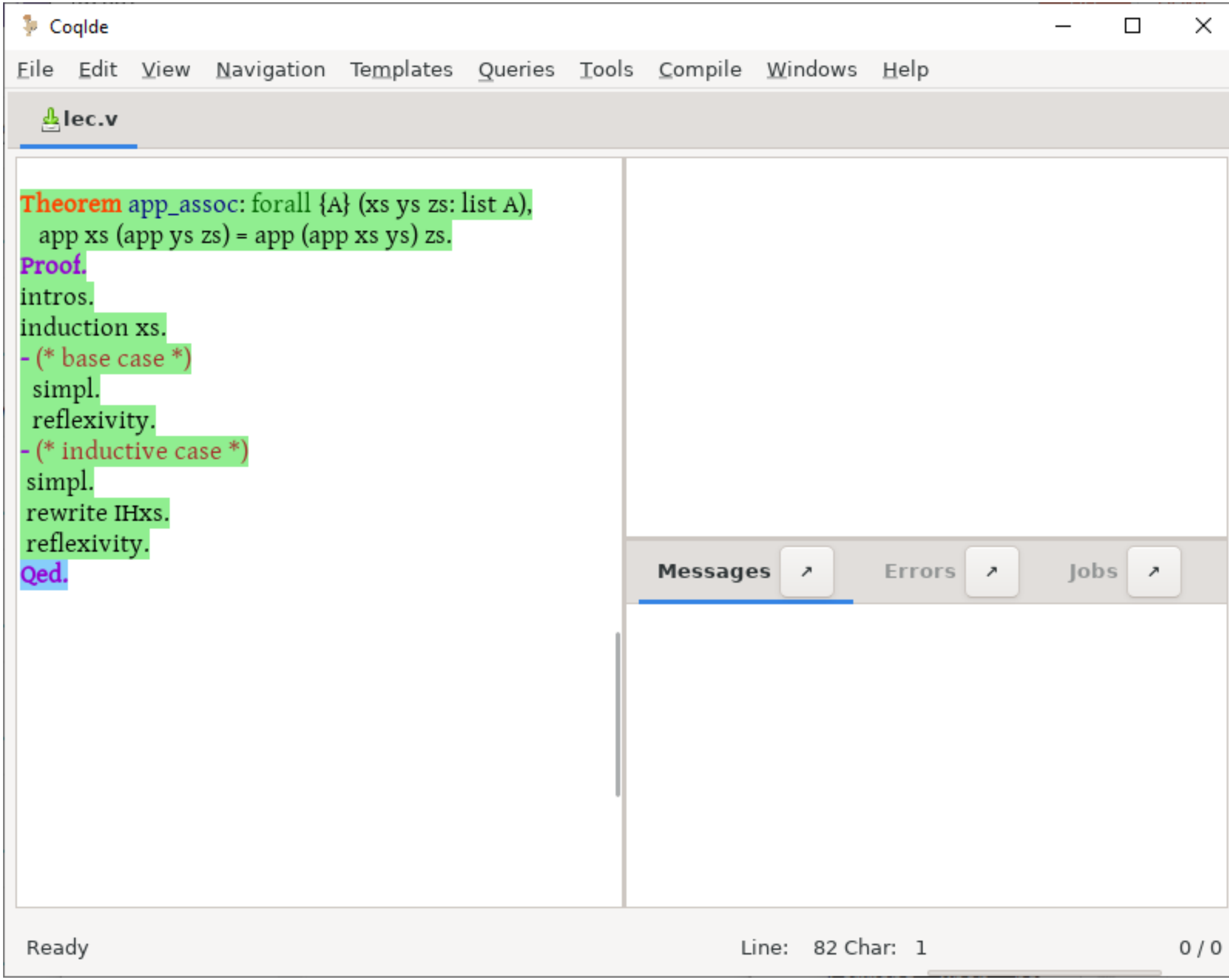
lec.v
Theorem app_assoc: forall {A} (xs ys zs: list A),
app xs (app ys zs) = app (app xs ys) zs.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
rewrite IHxs.
reflexivity.
Qed.
1 subgoal
(1/1)
forall (A : Type) (xs ys zs : list A),
app xs (app ys zs) = app (app xs ys) zs
Messages
Errors
Jobs
Ready, proving app_assoc
Line: 64 Char: 1
0 / 0


CoqIde
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lec.v
Theorem app_assoc: forall {A} (xs ys zs: list A),
app xs (app ys zs) = app (app xs ys) zs.
Proof.
intros.
induction xs.
- (* base case *)
simpl.
reflexivity.
- (* inductive case *)
simpl.
rewrite IHxs.
reflexivity.
Qed.
Messages
Errors
Jobs
Ready
Line: 82 Char: 1
0 / 0

Applications of Formal Methods

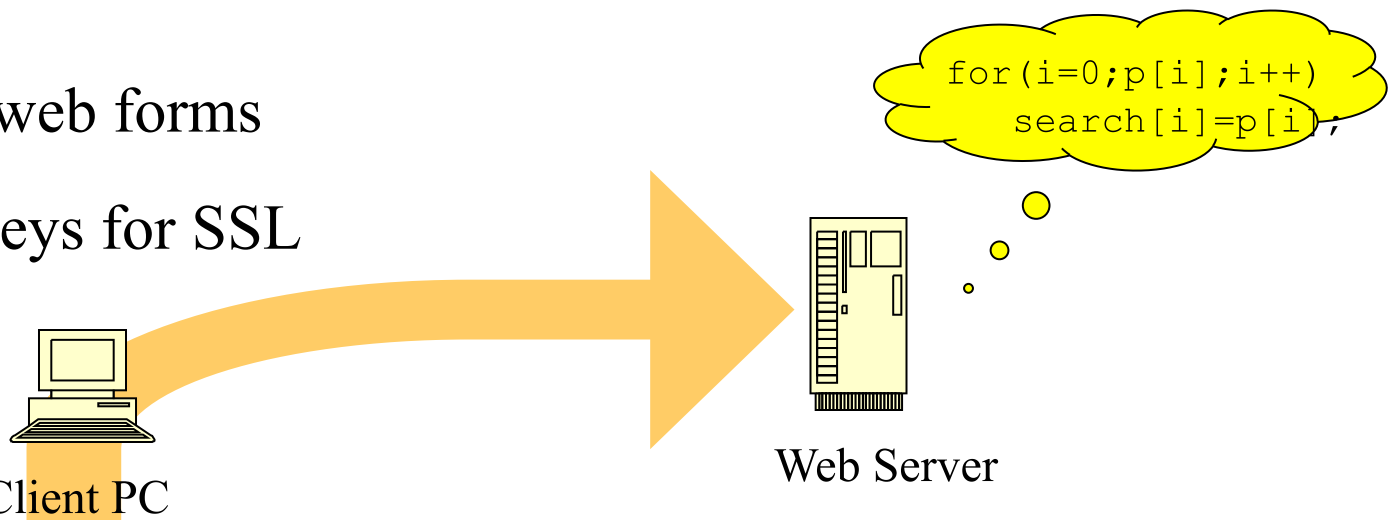
Attacking a web server

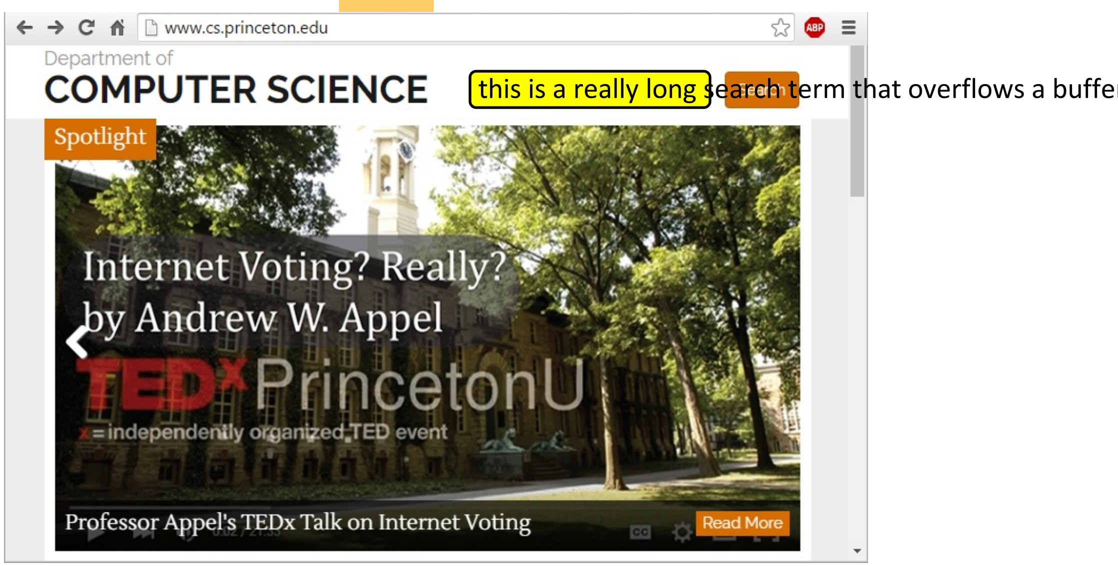
URLs

Input in web forms

Crypto keys for SSL

etc.





Attacking a web browser

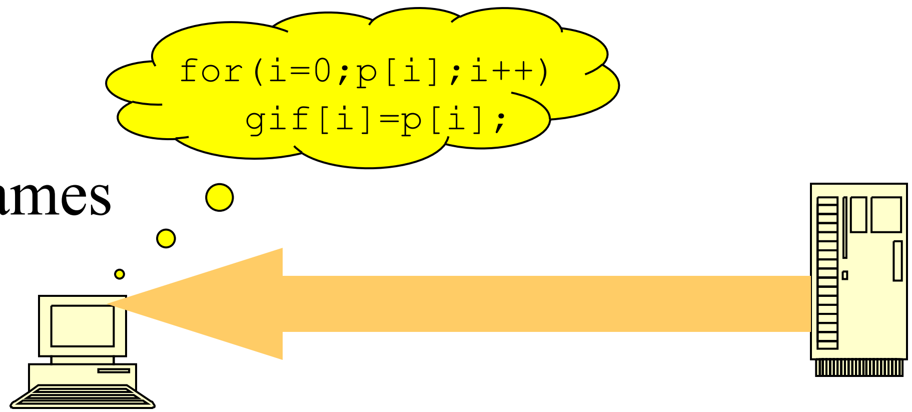
HTML keywords

Images

Image names

URLs

etc.



Client PC

Web Server
@ badguy.com

Attacking everything in sight

E-mail client

PDF viewer

Web browser

Operating-system kernel

TCP/IP stack

Any application that ever sees input directly from the outside

Solution: implement the outward-facing parts of software <u>without any bugs</u>!

E-mail client

PDF viewer

Web browser

Operating-system kernel

TCP/IP stack

Any application that ever sees input directly from the outside

In recent years, great progress in . . .

- Proved-correct optimizing C compiler (France)
- Proved-correct ML compiler (Sweden, Princeton)
- Proved-correct O.S. kernels (Australia, New Haven)
- Proved-correct crypto (Princeton NJ, Cambridge MA)
- Proved-correct distributed systems (Seattle, Israel)
- Proved-correct web server (Philadelphia)
- Proved-correct malloc/free library (Princeton, Hoboken)

Automated verification in industry

Amazon

Microsoft

Intel

Facebook

Google

Galois, HRL, Rockwell, Bedrock, …

Recent Princeton JIW / Sr. Thesis

- Katherine Ye ’16 verified crypto security
- Naphat Sanguansin ’16 verified crypto impl’n
- Brian McSwiggen ’18 verified B-trees
- Katja Vassilev ’19 verified dead-var elimination
- John Li ’19 verified uncurrying
- Jake Waksbaum ’20 verified Burrows-Wheeler
- Anvay Grover ’20 verified CPS-conversion

Verified Correctness and Security of mbedTLS HMAC-DRBG

Katherine Q. Ye ’16
Princeton U., Carnegie Mellon U.

Matthew Green
Johns Hopkins University

Naphat Sanguansin’16
Princeton University

Lennart Beringer
Princeton University

Adam Petcher
Oracle

Andrew W. Appel ’81
Princeton University

ABSTRACT

We have formalized the functional specification of HMAC-DRBG (NIST 800-90A), and we have proved its cryptographic security—that its output is pseudorandom—using a hybrid game-based proof. We have also proved that the mbedTLS implementation (C program) correctly implements this functional specification. That proof composes with an existing C compiler correctness proof to guarantee, end-to-end, that the machine language program gives strong pseudorandomness. All proofs (hybrid games, C program verification, compiler, and their composition) are machine-checked in the Coq proof assistant. Our proofs are modular: the hybrid game proof holds on any implementation of HMAC-DRBG that satisfies our functional specification. Therefore, our functional specification can serve as a high-assurance reference.

Prerequisites for COS 510

if you're an undergrad

1. COS 326 Functional Programming

2. Enjoy the proofs in COS 326