COS 226	Algorithms and Data Structures	Fall 2006

Midterm Solutions

1. **8 sorting algorithms.**

   4 6 3 8 2 1 7 5 9

2. **Algorithm Properties.**

   $\log N$ Binary heaps are perfectly balanced by definition.

   N An unbalanced BST can have height proportional to N.

   $N \log N$ The Sedgewick partitioning algorithm stops on equal keys. As a result, each partitioning step will create two subproblems of equal size, just like mergesort.

   N If all the keys are equal, 3-way quicksort will terminate after a single partioning step.

   N Traversing a tree using { inorder, preorder, postorder, level-order } takes linear time.

   N^2 If all keys hash to the same bin, the ith insertion will take time proportional to i.

3. **Sorting a linked list.**

   Mergesort is the algorithm of choice for linked lists (Sedgewick 8.7) since the merging can be done in-place. Quicksort is also a good choice since it's now easy to achieve stability.

Algorithm	Extra memory	Running time	Stability
Mergesort	$O(\log N)$	$O(N \log N)$	Y
Quicksort	$O(\log N)$	$O(N \log N)$	Y

Some variants of mergesort (bottom-up mergesort and natural mergesort) avoid recursion and only require $O(1)$ extra space.

4. **Comparable interface.**

Because of the epsilon fudge-factor, it's possible to have both `(a.compareTo(b) == 0)` and `(b.compareTo(c) == 0)`, but not `(a.compareTo(c) == 0)`. This breaks the contract.

5. **Java API.**

It's impossible because it would violate the $\Omega(N \log N)$ lower bound for sorting. The argument is almost identical to the one presented in class for why not all priority queue operations can take $O(1)$ time.

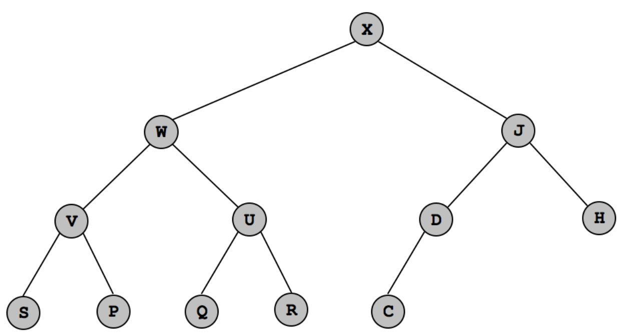
Here's a sorting algorithm that uses the `OrderStatistic` API.

- Add the N elements to the data structure.
- For each k from 1 to N, print the kth largest.

If all operations take $O(1)$ time, this is an $O(N)$ sorting algorithm. Since `OrderStatistic` only accesses the `Comparable` items through the `compareTo()` method, this contradicts the sorting lower bound.

6. **Binary heaps.**

(a)



(b) Inserting M causes array entries 13, 6, and 3 to change.

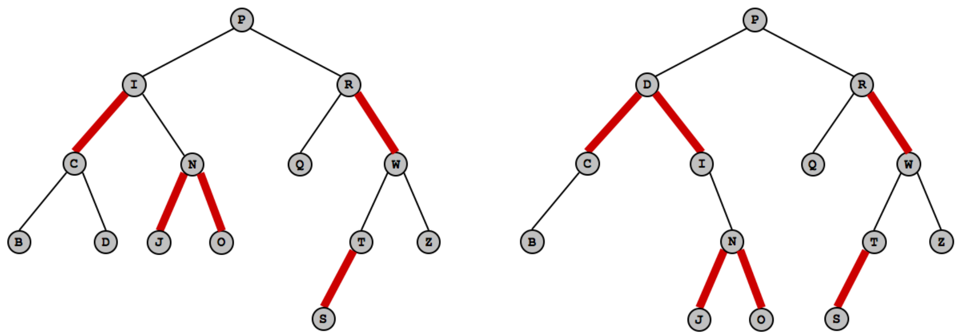
0	1	2	3	4	5	6	7	8	9	10	11	12	13
-	X	W	M	V	U	J	H	S	P	Q	R	C	D

(c) Inserting M causes array entries 12, 1, 2, 4, and 8 to change.

0	1	2	3	4	5	6	7	8	9	10	11	12	13
-	W	V	J	S	U	D	H	C	P	Q	R	-	-

7. **Red-black trees.**

Unfortunately, the figure shown is *not* a red-black tree! The 3-node containing I and C has only two children (instead of 3). As a result, we accepted either of the following two solutions (and awarded extra credit for correctly observing that the figure is not a red-black tree).



8. **Two-sum.**

There are two main approaches. (Note that we excluded 0 and -2^{63} since these are the only two `long` integers x such that $x + -x = 0$.)

- *Hashing.* Insert each integer x into a hash table (linear probing or separate chaining). When inserting x, check if $-x$ is already in the hash table. If so, you've found two integers that sum to 0.

  The running time is $O(N)$ on average, under the assumption that the hash function maps the keys uniformly. The running time is quadratic in the worst case, if all the keys hash to the same bin.

- *Sorting.* Sort the integers in ascending order into an array `a[]`. Maintain a pointer $i = 0$ to the most negative integer and a pointer $j = N - 1$ to the most positive integer. If `(a[i] + a[j] == 0)`, you have two integers that sum to 0. Otherwise, if the sum is negative, increment i; if the sum is positive, decrement j. Stop when $i = j$.

  The bottleneck operation is sorting. This takes $O(N)$ time in the average-case and worst-case using a radix sorting algorithm.