# 5.5 Data Compression

- basics
- run-length coding
- Huffman compression
- LZW compression

# Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.

Who needs compression?

- Moore's law: # transistors on a chip doubles every 18-24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, …

> “ *All of the books in the world contain no more information than is broadcast as video in a single large American city in a single year. Not all bits have equal value.* ” *— Carl Sagan*

Basic concepts ancient (1950s), best technology recently developed.

# Applications

**Generic file compression.**

- Files: GZIP, BZIP, BOA.
- Archivers: PKZIP.
- File systems: NTFS.

**Multimedia.**

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.

**Communication.**

- ITU-T T4 Group 3 Fax.
- V.42bis modem.

**Databases.** Google.

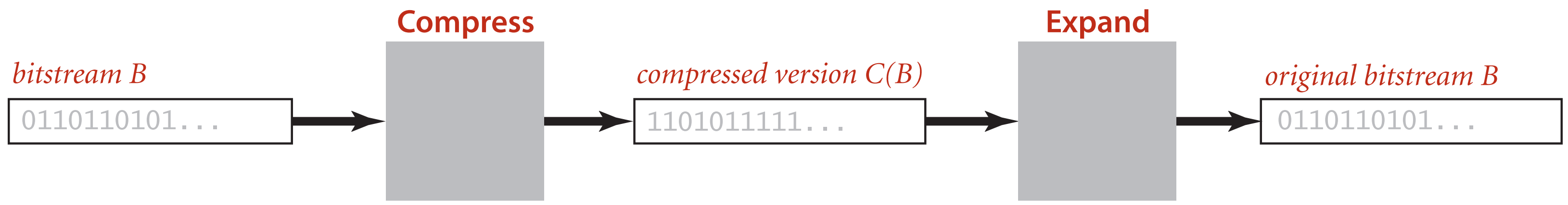
# Lossless compression and expansion

uses fewer bits (you hope)

**Message.** Binary data $B$ we want to compress.

**Compress.** Generates a "compressed" representation $C(B)$.

**Expand.** Reconstructs original bitstream $B$.



Basic model for data compression

**Compression ratio.** Bits in $C(B)$ / bits in $B$.

**Ex.** 50-75% or better compression ratio for natural language.

# Food for thought

Data compression has been omnipresent since antiquity:

- Number systems.
- Natural languages.
- Mathematical notation.

has played a central role in communications technology,

- Braille.
- Morse code.
- Telephone system.

and is part of modern life.

- MP3.
- MPEG.

Q. What role will it play in the future?

- **basics**
- run-length coding
- Huffman compression
- LZW compression

# Data representation: genomic code

Genome. String over the alphabet { A, C, T, G }.

Goal. Encode an $N$-character genome: **ATAGATGCATAG...**

Standard ASCII encoding.

- $8$ bits per char.
- $8\ N$ bits.

| char | hex | binary |
|---|---|---|
| **A** | 41 | 01000001 |
| **C** | 43 | 01000011 |
| **T** | 54 | 01010100 |
| **G** | 47 | 01000111 |

Two-bit encoding.

- $2$ bits per char.
- $2\ N$ bits.

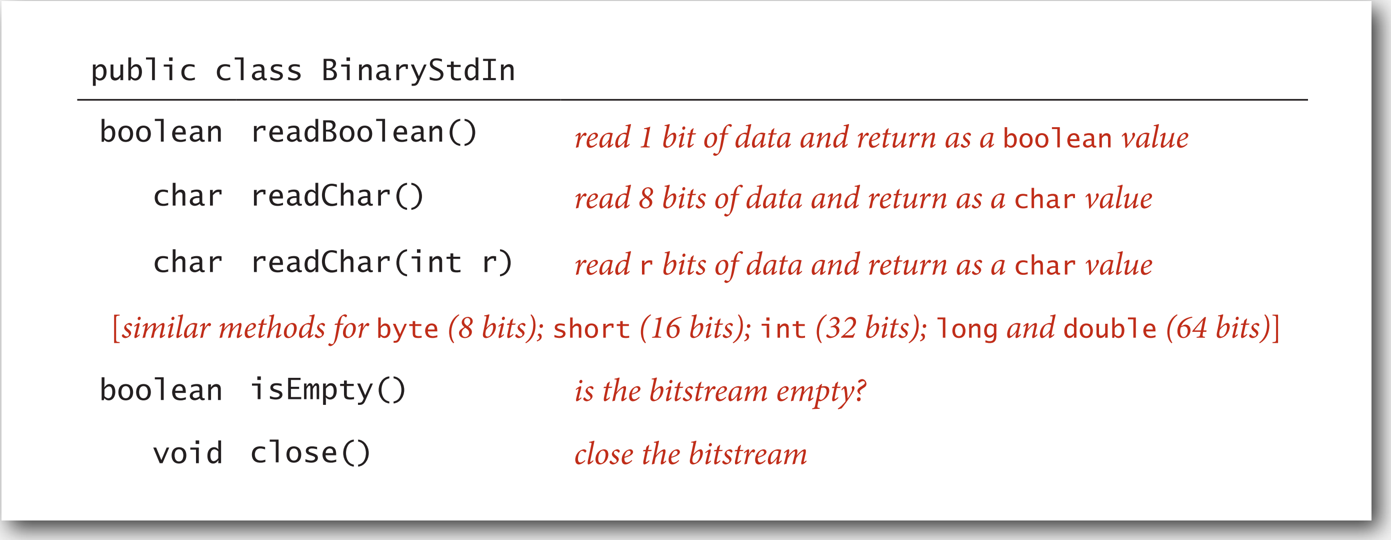
| char | binary |
|---|---|
| **A** | 00 |
| **C** | 01 |
| **T** | 10 |
| **G** | 11 |

Amazing but true. Initial genomic databases in 1990s did not use such a code!

Fixed-length code. $k$-bit code supports alphabet of size $2^k$.

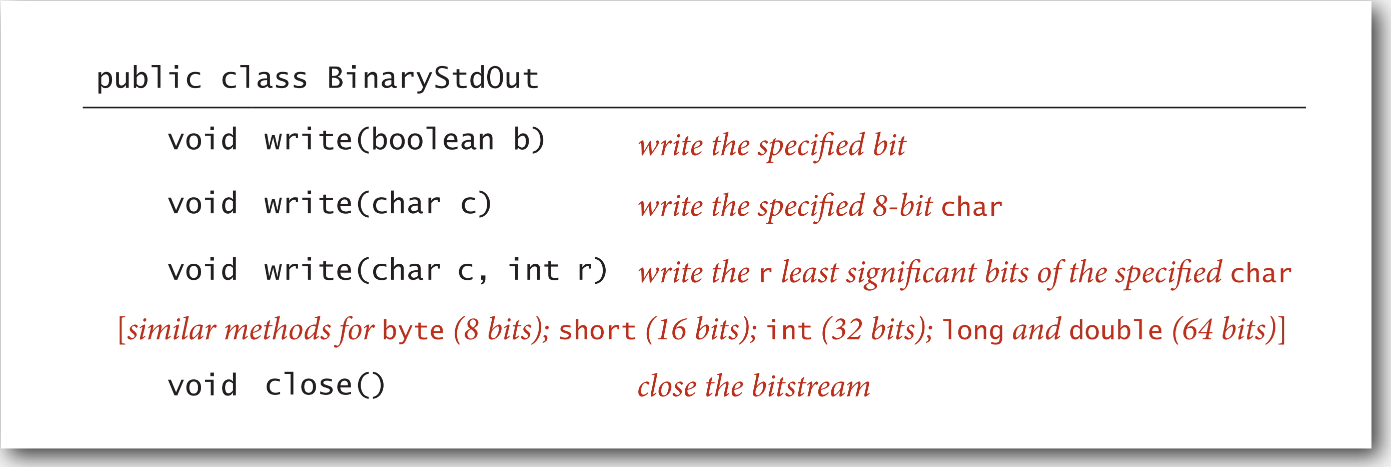
# Reading and writing binary data

Binary standard input and standard output. Libraries to read and write bits from standard input and to standard output.



public class BinaryStdIn

| | | |
|---|---|---|
| boolean | readBoolean() | *read 1 bit of data and return as a* boolean *value* |
| char | readChar() | *read 8 bits of data and return as a* char *value* |
| char | readChar(int r) | *read* r *bits of data and return as a* char *value* |
| *[similar methods for* byte *(8 bits);* short *(16 bits);* int *(32 bits);* long *and* double *(64 bits)]* | | |
| boolean | isEmpty() | *is the bitstream empty?* |
| void | close() | *close the bitstream* |

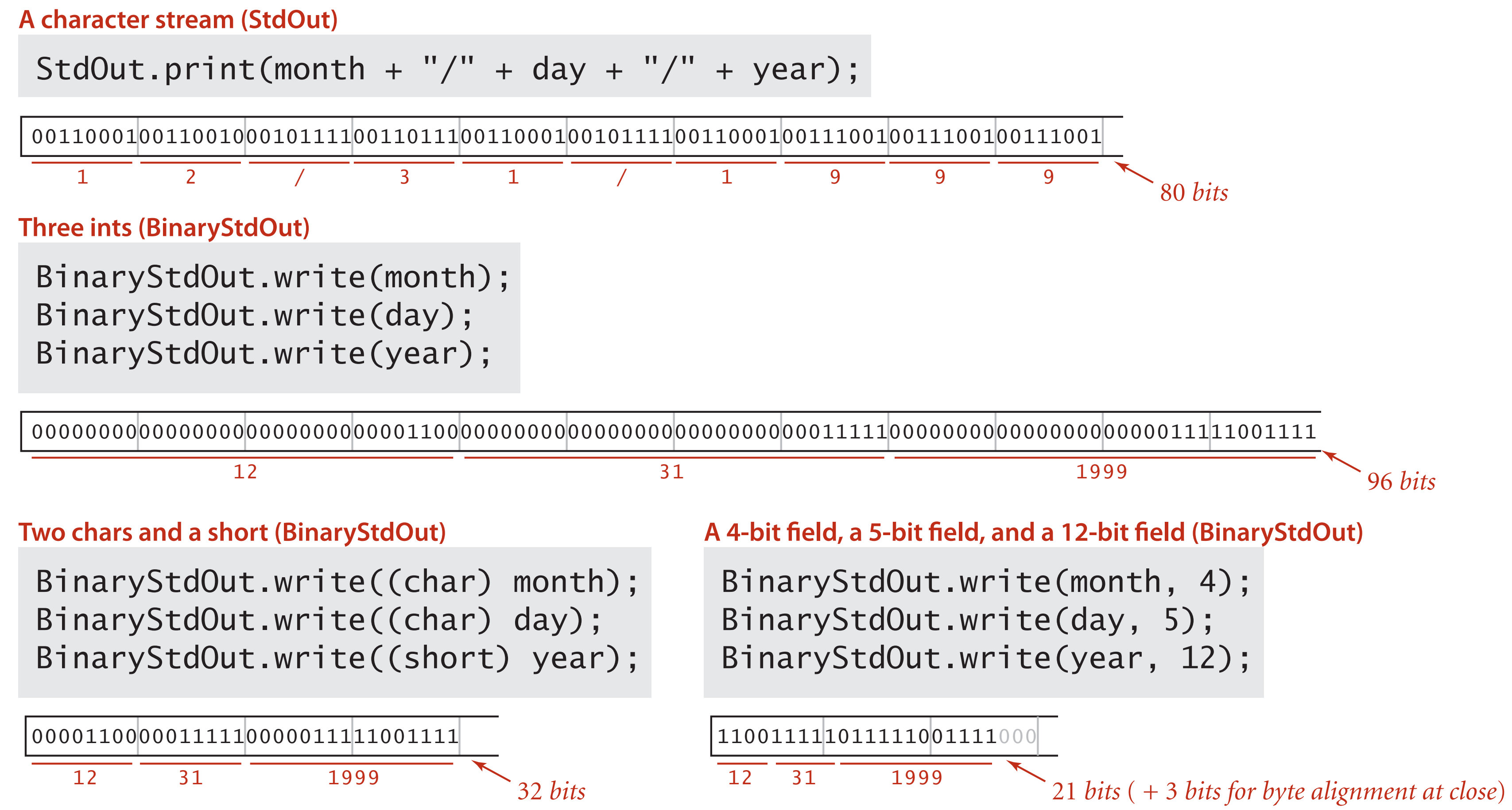


public class BinaryStdOut

| | | |
|---|---|---|
| void | write(boolean b) | *write the specified bit* |
| void | write(char c) | *write the specified 8-bit* char |
| void | write(char c, int r) | *write the* r *least significant bits of the specified* char |
| *[similar methods for* byte *(8 bits);* short *(16 bits);* int *(32 bits);* long *and* double *(64 bits)]* | | |
| void | close() | *close the bitstream* |

# Writing binary data

Date representation. Different ways to represent 12/31/1999.



A character stream (StdOut)

```
StdOut.print(month + "/" + day + "/" + year);
```

```
00110001001100100010111100110111001100010010111100110001001110010011100100111001
```

Three ints (BinaryStdOut)

```
BinaryStdOut.write(month);
BinaryStdOut.write(day);
BinaryStdOut.write(year);
```

```
000000000000000000000000000011000000000000000000000000000001111100000000000000000000011111001111
```

Two chars and a short (BinaryStdOut)

```
BinaryStdOut.write((char) month);
BinaryStdOut.write((char) day);
BinaryStdOut.write((short) year);
```

```
00001100000111110000011111001111
```

A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)

```
BinaryStdOut.write(month, 4);
BinaryStdOut.write(day, 5);
BinaryStdOut.write(year, 12);
```

```
110011111011111001111000
```

Four ways to put a date onto standard output

# Binary dumps

Q. How to examine the contents of a bitstream?

**Standard character stream**

```
% more abra.txt
ABRACADABRA!
```

**Bitstream represented as 0 and 1 characters**

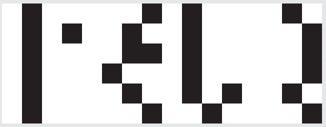
```
% java BinaryDump 16 < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

**Bitstream represented with hex digits**

```
%  java HexDump 4 < abra.txt
41 42 52 41
43 41 44 41
42 52 41 21
12 bytes
```

**Bitstream represented as pixels in a Picture**

```
% java PictureDump 16 6 < abra.txt
```



*16-by-6 pixel window, magnified*

```
96 bits
```

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | NUL | SOH | STX | ETX | EOT | ENQ | ACK | BEL | BS | HT | LF | VT | FF | CR | SO | SI |
| 1 | DLE | DC1 | DC2 | DC3 | DC4 | NAK | SYN | ETB | CAN | EM | SUB | ESC | FS | GS | RS | US |
| 2 | SP | ! | “ | # | $ | % | & | ‘ | ( | ) | * | + | , | - | . | / |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | : | ; | < | = | > | ? |
| 4 | @ | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| 5 | P | Q | R | S | T | U | V | W | X | Y | Z | [ | \ | ] | ^ | _ |
| 6 | ` | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o |
| 7 | p | q | r | s | t | u | v | w | x | y | z | { | \| | } | ~ | DEL |

**Hexadecimal to ASCII conversion table**

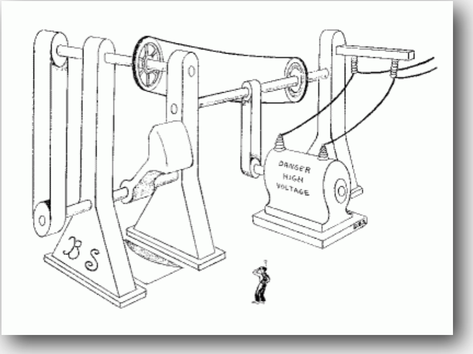
# Universal data compression

US Patent 5,533,051 on "Methods for Data Compression", which is capable of compression all files.

Slashdot reports of the Zero Space Tuner™ and BinaryAccelerator™.

> *“ ZeoSync has announced a breakthrough in data compression that allows for 100:1 lossless compression of random data. If this is true, our bandwidth problems just got a lot smaller.… ”*

Physical analog. Perpetual motion machines.



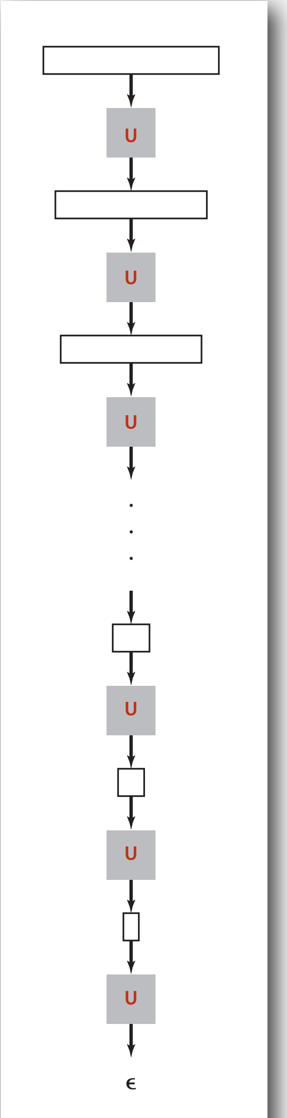
Gravity engine by Bob Schadewald

# Universal data compression

Proposition. No algorithm can compress every bitstring.

Pf 1. [by contradiction]

- Suppose you have a universal data compression algorithm $U$ that can compress every bitstream.
- Given bitstring $B_0$, compress it to get smaller bitstring $B_1$.
- Compress $B_1$ to get a smaller bitstring $B_2$.
- Continue until reaching bitstring of size $0$.
- Implication: all bitstrings can be compressed to $0$ bits!



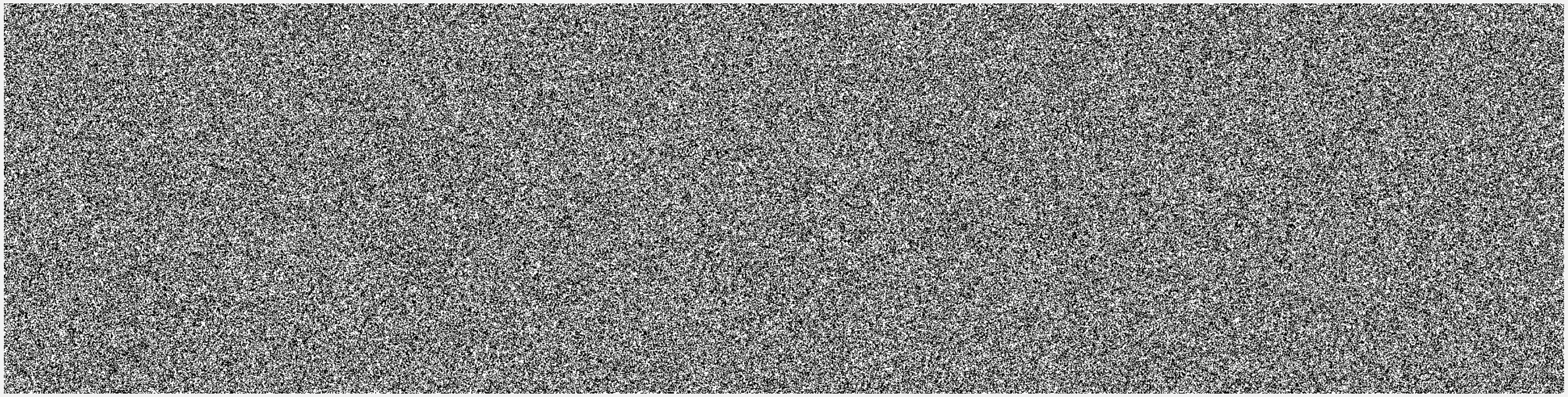
Universal
data compression?

Pf 2. [by counting]

- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- $2^{1000}$ possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999}$ can be encoded with $\leq 999$ bits.
- Similarly, only $1$ in $2^{499}$ bitstrings can be encoded with $\leq 500$ bits!

# Undecidability

```
% java RandomBits | java PictureDump 2000 500
```



```
1000000 bits
```

A difficult file to compress: one million (pseudo-) random bits

```
public class RandomBits
{
   public static void main(String[] args)
   {
      int x = 11111;
      for (int i = 0; i < 1000000; i++)
      {
         x = x * 314159 + 218281;
         BinaryStdOut.write(x > 0);
      }
      BinaryStdOut.close();
   }
}
```

# Rdenudcany in Enlgsih lnagugae

Q. How much redundancy is in the English language?

> “ ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang. ” — *Graham Rawlinson*

A. Quite a bit.

- basics
- run-length coding
- Huffman compression
- LZW compression

# Run-length encoding

Simple type of redundancy in a bitstream. Long runs of repeated bits.

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1
```

Representation. Use 4-bit counts to represent alternating runs of 0s and 1s: 15 0s, then 7 1s, then 7 0s, then 11 1s.

$$\underbrace{1\,1\,1\,1}_{15}\;\underbrace{0\,1\,1\,1}_{7}\;\underbrace{0\,1\,1\,1}_{7}\;\underbrace{1\,0\,1\,1}_{11} \longleftarrow \text{16 bits (instead of 40)}$$

Q. How many bits to store the counts?

A. We'll use 8.

Q. What to do when run length exceeds max count?

A. If longer than 255, intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

# Run-length encoding: Java implementation

```
public class RunLength
{
   private final static int R = 256;

   public static void compress()
   {  /* see textbook */  }

   public static void expand()
   {
      boolean b = false;
      while (!BinaryStdIn.isEmpty())
      {
         char run = BinaryStdIn.readChar();
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(b);
         b = !b;
      }
      BinaryStdOut.close();
   }

}
```

run-length limit
(needed for compress)

read 8-bit count from standard input

write 1 bit to standard output

# An application: compress a bitmap

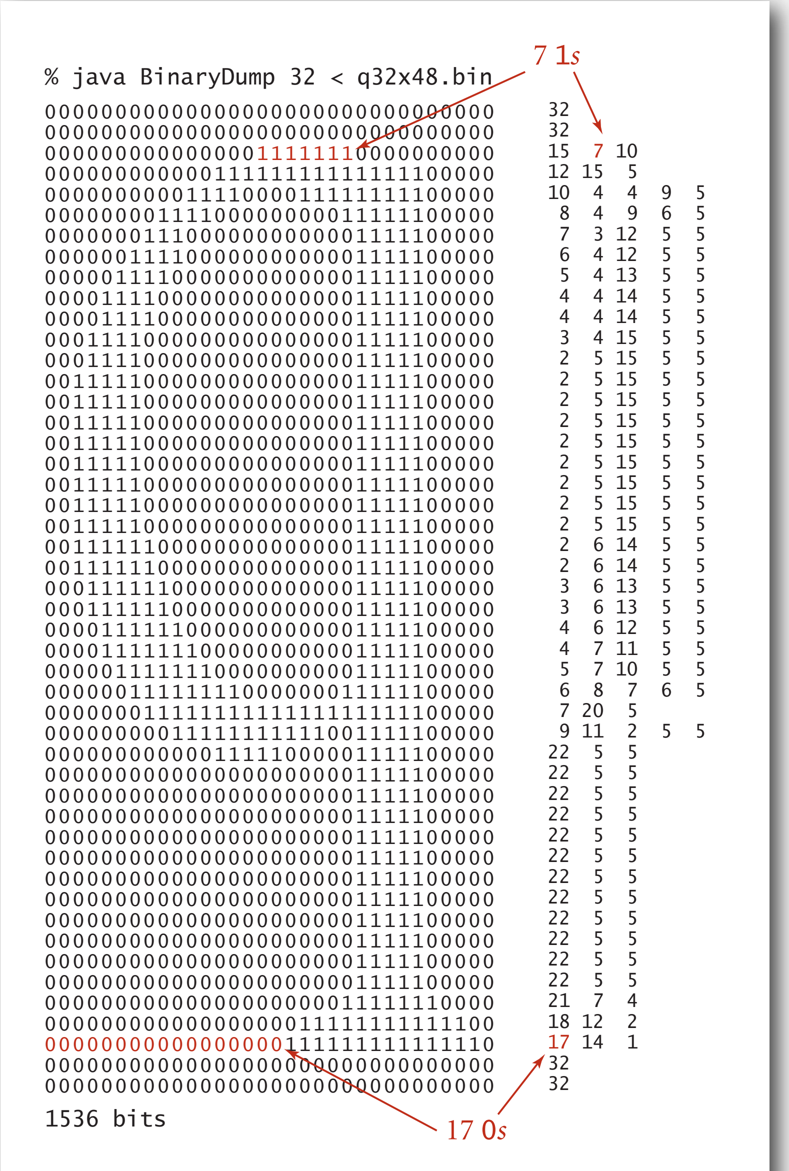
Typical black-and-white-scanned image.

- 300 pixels/inch.
- 8.5-by-11 inches.
- 300 × 8.5 × 300 × 11 = 8.415 million bits.

Observation. Bits are mostly white.

Typical amount of text on a page.

40 lines × 75 chars per line = 3,000 chars.



```
% java BinaryDump 32 < q32x48.bin
00000000000000000000000000000000     32
00000000000000000000000000000000     32
00000000000000011111110000000000     15  7 10
00000000000011111111111111100000     12 15  5
00000000001111000011111111100000     10  4  4   9   5
00000000111100000000011111100000      8  4  9   6   5
00000001110000000000001111100000      7  3 12   5   5
00000011110000000000001111100000      6  4 12   5   5
00000111100000000000001111100000      5  4 13   5   5
00001111000000000000001111100000      4  4 14   5   5
00001111000000000000001111100000      4  4 14   5   5
00011110000000000000001111100000      3  4 15   5   5
00011110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111110000000000000001111100000      2  5 15   5   5
00111111000000000000001111100000      2  6 14   5   5
00111111000000000000001111100000      2  6 14   5   5
00011111100000000000001111100000      3  6 13   5   5
00011111100000000000001111100000      3  6 13   5   5
00001111110000000000001111100000      4  6 12   5   5
00001111111000000000001111100000      4  7 11   5   5
00000111111100000000001111100000      5  7 10   5   5
00000011111111000000011111100000      6  8  7   6   5
00000001111111111111111111100000      7 20  5
00000000011111111111001111100000      9 11  2   5   5
00000000000011111000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000001111100000     22  5  5
00000000000000000000011111110000     21  7  4
00000000000000000011111111111100     18 12  2
00000000000000000111111111111110     17 14  1
00000000000000000000000000000000     32
00000000000000000000000000000000     32
1536 bits
```

7 1s

17 0s

A typical bitmap, with run lengths for each row

- basics
- run-length coding
- **Huffman compression**
- LZW compression

# Variable-length codes

Use different number of bits to encode different chars.

Ex. Morse code: • • • – – – • • •

Issue. Ambiguity.

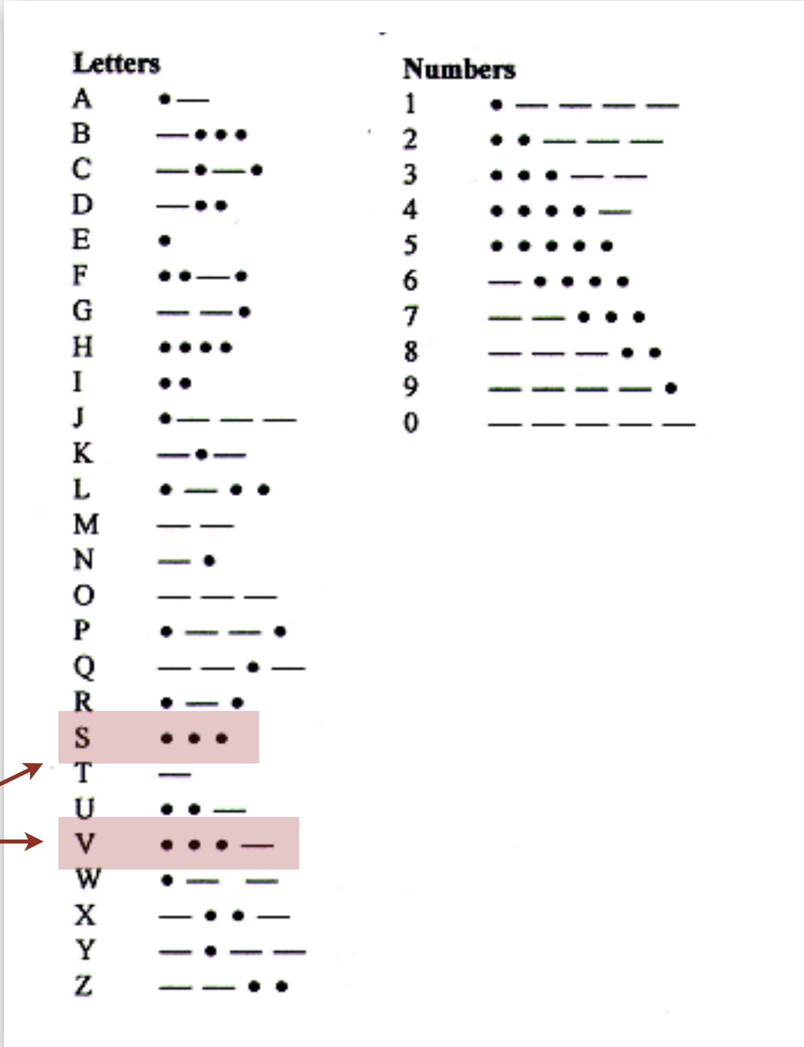
SOS ?

IAMIE ?

EEWNI ?

V7 ?

In practice. Use a medium gap to separate codewords.



| Letters | | Numbers | |
|---|---|---|---|
| A | •— | 1 | •———— |
| B | —••• | 2 | ••——— |
| C | —•—• | 3 | •••—— |
| D | —•• | 4 | ••••— |
| E | • | 5 | ••••• |
| F | ••—• | 6 | —•••• |
| G | ——• | 7 | ——••• |
| H | •••• | 8 | ———•• |
| I | •• | 9 | ————• |
| J | •——— | 0 | ————— |
| K | —•— | | |
| L | •—•• | | |
| M | —— | | |
| N | —• | | |
| O | ——— | | |
| P | •——• | | |
| Q | ——•— | | |
| R | •—• | | |
| S | ••• | | |
| T | — | | |
| U | ••— | | |
| V | •••— | | |
| W | •—— | | |
| X | —••— | | |
| Y | —•—— | | |
| Z | ——•• | | |

codeword for S is a prefix
of codeword for V

# Variable-length codes

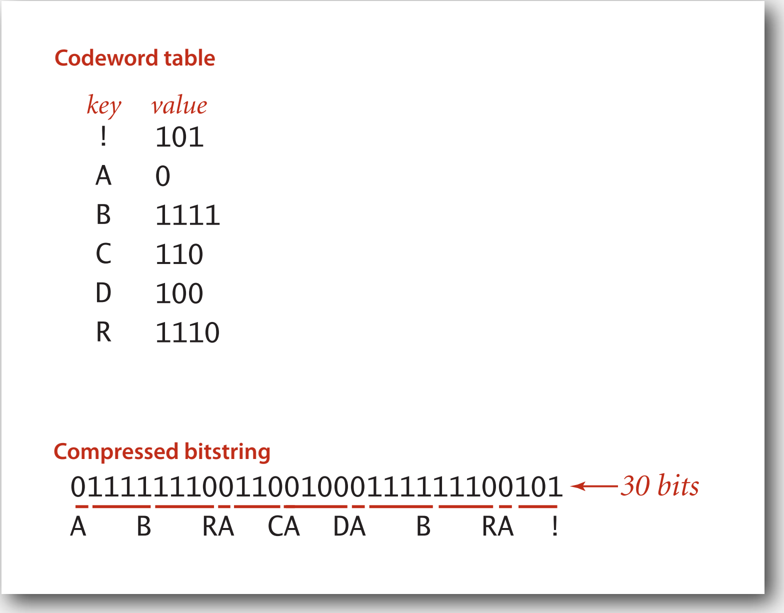
Q. How do we avoid ambiguity?

A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special stop char to each codeword.

Ex 3. General prefix-free code.



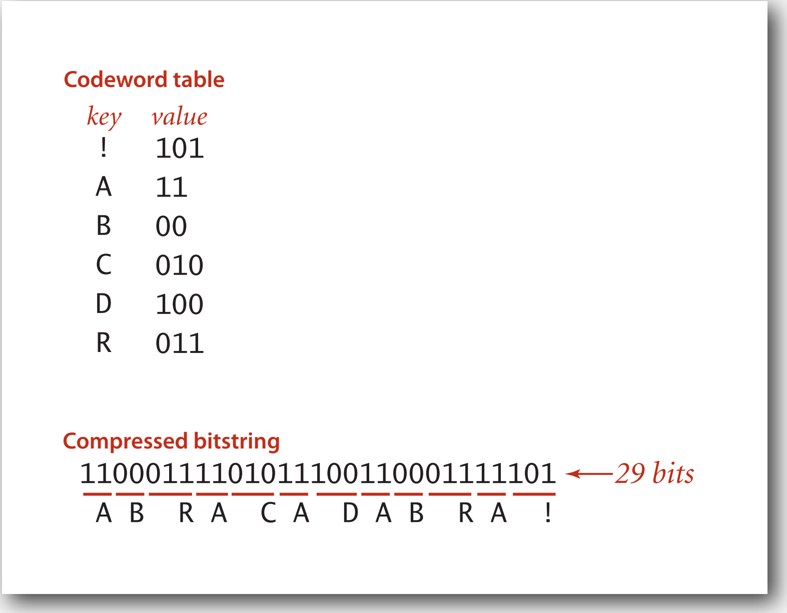
Codeword table

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 0 |
| B | 1111 |
| C | 110 |
| D | 100 |
| R | 1110 |

Compressed bitstring

011111110011001000111111100101 ← *30 bits*

A B RA CA DA B RA !



Codeword table

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 11 |
| B | 00 |
| C | 010 |
| D | 100 |
| R | 011 |

Compressed bitstring

11000111101011100110001111101 ← *29 bits*

A B R A C A D A B R A !

# Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

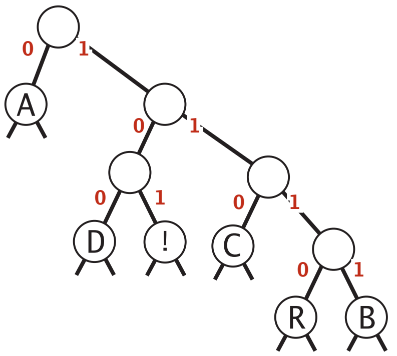
A. A binary trie!

- Chars in leaves.
- Codeword is path from root to leaf.

Codeword table

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 0 |
| B | 1111 |
| C | 110 |
| D | 100 |
| R | 1110 |

Trie representation



Compressed bitstring

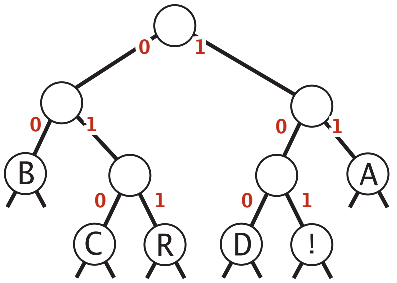
011111110011001000111111100101 ←30 bits

A B RA CA DA B RA !

Codeword table

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 11 |
| B | 00 |
| C | 010 |
| D | 100 |
| R | 011 |

Trie representation



Compressed bitstring

11000111101011100110001111101 ←29 bits

A B R A C A D A B R A !

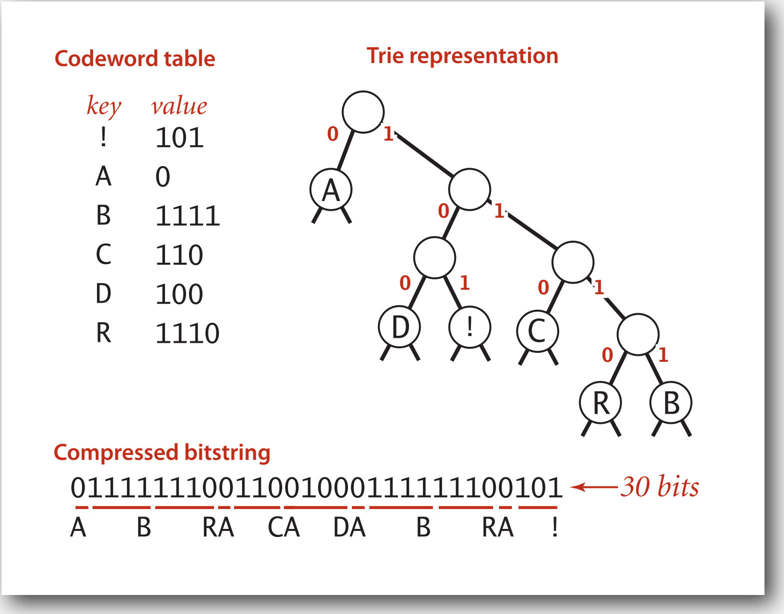
# Prefix-free codes: compression and expansion

## Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.
- Method 2: create ST of key-value pairs.

## Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, print char and return to root.



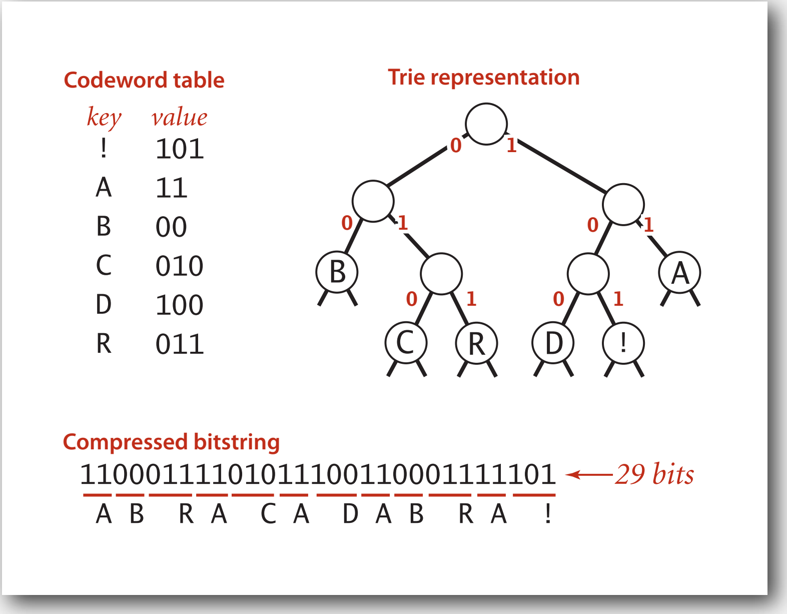
Codeword table

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 0 |
| B | 1111 |
| C | 110 |
| D | 100 |
| R | 1110 |

Trie representation

Compressed bitstring

```
011111110011001000111111100101 ←30 bits
A    B    RA   CA   DA    B    RA   !
```



Codeword table

| *key* | *value* |
|---|---|
| ! | 101 |
| A | 11 |
| B | 00 |
| C | 010 |
| D | 100 |
| R | 011 |

Trie representation

Compressed bitstring

```
11000111101011100110001111101 ←29 bits
 A B  R A  C A  D A B  R A  !
```

# Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
   private char ch;   // Unused for internal nodes.
   private int freq;  // Unused for expand.
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }
}
```

initializing constructor

is Node a leaf?

compare Nodes by frequency
(stay tuned)

# Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();
   int N = BinaryStdIn.readInt();

   for (int i = 0; i < N; i++)
   {
      Node x = root;
      while (!x.isLeaf())
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch);
   }
   BinaryStdOut.close();
}
```

read in encoding trie

read in number of chars

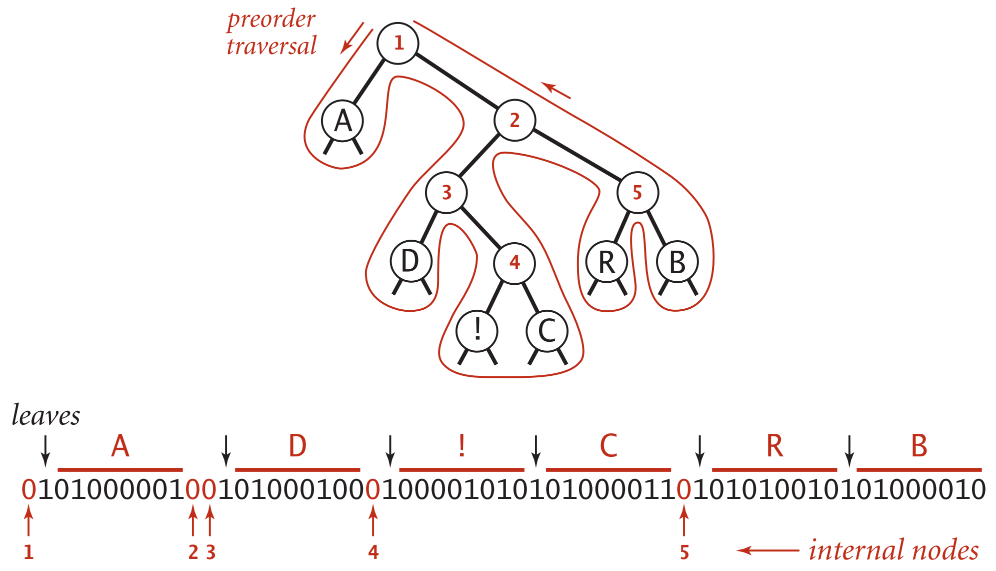
expand codeword for $i^{th}$ char

**Running time.** Linear in input size (constant amount of work per bit read).

# Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal of trie; mark leaf and internal nodes with a bit.



Using preorder traversal to encode a trie as a bitstream

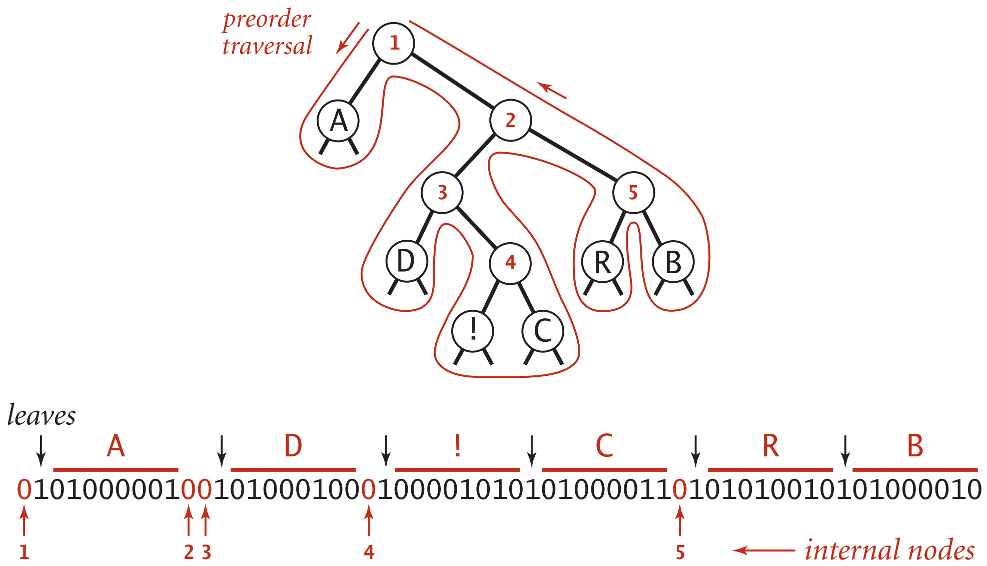
```
private static void writeTrie(Node x)
{
   if (x.isLeaf())
   {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

# Prefix-free codes: how to transmit

Q. How to read in the trie?

A. Reconstruct from preorder traversal of trie.



Using preorder traversal to encode a trie as a bitstream

```
private static Node readTrie()
{
   if (BinaryStdIn.readBoolean())
   {
      char c = BinaryStdIn.readChar();
      return new Node(c, 0, null, null);
   }
   Node x = readTrie();
   Node y = readTrie();
   return new Node('\0', 0, x, y);
}
```

not used

# Shannon-Fano codes

Q. How to find best prefix-free code?

Shannon-Fano algorithm:

- Partition symbols $S$ into two subsets $S_0$ and $S_1$ of (roughly) equal frequency.
- Codewords for symbols in $S_0$ start with $0$; for symbols in $S_1$ start with $1$.
- Recur in $S_0$ and $S_1$.

| char | freq | encoding |
|---|---|---|
| A | 5 | 0... |
| C | 1 | 0... |

$S_0$ = codewords starting with 0

| char | freq | encoding |
|---|---|---|
| B | 2 | 1... |
| D | 1 | 1... |
| R | 2 | 1... |
| ! | 1 | 1... |

$S_1$ = codewords starting with 1

Problem 1. How to divide up symbols?

Problem 2. Not optimal!

# Huffman codes

Q. How to find best prefix-free code?

David Huffman

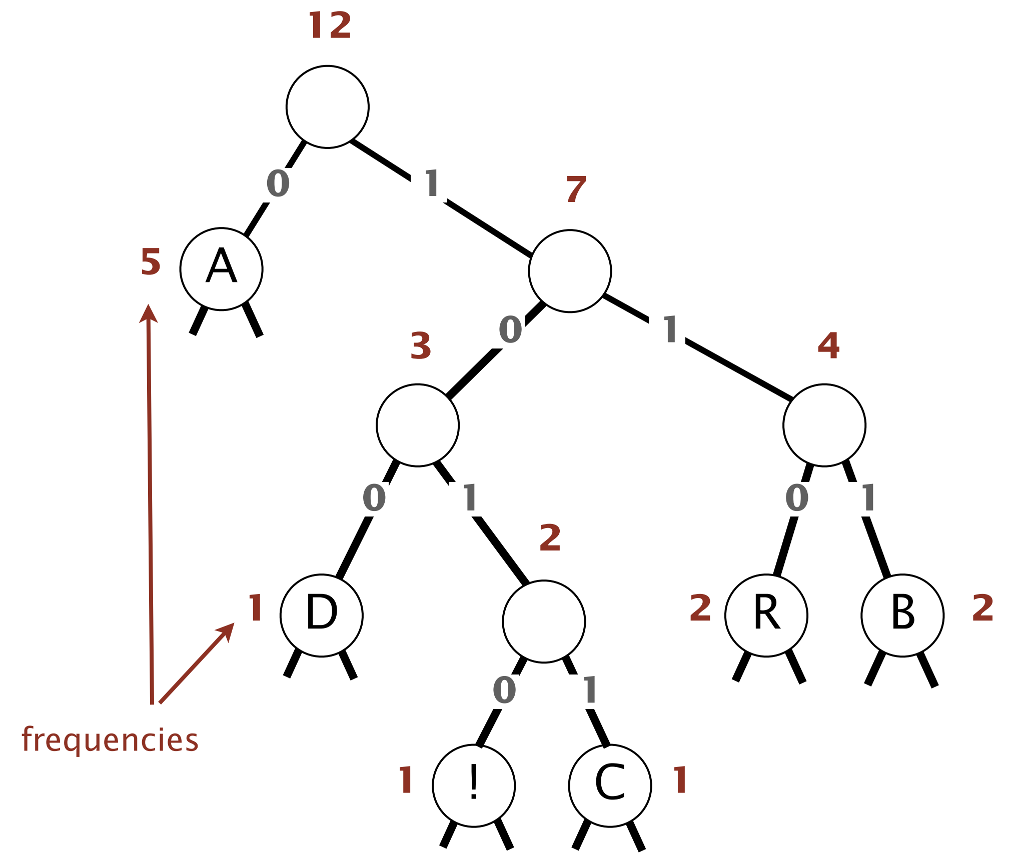
Huffman algorithm:

- Count frequency `freq[i]` for each char `i` in input.
- Start with one node corresponding to each char `i` (with weight `freq[i]`).
- Repeat until single trie formed:
  - select two tries with min weight `freq[i]` and `freq[j]`
  - merge into single trie with weight `freq[i] + freq[j]`

Applications. JPEG, MP3, MPEG, PKZIP, GZIP, PDF, …

# Constructing a Huffman encoding trie

| char | freq | encoding |
|---|---|---|
| A | 5 | 0 |
| B | 2 | 1 1 1 |
| C | 1 | 1 0 1 1 |
| D | 1 | 1 0 0 |
| R | 2 | 1 1 0 |
| ! | 1 | 1 0 1 0 |



Huffman code construction for A B R A C A D A B R A !

# Constructing a Huffman encoding trie: Java implementation

```java
private static Node buildTrie(int[] freq)
{
    MinPQ<Node> pq = new MinPQ<Node>();
    for (char i = 0; i < R; i++)
       if (freq[i] > 0)
          pq.insert(new Node(i, freq[i], null, null));

    while (pq.size() > 1)
    {
       Node x = pq.delMin();
       Node y = pq.delMin();
       Node parent = new Node('\0', x.freq + y.freq, x, y);
       pq.insert(parent);
    }

    return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used

total frequency

two subtries

# Huffman encoding summary

**Proposition.** [Huffman 1950s] Huffman algorithm produces an optimal prefix-free code.

no prefix-free code uses fewer bits

**Pf.** See textbook.

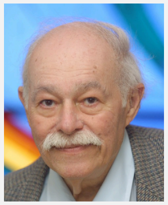
**Implementation.**

- Pass 1: tabulate char frequencies and build trie.
- Pass 2: encode file by traversing trie or lookup table.

**Running time.** Using a binary heap ⇒ $O(\underset{\substack{\uparrow \\ \text{input} \\ \text{size}}}{N} + R \log \underset{\substack{\uparrow \\ \text{alphabet} \\ \text{size}}}{R})$.

**Q.** Can we do better? [stay tuned]

- basics
- run-length coding
- Huffman compression
- **LZW compression**



Abraham Lempel

Jacob Ziv

## Statistical methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

# Lempel-Ziv-Welch compression example

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *input* | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
| *matches* | A | B | R | A | C | A | D | A B | | R A | | B R | | A B R | | | A |
| *value* | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | 82 | | 88 | | | 41 |

**LZW compression for ABRACADABRABRABRA**

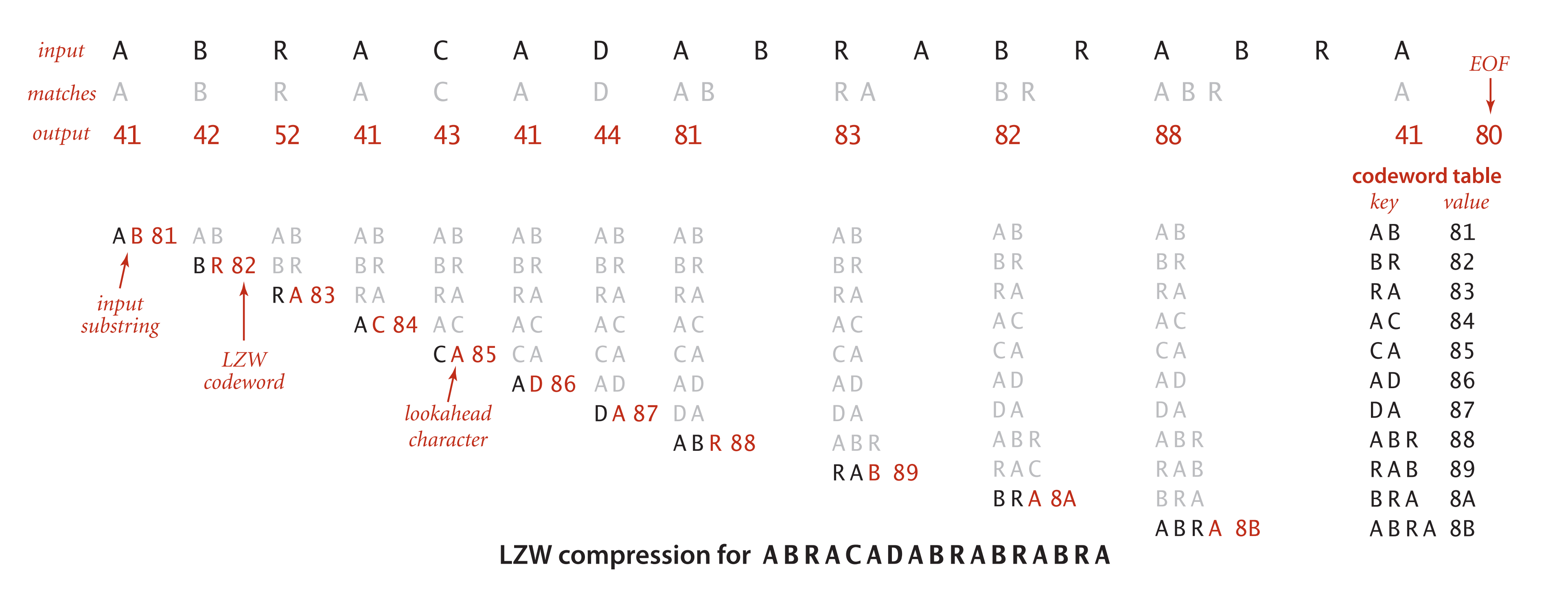
| key | value |
|---|---|
| ... | |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| ... | |
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |

**codeword table**

# Lempel-Ziv-Welch compression

## LZW compression.

- Create ST associating W-bit codewords with string keys.
- Initialize ST with codewords for single-char keys.
- Find longest string s in ST that is a prefix of unscanned part of input.
- Write the W-bit codeword associated with s.
- Add s + c to ST, where c is next char in the input.



| *input* | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A | *EOF* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *matches* | A | B | R | A | C | A | D | A B | | R A | | B R | | A B R | | | A | |
| *output* | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | 82 | | 88 | | | 41 | 80 |

**codeword table**

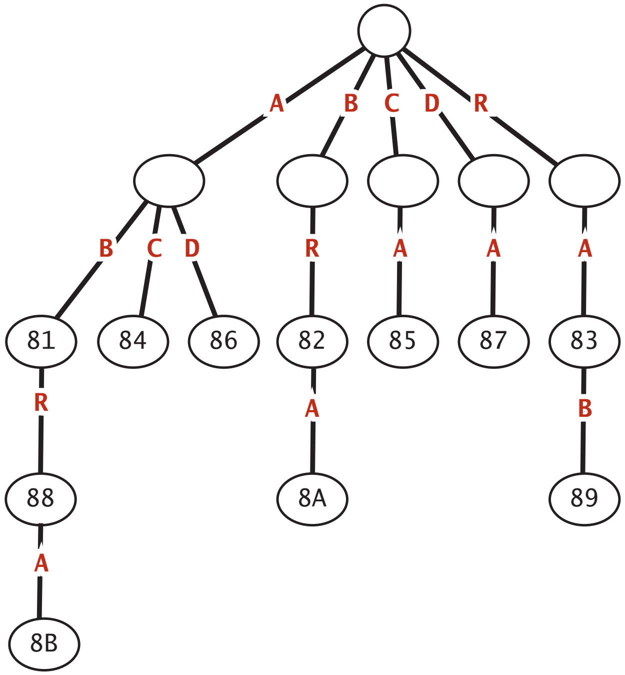
| *key* | *value* |
|---|---|
| A B | 81 |
| B R | 82 |
| R A | 83 |
| A C | 84 |
| C A | 85 |
| A D | 86 |
| D A | 87 |
| A B R | 88 |
| R A B | 89 |
| B R A | 8A |
| A B R A | 8B |

LZW compression for ABRACADABRABRABRA

# Representation of LZW code table

Q. How to represent LZW code table?

A. A trie: supports efficient longest prefix match.



Trie representation of LZW code table

Remark. Every prefix of a key in encoding table is also in encoding table.

# LZW compression: Java implementation

```
public static void compress()
{
  String input = BinaryStdIn.readString();

   TST<Integer> st = new TST<Integer>();
   for (int i = 0; i < R; i++)
       st.put("" + (char) i, i);
   int code = R+1;

   while (input.length() > 0)
   {
      String s = st.longestPrefixOf(input);
      BinaryStdOut.write(st.get(s), W);
      int t = s.length();
      if (t < input.length() && code < L)
          st.put(input.substring(0, t+1), code++);
      input = input.substring(t);
   }

   BinaryStdOut.write(R, W);
   BinaryStdOut.close();
}
```

read in input as a string

codewords for single-char, radix R keys

find longest prefix match s

write W-bit codeword for s

add new codeword

scan past s in input

write last codeword and close input stream

# Lempel-Ziv-Welch expansion example

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *value* | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
| *output* | A | B | R | A | C | A | D | A B | R A | B R | A B R | A | |

**LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80**

| value | key |
|---|---|
| ... | ... |
| 41 | A |
| 42 | B |
| 43 | C |
| 44 | D |
| ... | ... |

| value | key |
|---|---|
| 81 | AB |
| 82 | BR |
| 83 | RA |
| 84 | AC |
| 85 | CA |
| 86 | AD |

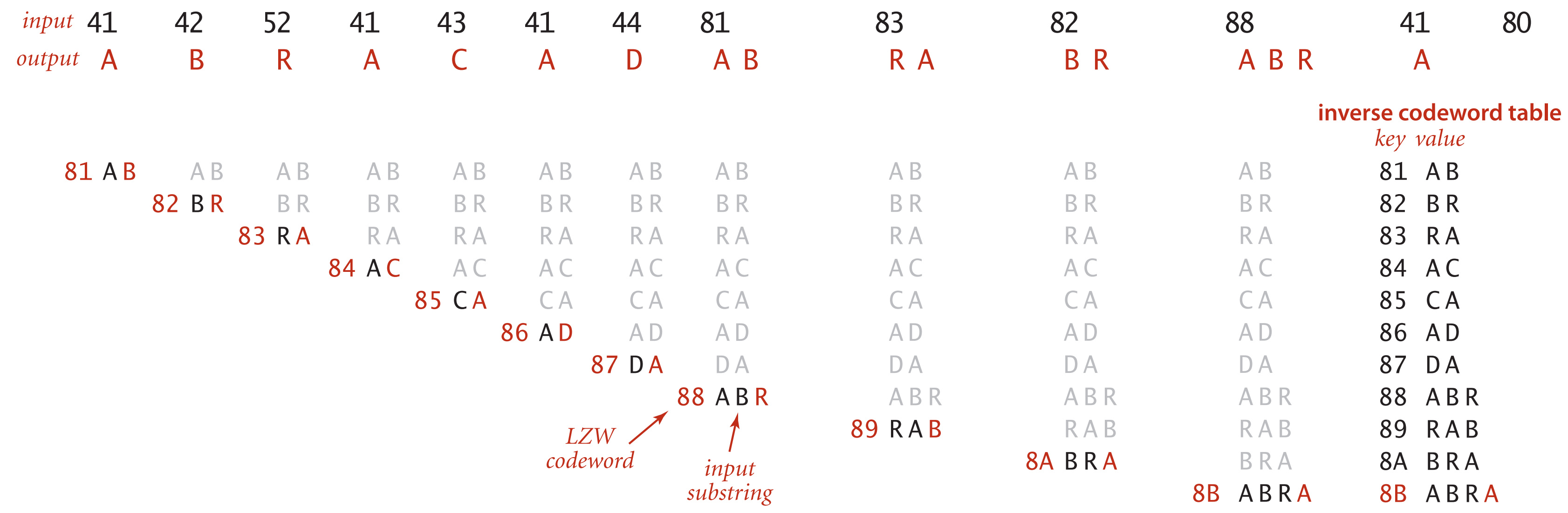
| value | key |
|---|---|
| 87 | DA |
| 88 | ABR |
| 89 | RAB |
| 8A | BRA |
| 8B | ABRA |
| | |

**codeword table**

# LZW expansion

## LZW expansion.

- Create ST associating string values with $W$-bit keys.
- Initialize ST to contain with single-char values.
- Read a $W$-bit key.
- Find associated string value in ST and write it out.
- Update ST.



| *input* | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *output* | A | B | R | A | C | A | D | A B | R A | B R | A B R | A | |
| | | | | | | | | | | | | **inverse codeword table**<br>*key value* | |
| | 81 A B | A B | A B | A B | A B | A B | A B | A B | A B | A B | A B | 81 A B | |
| | | 82 B R | B R | B R | B R | B R | B R | B R | B R | B R | B R | 82 B R | |
| | | | 83 R A | R A | R A | R A | R A | R A | R A | R A | R A | 83 R A | |
| | | | | 84 A C | A C | A C | A C | A C | A C | A C | A C | 84 A C | |
| | | | | | 85 C A | C A | C A | C A | C A | C A | C A | 85 C A | |
| | | | | | | 86 A D | A D | A D | A D | A D | A D | 86 A D | |
| | | | | | | | 87 D A | D A | D A | D A | D A | 87 D A | |
| | | | | | | | | 88 A B R | A B R | A B R | A B R | 88 A B R | |
| | | | | | | | | | 89 R A B | R A B | R A B | 89 R A B | |
| | | | | | | | | | | 8A B R A | B R A | 8A B R A | |
| | | | | | | | | | | | 8B A B R A | 8B A B R A | |

**LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80**

# LZW example: tricky situation

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *input* | A | B | A | B | A | B | A | |
| *matches* | A | B | A B | | A B A | | | |
| *value* | 41 | 42 | 81 | | 83 | | | 80 |

**LZW compression for ABABABA**

| key | value | key | value |
|---|---|---|---|
| … | | AB | 81 |
| A | 41 | BA | 82 |
| B | 42 | ABA | 83 |
| C | 43 | | |
| D | 44 | | |
| … | | | |

**codeword table**

# LZW example:  tricky situation

| | | | | | |
|---|---|---|---|---|---|
| *value* | 41 | 42 | 81 | 83 | 80 |
| *output* | A | B | A B | A B A | |

need to know which key has value 83 before it is in ST!

LZW expansion for 41 42 81 83 80

| value | key |
|---|---|
| … | … |
| 41 | A |
| 42 | B |
| 43 | C |
| 44 | D |
| … | … |

| value | key |
|---|---|
| 81 | AB |
| 82 | BA |
| 83 | ABA |
| | |
| | |
| | |

codeword table

# LZW implementation details

## How big to make ST?

- How long is message?
- Whole message similar model?
- [many variations have been developed]

## What to do when ST fills up?

- Throw away and start over. [GIF]
- Throw away when not effective. [Unix compress]
- [many other variations]

## Why not put longer substrings in ST?

- [many variations have been developed]

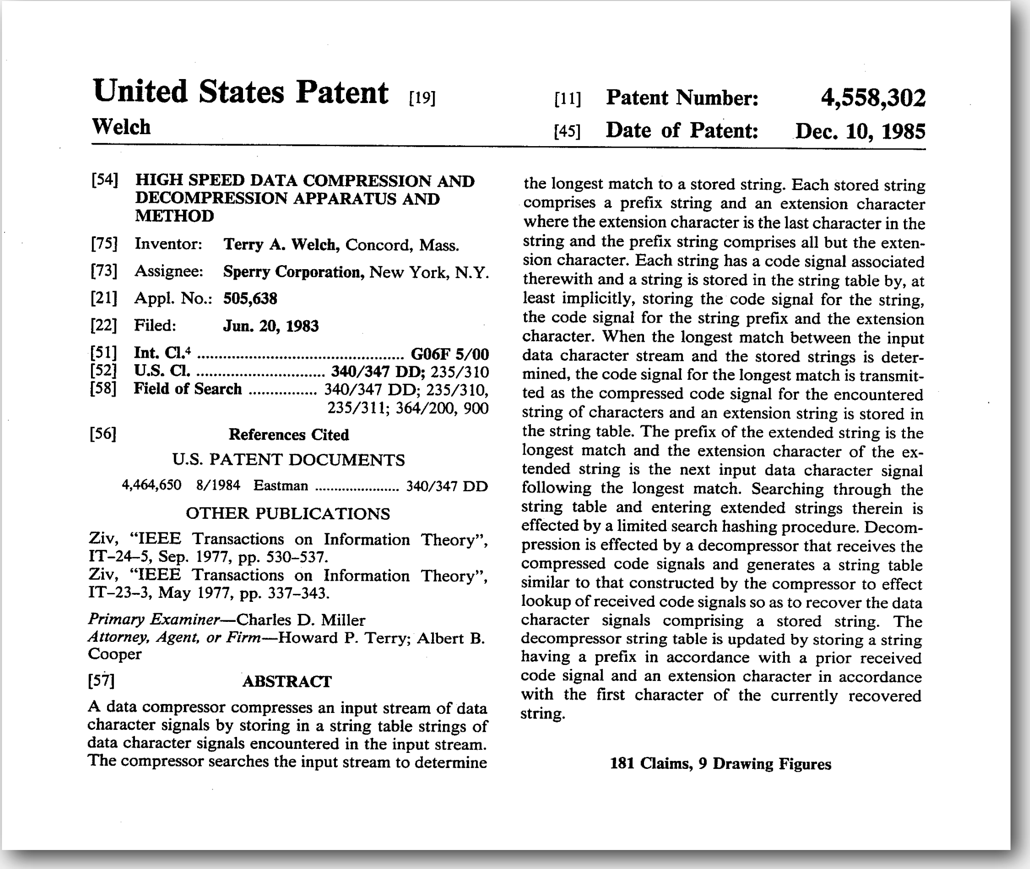
# LZW in the real world

## Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate = LZ77 variant + Huffman.

LZ77 not patented ⇒ widely used in open source

LZW patent #4,558,302 expired in US on June 20, 2003



**United States Patent** [19]
**Welch**

[11] **Patent Number:** **4,558,302**
[45] **Date of Patent:** **Dec. 10, 1985**

[54] **HIGH SPEED DATA COMPRESSION AND DECOMPRESSION APPARATUS AND METHOD**

[75] Inventor: **Terry A. Welch,** Concord, Mass.

[73] Assignee: **Sperry Corporation,** New York, N.Y.

[21] Appl. No.: **505,638**

[22] Filed: **Jun. 20, 1983**

[51] **Int. Cl.$^4$** ............................................ **G06F 5/00**
[52] **U.S. Cl.** ............................ **340/347 DD;** 235/310
[58] **Field of Search** ............... 340/347 DD; 235/310, 235/311; 364/200, 900

[56] **References Cited**

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Ziv, "IEEE Transactions on Information Theory", IT-23-3, May 1977, pp. 337-343.

*Primary Examiner*—Charles D. Miller
*Attorney, Agent, or Firm*—Howard P. Terry; Albert B. Cooper

[57] **ABSTRACT**

A data compressor compresses an input stream of data character signals by storing in a string table strings of data character signals encountered in the input stream. The compressor searches the input stream to determine the longest match to a stored string. Each stored string comprises a prefix string and an extension character where the extension character is the last character in the string and the prefix string comprises all but the extension character. Each string has a code signal associated therewith and a string is stored in the string table by, at least implicitly, storing the code signal for the string, the code signal for the string prefix and the extension character. When the longest match between the input data character stream and the stored strings is determined, the code signal for the longest match is transmitted as the compressed code signal for the encountered string of characters and an extension string is stored in the string table. The prefix of the extended string is the longest match and the extension character of the extended string is the next input data character signal following the longest match. Searching through the string table and entering extended strings therein is effected by a limited search hashing procedure. Decompression is effected by a decompressor that receives the compressed code signals and generates a string table similar to that constructed by the compressor to effect lookup of received code signals so as to recover the data character signals comprising a stored string. The decompressor string table is updated by storing a string having a prefix in accordance with a prior received code signal and an extension character in accordance with the first character of the currently recovered string.

**181 Claims, 9 Drawing Figures**

# LZW in the real world

Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate = LZ77 variant + Huffman.

PNG: LZ77.

7zip, gzip, jar, pdf, java.util.zip: deflate.

Unix compress: LZW.

Pkzip: LZW + Shannon-Fano.

GIF, TIFF, V.42bis modem: LZW.

Google: zlib which is based on deflate.

never expands a file

# Lossless data compression benchmarks

| year | scheme | bits / char |
|---|---|---|
| 1967 | ASCII | 7.00 |
| 1950 | Huffman | 4.70 |
| 1977 | LZ77 | 3.94 |
| 1984 | LZMW | 3.32 |
| 1987 | LZH | 3.30 |
| 1987 | move-to-front | 3.24 |
| 1987 | LZB | 3.18 |
| 1987 | gzip | 2.71 |
| 1988 | PPMC | 2.48 |
| 1994 | SAKDC | 2.47 |
| 1994 | PPM | 2.34 |
| 1995 | Burrows-Wheeler | 2.29 |
| 1997 | BOA | 1.99 |
| 1999 | RK | 1.89 |

← next programming assignment

**data compression using Calgary corpus**

# Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes. [Huffman]
- Represent variable-length symbols with fixed-length codes. [LZW]

Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3, …
- FFT, wavelets, fractals, …

Theoretical limits on compression. Shannon entropy.

Practical compression. Use extra knowledge whenever possible.