

Simple Data

COS 326

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Logistics

- Sign up for Piazza, our Q&A forum:
 - <https://piazza.com/princeton/fall2016/cos326/home>
- Assignment #1 is due on Wednesday at 11:59pm

OCaml

OCaml is a *functional* programming language

- Java gets most work done by *modifying* data
- OCaml gets most work done by producing *new, immutable* data

OCaml is a *typed* programming language

- the *type* of an expression *correctly predicts* the kind of *value* the expression will generate when it is executed
- the type system is *sound*; the language is *safe*
- types help us *understand* and *write* our programs
- there are hard and fast type checking *rules*

Example Type-checking Rules

if $e1 : \text{int}$
and $e2 : \text{int}$
then $e1 + e2 : \text{int}$

Type Checking Rules

- Violating the rules:

```
# "hello" + 1;;
```

```
Error: This expression has type string but an  
expression was expected of type int
```

- The type error message tells you the type that was **expected** and the type that it **inferred** for your subexpression
- Notice that there is no way to evaluate this expression – it is undefined (has no semantics according to the language definition)
- Type checking rules out such non-sensical expressions

Type Checking Rules

- Violating the rules:

```
# "hello" + 1;;
```

```
Error: This expression has type string but an  
expression was expected of type int
```

- A possible fix:

```
# "hello" ^ (string_of_int 1);;  
- : string = "hello1"
```

- *One of the keys to becoming a good ML programmer is to understand type error messages.*

Example Type-checking Rules

if $e_1 : \text{bool}$

and $e_2 : t$ and $e_3 : t$ (the same type t , for some type t)

then $\text{if } e_1 \text{ then } e_2 \text{ else } e_3 : t$ (that same type t)

Type Checking Rules

- Type errors for if statements can be confusing sometimes.
Example. We create a string from s , concatenating it n times:

```
let rec concatn s n =  
  if n <= 0 then  
    ...  
  else  
    s ^ (concatn s (n-1))
```


Type Checking Rules

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Example. We create a string from `s`, concatenating it `n` times:

```
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ocamlbuild says:

```
Error: This expression has type int but an  
expression was expected of type string
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Type Checking Rules

- Type errors for if statements can be confusing sometimes.
Example. We create a string from `s`, concatenating it `n` times:

they don't
agree!

```
let rec concatn s n =
  if n <= 0 then
    0
  else
    s ^ (concatn s (n-1))
```

???

ocamlbuild says:

Error: This expression has type int but an expression was expected of type string

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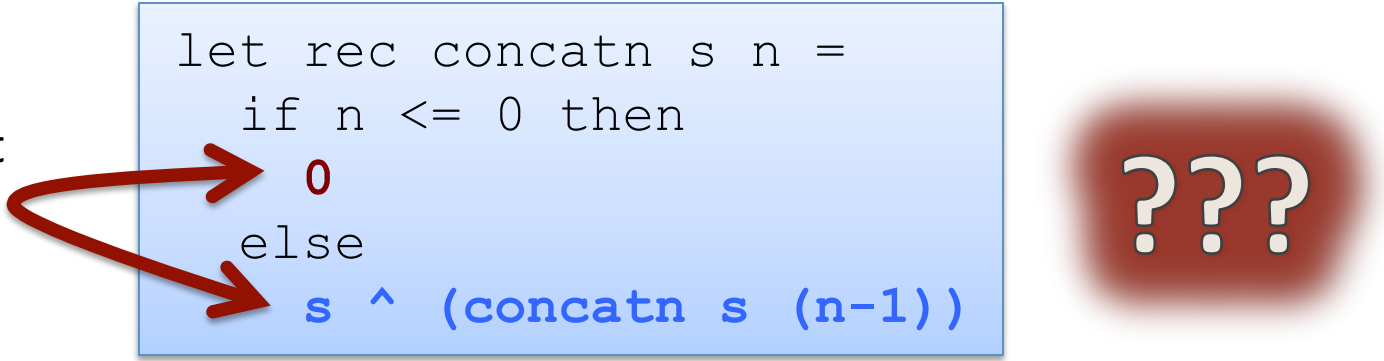
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- Type errors for if statements can be confusing sometimes.
Example. We create a string from s , concatenating it n times:

they don't
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```
let rec concatn s n =
  if n <= 0 then
    0
  else
    s ^ (concatn s (n-1))
```




The type checker points to the correct branch as the cause of an error because it does not AGREE with the type of an earlier branch.
Really, the error is in the earlier branch.

Moral: Sometimes need to look in an earlier branch for the error even though the type checker points to a later branch.
The type checker doesn't know what the user wants.

A Tactic: Add Typing Annotations

```
let rec concatn (s:string) (n:int) : string =  
  if n <= 0 then  
    0  
  else  
    s ^ (concatn s (n-1))
```



Error: This expression has type int but an expression was expected of type string

ONWARDS!

What is the single most important mathematical concept ever developed in human history?

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An answer: The mathematical variable

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An answer: The mathematical variable

(runner up: natural numbers/induction)

Why is the mathematical variable so important?

The mathematician says:

“Let x be some integer, we define a polynomial over x ...”

Why is the mathematical variable so important?

The mathematician says:

“Let x be some integer, we define a polynomial over x ...”

What is going on here? The mathematician has separated a *definition* (of x) from its *use* (in the polynomial).

This is the most primitive kind of *abstraction* (x is *some* integer)

Abstraction is the key to controlling complexity and without it, modern mathematics, science, and computation would not exist.

OCAML BASICS: LET DECLARATIONS

Abstraction

- Good programmers identify repeated patterns in their code and factor out the repetition into meaningful components
- In O'CamL, the most basic technique for factoring your code is to use **let expressions**
- Instead of writing this expression:

```
(2 + 3) * (2 + 3)
```

Abstraction & Abbreviation

- Good programmers identify repeated patterns in their code and factor out the repetition into meaning components
- In O'Caml, the most basic technique for factoring your code is to use **let expressions**
- Instead of writing this expression:

```
(2 + 3) * (2 + 3)
```

- We write this one:

```
let x = 2 + 3 in  
x * x
```

A Few More Let Expressions

```
let x = 2 in
let squared = x * x in
let cubed = x * squared in
squared * cubed
```


A Few More Let Expressions

```
let x = 2 in
let squared = x * x in
let cubed = x * squared in
squared * cubed
```

```
let a = "a" in
let b = "b" in
let as = a ^ a ^ a in
let bs = b ^ b ^ b in
as ^ bs
```


Abstraction & Abbreviation

- Two kinds of let:

```

if tuesday() then
    let x = 2 + 3 in
    x + x
else
    0
;;

```




let ... in ... is an *expression* that can appear inside any other *expression*

The scope of x does not extend outside the enclosing “in”

```

let x = 2 + 3 ;;
let y = x + 17 / x ;;

```



let ... ;; without “in” is a top-level *declaration*

Variables x and y may be exported; used by other modules


(Don't need ;; if another let comes next; do need it the next top-level declaration is an expression)

Binding Variables to Values

- Each OCaml variable is *bound* to 1 value
- *The value to which a variable is bound to never changes!*

```
let x = 3 ;;
```

```
let add_three (y:int) : int = y + x ;;
```



Binding Variables to Values

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```
let x = 3 ;;
```

```
let add_three (y:int) : int = y + x ;;
```

*It does not
matter what
I write next.
add_three
will always
add 3!*

Binding Variables to Values

- Each OCaml variable is bound to 1 value
- *The value a variable is bound to never changes!*

a distinct
variable that
"happens to
be spelled the
same"

```
let x = 3 ;;
```

```
let add_three (y:int) : int = y + x ;;
```

```
let x = 4 ;;
```

```
let add_four (y:int) : int = y + x ;;
```

Binding Variables to Values

- Since the 2 variables (both happened to be named x) are actually different, unconnected things, we can rename them

rename x
to zzz
if you want
to, replacing
its uses

```
let x = 3 ;;
```

```
let add_three (y:int) : int = y + x ;;
```

```
let zzz = 4 ;;
```

```
let add_four (y:int) : int = y + zzz ;;
```

```
let add_seven (y:int) : int =  
  add_three (add_four y)  
;;
```

Binding Variables to Values

- Each OCaml variable is bound to 1 value
- OCaml is a **statically scoped** language

```
let x = 3 ;;
```

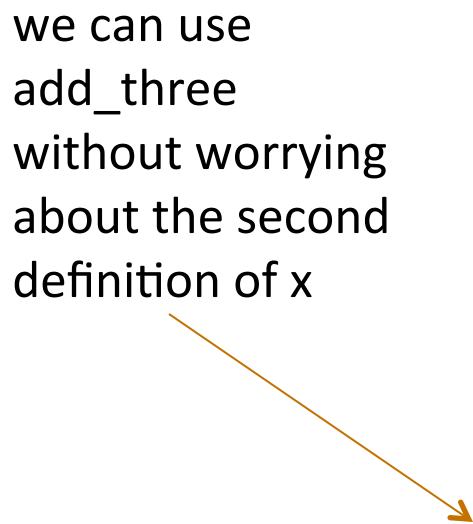
```
let add_three (y:int) : int = y + x ;;
```

```
let x = 4 ;;
```

```
let add_four (y:int) : int = y + x ;;
```

```
let add_seven (y:int) : int =  
  add_three (add_four y)  
;;
```

we can use
add_three
without worrying
about the second
definition of x



How do let expressions operate?

```
let x = 2 + 1 in x * x
```


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```
let x = 2 + 1 in x * x
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-->

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let x = 3 in x * x
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```
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```


-->

```
let x = 3 in x * x
```

-->

```
3 * 3
```

substitute
3 for x



How do let expressions operate?

```
let x = 2 + 1 in x * x
```

-->

```
let x = 3 in x * x
```


-->

```
3 * 3
```

-->

```
9
```

substitute
3 for x



How do let expressions operate?

```
let x = 2 + 1 in x * x
```

-->

```
let x = 3 in x * x
```

-->

```
3 * 3
```

-->

```
9
```

substitute
3 for x



Note: I write
 $e1 \rightarrow e2$
when $e1$ evaluates
to $e2$ in one step

Did you see what I did there?

Did you see what I did there?

I defined the language in terms of itself:

let $x = 2$ in $x + 3$ \rightarrow $2 + 3$

I'm trying to train you to think at a high level of abstraction.

I didn't have to mention low-level abstractions like assembly code or registers or memory layout


Another Example

```
let x = 2 in  
let y = x + x in  
y * x
```

Another Example

```
let x = 2 in  
let y = x + x in  
y * x
```

substitute
2 for x




-->

```
let y = 2 + 2 in  
y * 2
```


Another Example

```
let x = 2 in  
let y = x + x in  
y * x
```

substitute
2 for x



-->

```
let y = 2 + 2 in  
y * 2
```

-->

```
let y = 4      in  
y * 2
```

Another Example

```
let x = 2 in  
let y = x + x in  
y * x
```

substitute
2 for x




-->

```
let y = 2 + 2 in  
y * 2
```

-->

```
let y = 4      in  
y * 2
```

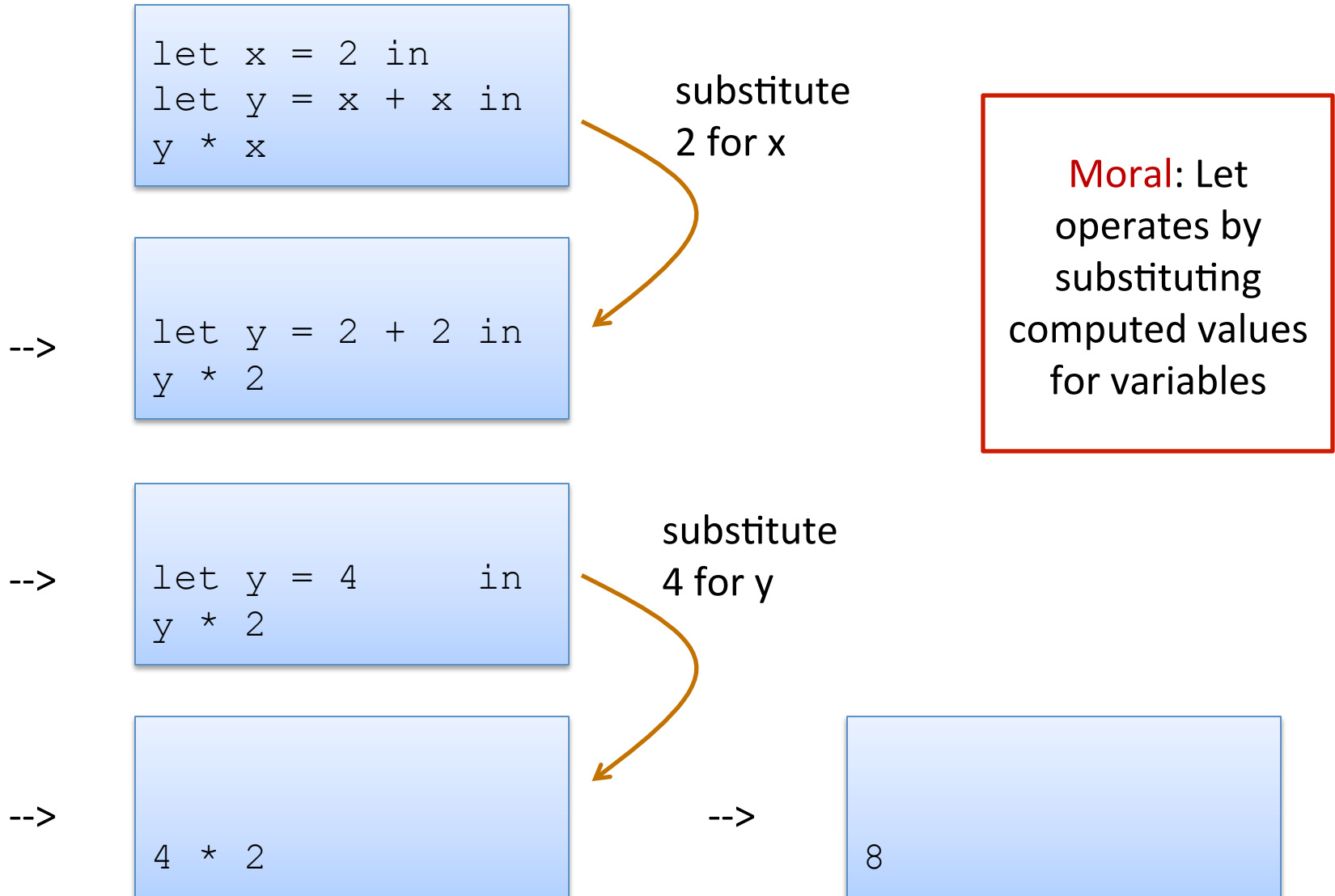
substitute
4 for y



-->

```
4 * 2
```

Another Example



What would happen in an imperative language?

C program:

```
x = 2;  
x += x;  
return x*2;
```

substitute
2 for x

-->

```
x += 2 ???  
return x*2;
```

substituting
computed values
for variables

This principle works in
functional languages, not
so well in imperative
languages

OCAML BASICS: TYPE CHECKING AGAIN

Type-checking Rules

There are simple rules that tell you what the type of an expression is.

Those rules compute a type for an expression based on the *types* of its subexpressions (and the types of the variables that are in scope).

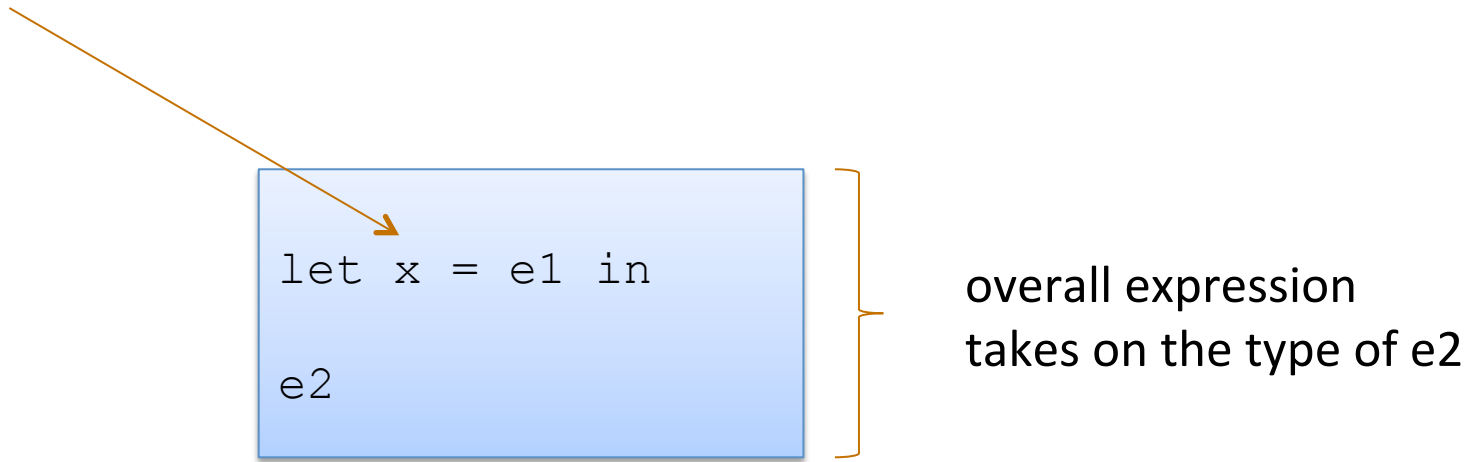
You don't have to know the details of how a subexpression is implemented to do type checking. You just need to know its type.

That's what makes OCaml type checking *modular*.

We write “ $e : t$ ” to say that expression e has type t

Back to Let Expressions ... Typing

x granted type of e1 for use in e2



```
let x = e1 in  
e2
```

overall expression
takes on the type of e2

Back to Let Expressions ... Typing

x granted type of e1 for use in e2

```
let x = e1 in  
e2
```

overall expression
takes on the type of e2

x has type int
for use inside the
let body

```
let x = 3 + 4 in  
string_of_int x
```

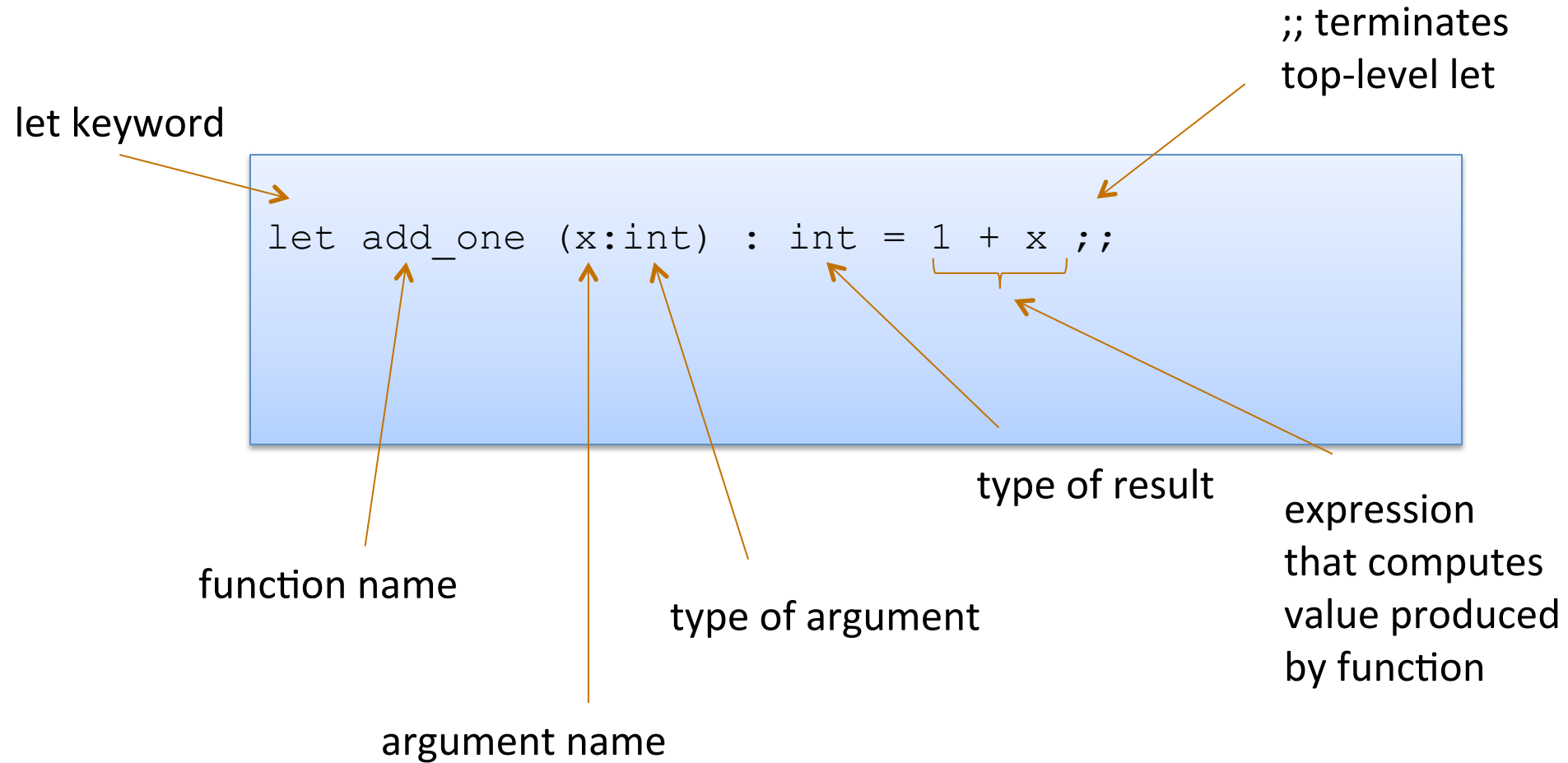
overall expression
has type string

OCAML BASICS: FUNCTIONS

Defining functions

```
let add_one (x:int) : int = 1 + x ;;
```

Defining functions

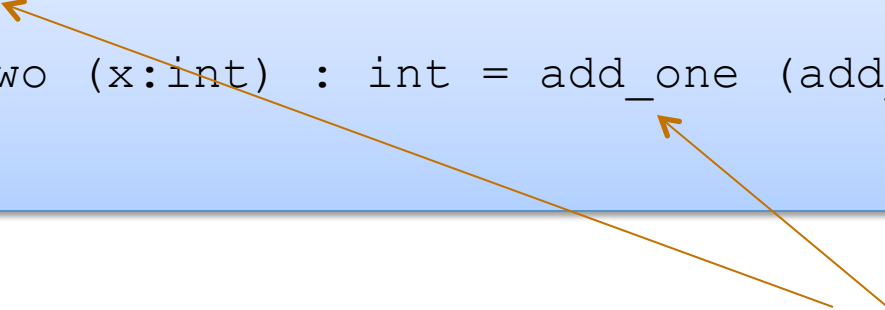


Note: recursive functions with begin with **"let rec"**

Defining functions

- Nonrecursive functions:

```
let add_one (x:int) : int = 1 + x ;;  
let add_two (x:int) : int = add_one (add_one x) ;;
```



definition of add_one
must come before use

Defining functions

- Nonrecursive functions:

```
let add_one (x:int) : int = 1 + x ;;  
let add_two (x:int) : int = add_one (add_one x) ;;
```

- With a local definition:

```
let add_two' (x:int) : int =  
  let add_one x = 1 + x in  
  add_one (add_one x)  
;;
```

local function definition
hidden from clients

I left off the types.
O'Caml figures them out

Good style: types on
top-level definitions

Types for Functions

Some functions:

```
let add_one (x:int) : int = 1 + x ;;  
let add_two (x:int) : int = add_one (add_one x) ;;  
let add (x:int) (y:int) : int = x + y ;;
```

function with two arguments



Types for functions:

```
add_one : int -> int  
add_two : int -> int  
add : int -> int -> int
```

Rule for type-checking functions

General Rule:

If a function $f : T1 \rightarrow T2$
and an argument $e : T1$
then $f e : T2$

Example:

```
add_one : int -> int
```

```
3 + 4 : int
```

```
add_one (3 + 4) : int
```

Rule for type-checking functions

- Recall the type of add:

Definition:

```
let add (x:int) (y:int) : int =  
  x + y  
;;
```

Type:

```
add : int -> int -> int
```


Rule for type-checking functions

- Recall the type of add:

Definition:

```
let add (x:int) (y:int) : int =  
  x + y  
;;
```

Type:

```
add : int -> int -> int
```

Same as:

```
add : int -> (int -> int)
```

Rule for type-checking functions

General Rule:

If a function $f : T1 \rightarrow T2$
and an argument $e : T1$
then $f e : T2$

$$\frac{f : T1 \rightarrow T2 \quad e : T1}{f e : T2}$$

Example:

```
add : int -> int -> int
```

```
3 + 4 : int
```

```
add (3 + 4) : ???
```

Note:

$A \rightarrow B \rightarrow C$
is the same as
 $A \rightarrow (B \rightarrow C)$

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Example:

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add : int -> (int -> int)
```

```
3 + 4 : int
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add (3 + 4) :
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Example:

```

add : int -> (int -> int)
3 + 4 : int
add (3 + 4) : int -> int
  
```

Rule for type-checking functions

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Example:

```
add : int -> int -> int
```

```
3 + 4 : int
```

```
add (3 + 4) : int -> int
```

```
(add (3 + 4)) 7 : int
```



Rule for type-checking functions

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Remember:

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Example:

```
add : int -> int -> int
```

```
3 + 4 : int
```

```
add (3 + 4) : int -> int
```

```
add (3 + 4) 7 : int
```

Rule for type-checking functions

Example:

```
let munge (b:bool) (x:int) : ?? =  
  if not b then  
    string_of_int x  
  else  
    "hello"  
;;  
  
let y = 17;;
```

```
munge (y > 17) : ??  
  
munge true (f (munge false 3)) : ??  
  f : ??  
  
munge true (g munge) : ??  
  g : ??
```

Rule for type-checking functions

Example:

```
let munge (b:bool) (x:int) : ?? =  
  if not b then  
    string_of_int x  
  else  
    "hello"  
;;  
  
let y = 17;;
```

```
munge (y > 17) : ??  
  
munge true (f (munge false 3)) : ??  
  f : string -> int  
  
munge true (g munge) : ??  
  g : (bool -> int -> string) -> int
```


One key thing to remember

- If you have a function f with a type like this:

$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$

- Then each time you add an argument, you can get the type of the result by knocking off the first type in the series

$f\ a1 : B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$ (if $a1 : A$)

$f\ a1\ a2 : C \rightarrow D \rightarrow E \rightarrow F$ (if $a2 : B$)

$f\ a1\ a2\ a3 : D \rightarrow E \rightarrow F$ (if $a3 : C$)

$f\ a1\ a2\ a3\ a4\ a5 : F$ (if $a4 : D$ and $a5 : E$)

OUR FIRST* COMPLEX DATA STRUCTURE! THE TUPLE

* it is really our second complex data structure since functions are data structures too!

Tuples

- A tuple is a fixed, finite, ordered collection of values
- Some examples with their types:

```
(1, 2) : int * int
```

```
("hello", 7 + 3, true) : string * int * bool
```

```
('a', ("hello", "goodbye")) : char * (string * string)
```

Tuples

- To use a tuple, we extract its components
- General case:

```
let (id1, id2, ..., idn) = e1 in e2
```

- An example:

```
let (x, y) = (2, 4) in x + x + y
```

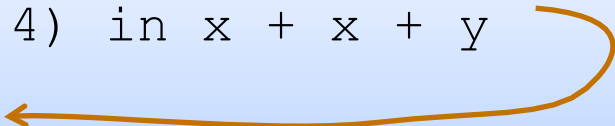
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```

- An example:

```
let (x, y) = (2, 4) in x + x + y  
--> 2 + 2 + 4
```



substitute!

Tuples

- To use a tuple, we extract its components
- General case:

```
let (id1, id2, ..., idn) = e1 in e2
```

- An example:

```
let (x, y) = (2, 4) in x + x + y
--> 2 + 2 + 4
--> 8
```

Rules for Typing Tuples

$$\frac{e1 : t1 \quad e2 : t2}{(e1, e2) : t1 * t2}$$

Rules for Typing Tuples

$$\frac{e1 : t1 \quad e2 : t2}{(e1, e2) : t1 * t2}$$

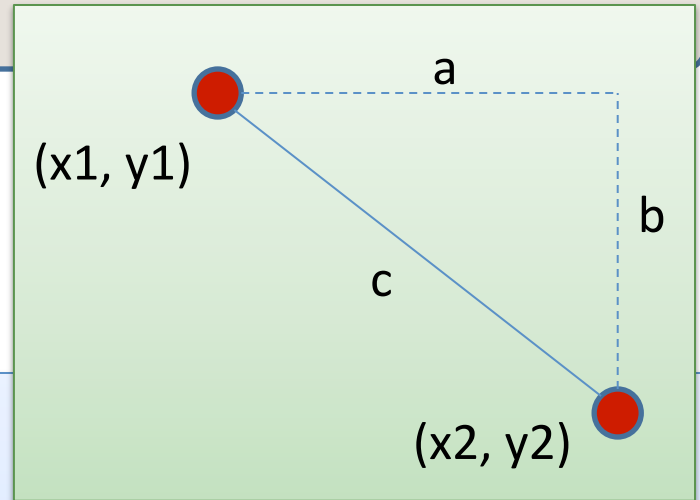
if $e1 : t1 * t2$ then
 $x1 : t1$ and $x2 : t2$
inside the expression $e2$

```
let (x1, x2) = e1 in  
e2
```

overall expression
takes on the type of $e2$

Distance between two points

$$c^2 = a^2 + b^2$$



Problem:

- A point is represented as a pair of floating point values.
- Write a function that takes in two points as arguments and returns the distance between them as a floating point number

Writing Functions Over Typed Data

Steps to writing functions over typed data:

1. **Write down** the function and argument names
2. **Write down** argument and result **types**
3. **Write down** some examples (in a comment)

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 - *the **argument types** suggests how to do it*
5. **Build** new output values
 - *the **result type** suggests how you do it*

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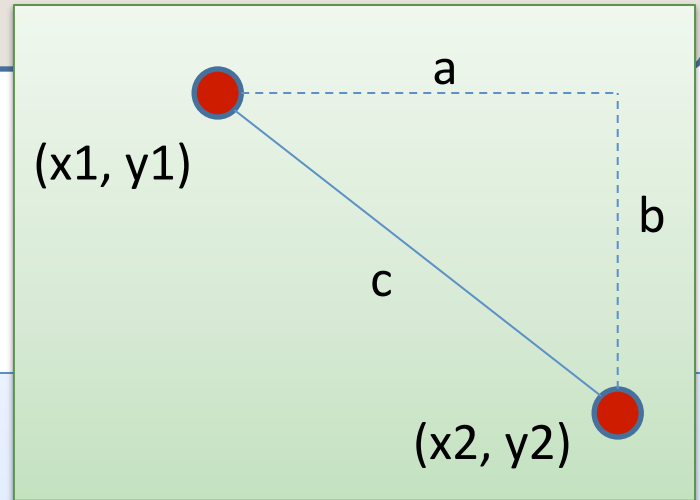
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Types help structure your thinking about how to write programs.

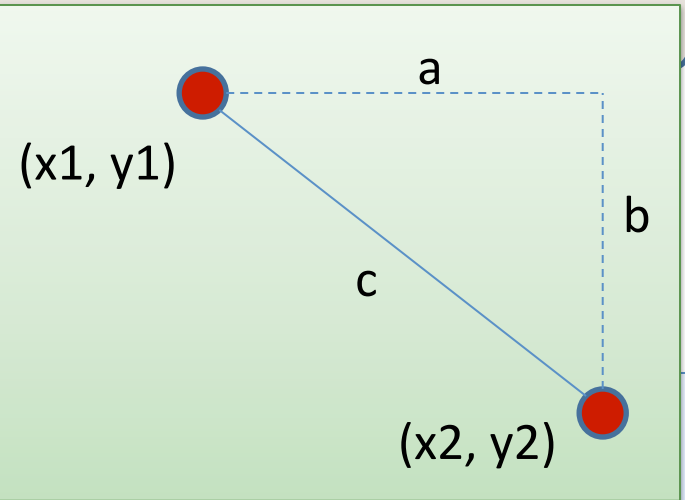
Distance between two points

a type abbreviation

```
type point = float * float
```



Distance between two points



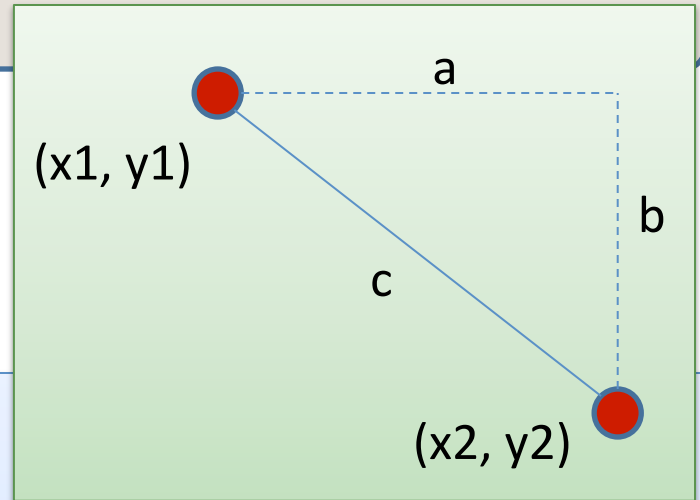
```
type point = float * float
```

```
let distance (p1:point) (p2:point) : float =
```

```
;;
```

write down function name
argument names and types

Distance between two points



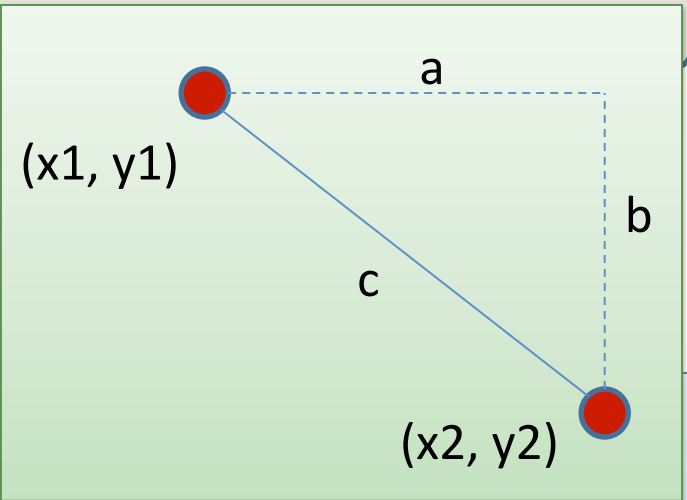
examples

```
type point = float * float
```

```
(* distance (0.0,0.0) (0.0,1.0) == 1.0
 * distance (0.0,0.0) (1.0,1.0) == sqrt(1.0 + 1.0)
 *
 * from the picture:
 * distance (x1,y1) (x2,y2) == sqrt(a^2 + b^2)
 *)
```

```
let distance (p1:point) (p2:point) : float =
```


Distance between two points



```
type point = float * float
```

```
let distance (p1:point) (p2:point) : float =
```

```
  let (x1,y1) = p1 in
```

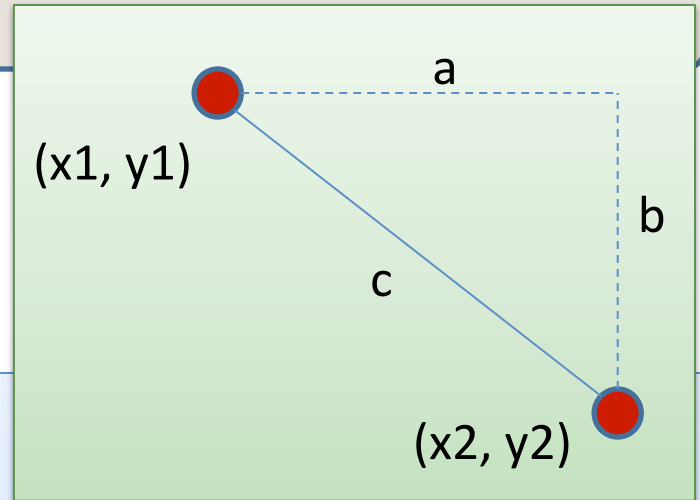
```
  let (x2,y2) = p2 in
```

```
  ...
```

```
;;
```

deconstruct
function inputs

Distance between two points



```
type point = float * float
```

```
let distance (p1:point) (p2:point) : float =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  sqrt ((x2 -. x1) *. (x2 -. x1) +.
```

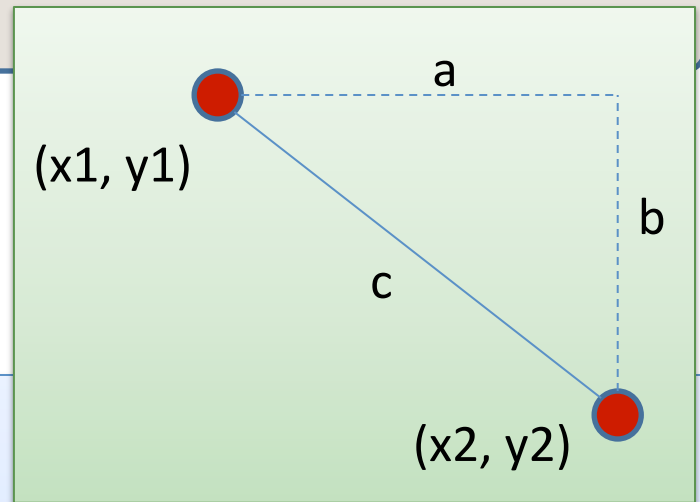
```
        (y2 -. y1) *. (y2 -. y1))
```

```
;;
```

} compute
function
results

notice operators on
floats have a "." in them

Distance between two points



```
type point = float * float
```

```
let distance (p1:point) (p2:point) : float =
```

```
  let square x = x *. x in
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

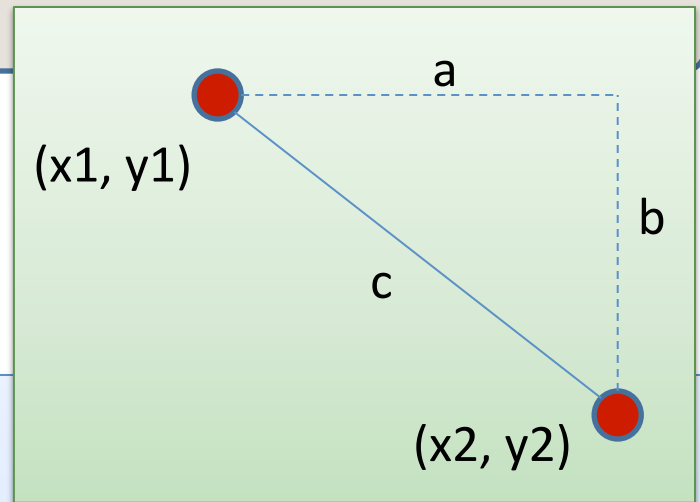
```
  sqrt (square (x2 -. x1)) +.
```

```
        square (y2 -. y1))
```

```
;;
```

define helper functions to
avoid repeated code

Distance between two points



```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;

let pt1 = (2.0,3.0);;
let pt2 = (0.0,1.0);;
let dist12 = distance pt1 pt2;;
```

testing

MORE TUPLES

Tuples

- Here's a tuple with 2 fields:

`(4.0, 5.0) : float * float`

Tuples

- Here's a tuple with 2 fields:

`(4.0, 5.0) : float * float`

- Here's a tuple with 3 fields:

`(4.0, 5, "hello") : float * int * string`

Tuples

- Here's a tuple with 2 fields:

`(4.0, 5.0) : float * float`

- Here's a tuple with 3 fields:

`(4.0, 5, "hello") : float * int * string`

- Here's a tuple with 4 fields:

`(4.0, 5, "hello", 55) : float * int * string * int`

Tuples

- Here's a tuple with 2 fields:

`(4.0, 5.0) : float * float`

- Here's a tuple with 3 fields:

`(4.0, 5, "hello") : float * int * string`

- Here's a tuple with 4 fields:

`(4.0, 5, "hello", 55) : float * int * string * int`

- Have you ever thought about what a tuple with 0 fields might look like?

Unit

- **Unit** is the tuple with zero fields!

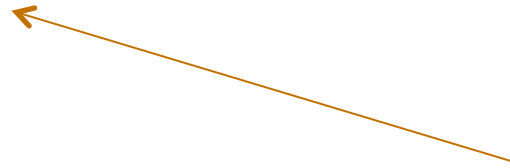
`() : unit`

- the unit value is written with an pair of parens
- there are no other values with this type!

Unit

- **Unit** is the tuple with zero fields!

`() : unit`



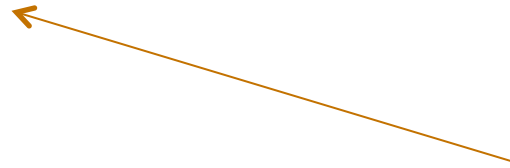
- the unit value is written with an pair of parens
 - there are no other values with this type!
-
- Why is the unit type and value useful?
 - Every expression has a type:

`(print_string "hello world\n") : ???`

Unit

- **Unit** is the tuple with zero fields!

`() : unit`



- the unit value is written with an pair of parens
 - there are no other values with this type!
-
- Why is the unit type and value useful?
 - Every expression has a type:

`(print_string "hello world\n") : unit`

- Expressions executed for their *effect* return the unit value

SUMMARY:
BASIC FUNCTIONAL PROGRAMMING

Writing Functions Over Typed Data

- Steps to writing functions over typed data:
 1. Write down the function and argument names
 2. Write down argument and result types
 3. Write down some examples (in a comment)
 4. **Deconstruct** input data structures
 5. **Build** new output values
 6. Clean up by identifying repeated patterns
- For unit type:
 - when the **input** has type **unit**
 - use `let () = ... in ...` to **deconstruct**
 - or better use `e1; ...` to deconstruct if `e1` has type **unit**
 - when the **output** has type **unit**
 - use `()` to **construct**

Writing Functions Over Typed Data

Steps to writing functions over typed data:

1. Write down the function and argument names
2. **Write down** argument and result **types**
3. Write down some examples (in a comment)
4. **Deconstruct** input data structures
 - the *argument types* suggest how to do it
5. **Build** new output values
 - the *result type* suggest how you do it
6. Clean up by identifying repeated patterns
 - define and reuse helper functions
 - your code should be elegant and easy to read

Writing Functions Over Typed Data

Steps to writing functions over typed data:

1. Write down the function and argument names
2. Write down argument and result types
3. Write down some examples (in a comment)
4. **Deconstruct** input data structures
5. **Build** new output values
6. Clean up by identifying repeated patterns

For tuple types:

- when the **input** has type $t1 * t2$
 - use `let (x,y) = ...` to **deconstruct**
- when the **output** has type $t1 * t2$
 - use `(e1, e2)` to **construct**

We will see this paradigm repeat itself over and over

Options

A value v has type t **option** if it is either:

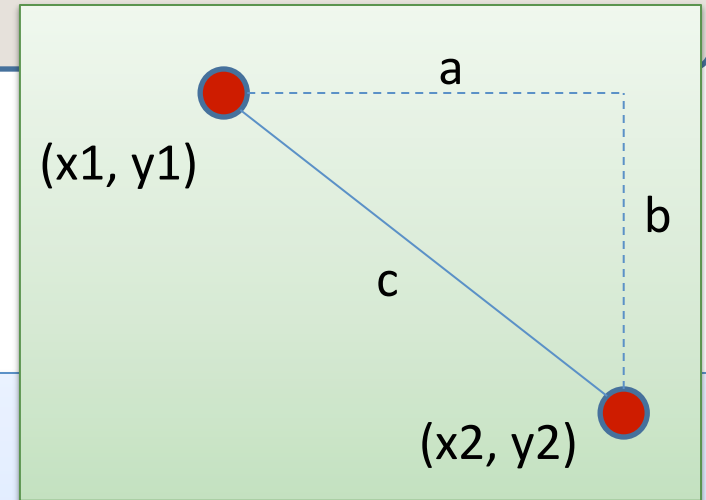
- the value **None**, or
- a value **Some v'** , and v' has type t

Options can signal there is no useful result to the computation

Example: we look up a value in a hash table using a key.

- **If the key is present**, return **Some v** where v is the associated value
- **If the key is not present**, we return **None**

Slope between two points

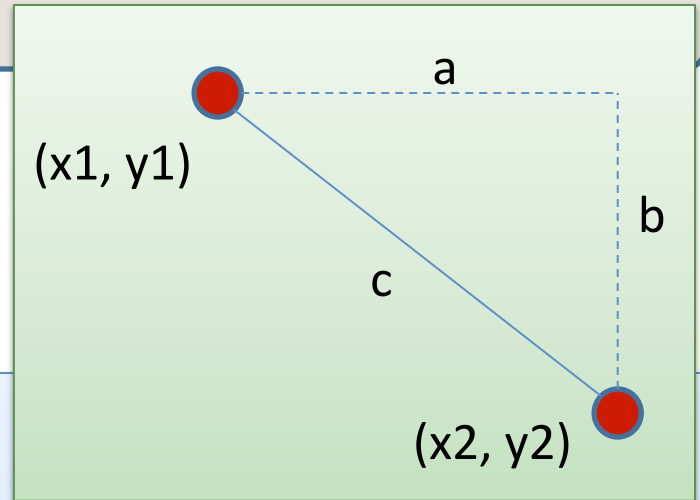


```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float =
```

```
;;
```

Slope between two points



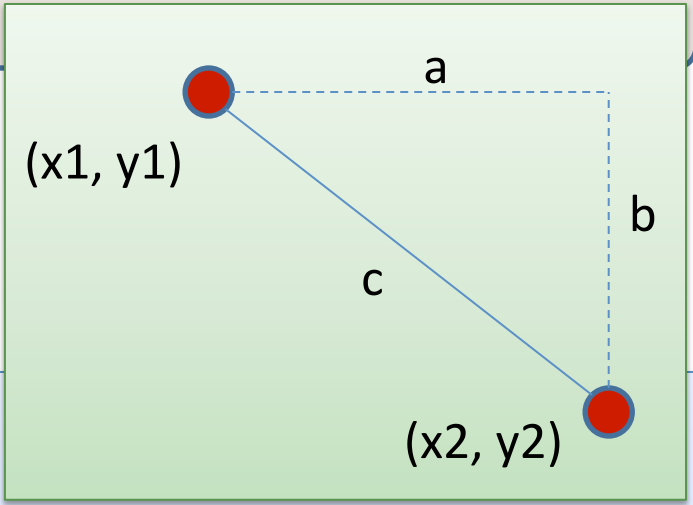
```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float =  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in
```

```
;;
```

deconstruct tuple

Slope between two points



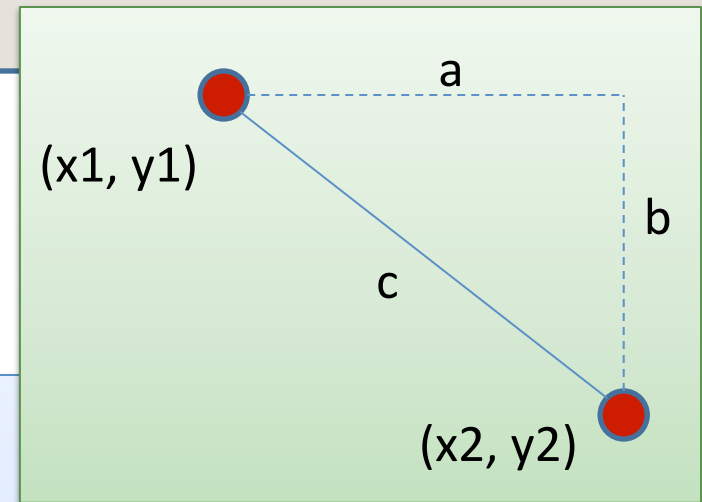
```
type point = float * float

let slope (p1:point) (p2:point) : float =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    ???
;;
```

avoid divide by zero

what can we return?

Slope between two points

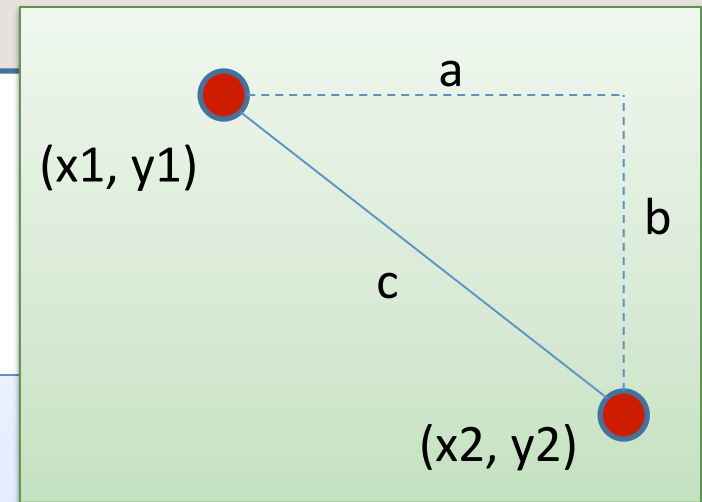


```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in  
  let xd = x2 -. x1 in  
  if xd != 0.0 then  
    ???  
  else  
    ???  
;;
```

we need an option
type as the result type

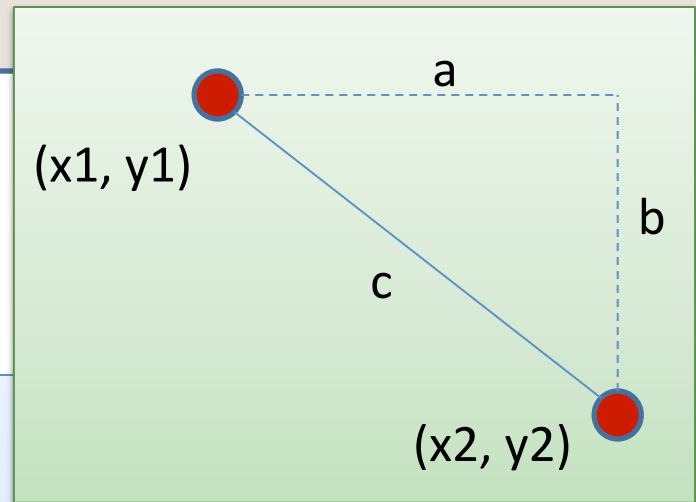
Slope between two points



```
type point = float * float

let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    Some ((y2 -. y1) /. xd)
  else
    None
;;
```

Slope between two points



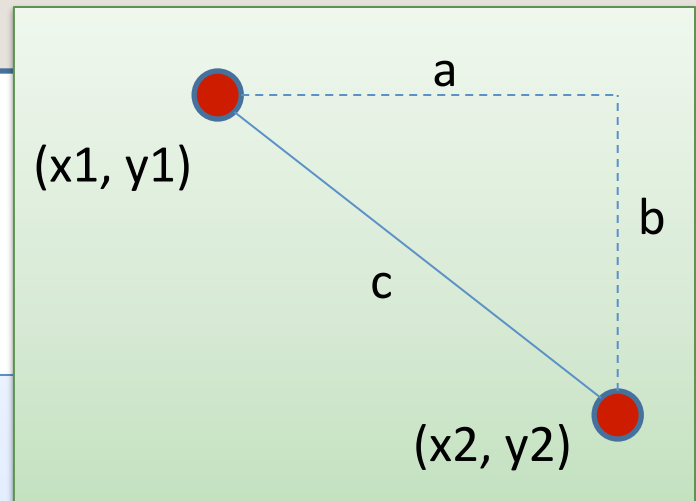
```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    None
;;
```

Has type **float**

Can have type **float option**

Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  let xd = x2 -. x1 in
```

```
  if xd != 0.0 then
```

```
    (y2 -. y1) /. xd
```

```
  else
```

```
    None
```

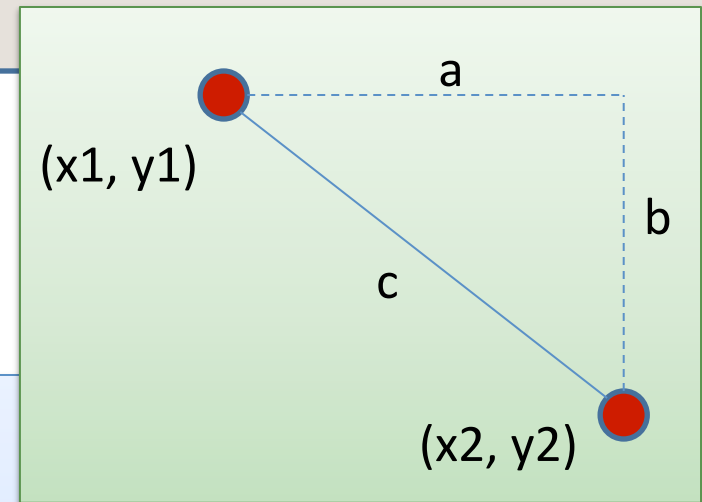
```
;;
```

Has type **float**

Can have type **float option**

WRONG: Type mismatch

Slope between two points



```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
```

```
  let (x1,y1) = p1 in
```

```
  let (x2,y2) = p2 in
```

```
  let xd = x2 -. x1 in
```

```
  if xd != 0.0 then
```

```
    (y2 -. y1) /. xd
```

```
  else
```

```
    None
```

```
;;
```

Has type **float**

doubly WRONG:
result does not
match declared result

Remember the typing rule for if

$$\frac{e1 : \text{bool} \quad e2 : T \quad e3 : T}{\text{if } e1 \text{ then } e2 \text{ else } e3 : T}$$

$$\frac{}{\text{None} : T \text{ option}}$$

$$\frac{e : T}{\text{Some } e : T \text{ option}}$$

- Returning an optional value from an if statement:

if ... then

None : t option

else

Some (...) : t option

How do we use an option?

```
slope : point -> point -> float option
```

returns a float option



How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
```

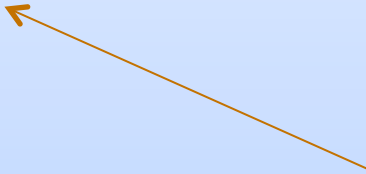
```
;;
```

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
    slope p1 p2
```

```
;;
```



returns a float option;
to print we must discover if it is
None or Some

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with
```

```
;;
```

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with  
  | Some s ->  
  | None ->  
;;
```

There are two possibilities

Vertical bar separates possibilities

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with  
  | Some s =>  
  | None ->  
;;
```

The "Some s" pattern includes the variable s

The object between | and -> is called a pattern

How do we use an option?

```
slope : point -> point -> float option

let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
  | Some s ->
    print_string ("Slope: " ^ string_of_float s)
  | None ->
    print_string "Vertical line.\n"
;;
```

Writing Functions Over Typed Data

- Steps to writing functions over typed data:
 1. Write down the function and argument names
 2. Write down argument and result types
 3. Write down some examples (in a comment)
 4. **Deconstruct** input data structures
 5. **Build** new output values
 6. Clean up by identifying repeated patterns
- For option types:

when the **input** has type **t option**,
deconstruct with:

```
match ... with
| None -> ...
| Some s -> ...
```

when the **output** has type **t option**,
construct with:

Some (...)

None

MORE PATTERN MATCHING

Recall the Distance Function

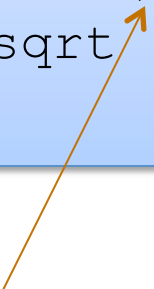
```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```



(x_2, y_2) is an example of a pattern – a pattern for tuples.


So let declarations can contain patterns just like match statements

The difference is that a match allows you to consider multiple different data shapes

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
    let (x2,y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```




There is only 1 possibility when matching a pair

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
    match p2 with
    | (x2,y2) ->
      sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```



We can nest one match expression inside another.

(We can nest any expression inside any other, if the expressions have the right types)

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | ((x1, y1), (x2, y2)) ->
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

Pattern for a pair of pairs: **((variable, variable), (variable, variable))**

All the variable names in the pattern must be different.

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | (p3, p4) ->
    let (x1, y1) = p3 in
    let (x2, y2) = p4 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

A pattern must be **consistent with** the type of the expression
in between **match ... with**

We use (p3, p4) here instead of ((x1, y1), (x2, y2))

Pattern-matching in function parameters

```
type point = float * float

let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

Function parameters are patterns too!

What's the best style?

```
let distance (p1:point) (p2:point) : float =  
  let square x = x *. x in  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in  
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

```
let distance ((x1,y1):point) ((x2,y2):point) : float =  
  let square x = x *. x in  
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Either of these is reasonably clear and compact.

Code with unnecessary nested matches/lets is particularly ugly to read.

You'll be judged on code style in this class.

Combining patterns

```
type point = float * float
```

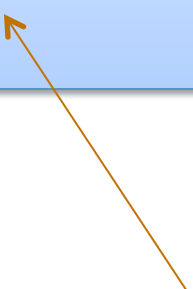
```
(* returns a nearby point in the graph if one exists *)  
nearby : graph -> point -> point option
```

```
let printer (g:graph) (p:point) : unit =  
  match nearby g p with  
  | None -> print_string "could not find one\n"  
  | Some (x,y) ->  
    print_float x;  
    print_string ", ";  
    print_float y;  
    print_newline();  
;;
```

Other Patterns

- Constant values can be used as patterns

```
let small_prime (n:int) : bool =  
  match n with  
  | 2 -> true  
  | 3 -> true  
  | 5 -> true  
  | _ -> false  
;;
```



```
let iffy (b:bool) : int =  
  match b with  
  | true -> 0  
  | false -> 1  
;;
```

the underscore pattern
matches anything
it is the "don't care" pattern

A SHORT JAVA RANT

Definition and Use of Java Pairs

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a, int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
  
        return p2;  
    }  
}
```

What could go wrong?

A Paucity of Types

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a, int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
  
        return p2;  
    }  
}
```

The input `p1` to `swap` may be `null` and we forgot to check.

Java has no way to define a pair data structure that is *just a pair*.

How many students in the class have seen an accidental null pointer exception thrown in their Java code?

From Java Pairs to O'Caml Pairs

In O'Caml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

From Java Pairs to O'Caml Pairs

In O'Caml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =  
  let (x,y) = p in  
  (y,x)
```

From Java Pairs to O'Caml Pairs

In O'Caml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =  
  let (x,y) = p in  
  (y,x)
```

You get a *helpful* error message like this:

```
# ... Characters 91-92:  
  let (x,y) = p in (y,x);;  
                ^
```

```
Error: This expression has type java_pair = (int * int) option  
      but an expression was expected of type 'a * 'b
```

From Java Pairs to O'Caml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | Some (x,y) -> Some (y,x)
```

From Java Pairs to O'Caml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | Some (x,y) -> Some (y,x)
```

OCaml to the rescue!

```
..match p with  
  | Some (x,y) -> Some (y,x)  
Warning 8: this pattern-matching is not exhaustive.  
Here is an example of a value that is not matched:  
None
```

From Java Pairs to O'Caml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | Some (x,y) -> Some (y,x)
```



An easy fix!



```
let swap_java_pair (p:java_pair) : java_pair =  
  match p with  
  | None -> None  
  | Some (x,y) -> Some (y,x)
```

From Java Pairs to O'Caml Pairs

Moreover, your pairs are probably almost never null!

Defensive programming & always checking for null is
AnNOyinG

From Java Pairs to O'Caml Pairs

There just isn't always some "good thing" for a function to do when it receives a bad input, like a null pointer

In O'Caml, all these issues disappear when you use the proper type for a pair and that type contains no "extra junk"

```
type pair = int * int
```

Once you know O'Caml, it is *hard* to write swap incorrectly

Your *bullet-proof* code is much simpler than in Java.

```
let swap (p:pair) : pair =  
  let (x,y) = p in (y,x)
```


Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe just the triples
- There is no type to describe the pairs of pairs
- There is no type ...

OCaml has many more types

- use option when things may be null
- do not use option when things are not null
- OCaml types describe data structures more precisely
 - programmers have fewer cases to worry about
 - entire classes of errors just go away
 - type checking and pattern analysis help prevent programmers from ever forgetting about a case

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe nested pairs
- There is no type to describe lists
- There is no type to describe...

OCaml

- use of...
- d...

SCORE: OCAML 1, JAVA 0

- type checking and pattern analysis help prevent programmers from ever forgetting about a case

C, C++ Rant

Java has a paucity of types

- but at least when you forget something, it ***throws an exception*** instead of silently going off the trolley!

If you forget to check for null pointer in a C program,

- no type-check error at compile time
- no exception at run time
- it might crash right away (that would be best), or
- it might permit a buffer-overflow (or similar) vulnerability
- so the hackers pwn you!

Summary of C, C++ rant

Java has a paucity of types

- but at least when you forget something it **throws an exception** instead of going off the trolley!

If you

- no type

SCORE:

OCAML 1, JAVA 0, C -1

- it is
- it is similar, vulnerability
- so the hacker can

**OVERALL SUMMARY:
A SHORT INTRODUCTION TO
FUNCTIONAL PROGRAMMING**

Functional Programming

Steps to writing functions over typed data:

1. **Write down** the function and argument **names**
2. **Write down** argument and result **types**
3. **Write down** some examples
4. **Deconstruct** input data structures
 - the argument types suggest how you do it
 - the types tell you which cases you must cover
5. **Build** new output values
 - the result type suggests how you do it
6. **Clean up** by identifying repeated patterns
 - define and reuse helper functions
 - refactor code to use your helpers
 - your code should be elegant and easy to read

Summary: Constructing/Deconstructing Values

Type	Construct Values	Number of Cases	Deconstruct Values
int	0, -1, 2, ...	$2^{31}-1$	match i with 0 -> ... -1 -> x -> ...
bool	true, false	2	match b with true -> ... false -> ...
t1 * t2	(2, "hi")	(# of t1) * (# of t2)	let (x,y) = ... in ... match p with (x,y) -> ...
unit	()	1	e1; ...
t option	None, Some 3	$1 + (\# \text{ of } t1)$	match opt with None -> ... Some x -> ...