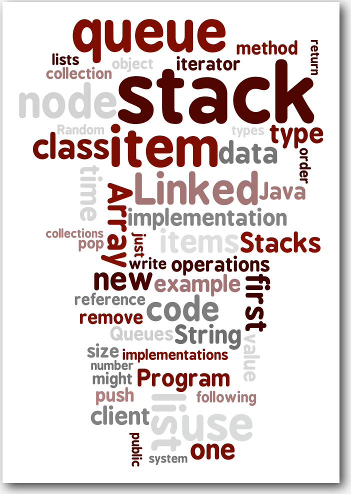
2.4 Stacks and Queues



- stacks
- dynamic resizing
- queues
- generics
- iterators
- applications

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Stacks and queues

Fundamental data types.
- Values: sets of objects
- Operations: insert, remove, test if empty.
- Intent is clear when we insert.
- Which item do we remove?

LIFO = "last in first out"

Stack. Remove the item most recently added.
Analogy. Cafeteria trays, Web surfing.

FIFO = "first in first out"

Queue. Remove the item least recently added.
Analogy. Registrar's line.

Client, implementation, interface

Separate interface and implementation so as to:
- Build layers of abstraction.
- Reuse software.
- Ex: stack, queue, symbol table, union-find, ...

Client: program using operations defined in interface.
Implementation: actual code implementing operations.
Interface: description of data type, basic operations.

Client, Implementation, Interface

Benefits.
- Client can't know details of implementation ⇒ client has many implementation from which to choose.
- Implementation can't know details of client needs ⇒ many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

Client: program using operations defined in interface.
Implementation: actual code implementing operations.
Interface: description of data type, basic operations.

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Stacks

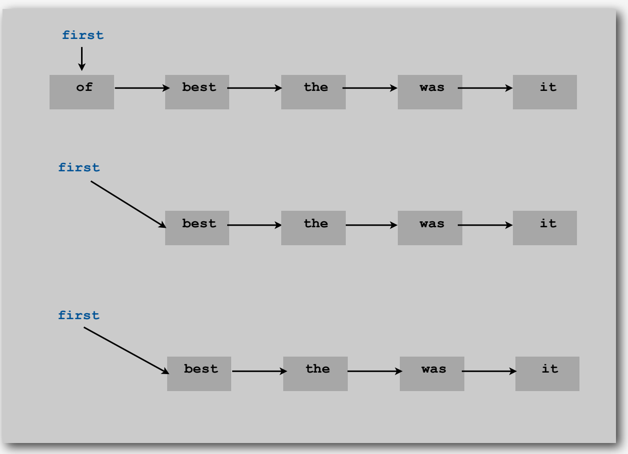
Stack operations.

- `push()` Insert a new item onto stack.
- `pop()` Remove and return the item most recently added.
- `isEmpty()` Is the stack empty?


```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-")) StdOut.print(stack.pop());
      else                  stack.push(item);
   }
}
```

```
% more tobe.txt
to be or not to - be - - that - - - is

% java StackOfStrings < tobe.txt
to be not that or be
```


Stack pop: linked-list implementation




```
String item = first.item;
```

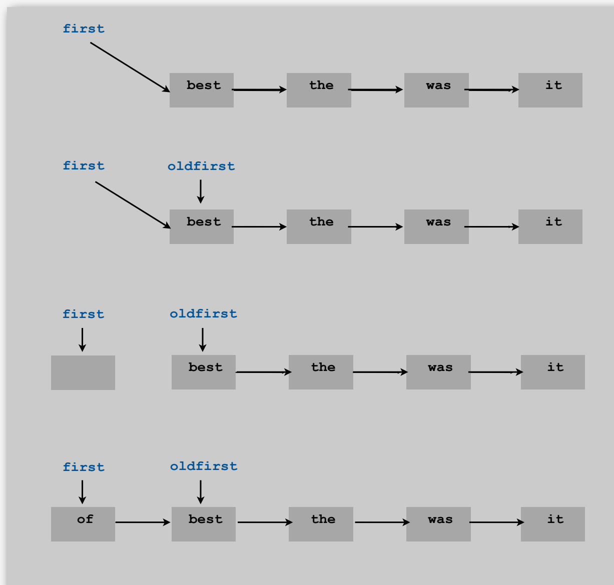
"of"

```
first = first.next;
```

```
return item;
```

"of"

Stack push: linked-list implementation




```
Node oldfirst = first;
```

```
first = new Node();
```

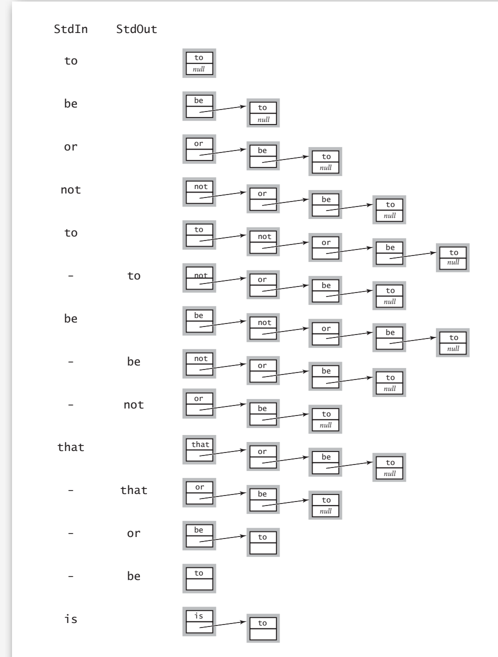
```
first.item = "of";
first.next = oldfirst;
```

Stack: linked-list implementation

```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      if (isEmpty()) throw new RuntimeException();
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

"inner class"

Stack: linked-list trace



Stack: array implementation

Array implementation of a stack.

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.

s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9
							N			capacity = 10

Stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

a cheat (stay tuned)

decrement N; then use to index into array

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   return item;
}
```

this version avoids "loitering"

garbage collector only reclaims memory if no outstanding references

- stacks
- **dynamic resizing**
- queues
- generics
- iterators
- applications

Problem. Requiring client to provide capacity does not implement API!
Q. How to grow and shrink array?

First try.

- `push()`: increase size of `s[]` by 1.
- `pop()`: decrease size of `s[]` by 1.

Too expensive.

- Need to copy all item to a new array.
- Inserting first N items takes time proportional to $1 + 2 + \ldots + N \sim N^2/2$.

infeasible for large N

Goal. Ensure that array resizing happens infrequently.

Q. How to grow array?
A. If array is full, create a new array of twice the size, and copy items.

"repeated doubling"

```
public StackOfStrings() {  s = new String[2];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] dup = new String[capacity];
   for (int i = 0; i < N; i++)
      dup[i] = s[i];
   s = dup;
}
```

$1 + 2 + 4 + \ldots + N/2 + N \sim 2N$

Consequence. Inserting first N items takes time proportional to N (not N^2).

Q. How to shrink array?

First try.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is half full.

Too expensive

- Consider push-pop-push-pop-... sequence when array is full.
- Time proportional to N per operation.

"thrashing"

N = 5	it	was	the	best	of	*null*	*null*	*null*
N = 4	it	was	the	best				
N = 5	it	was	the	best	of	*null*	*null*	*null*
N = 4	it	was	the	best				

Stack: dynamic array implementation

Q. How to shrink array?

Efficient solution.

- `push()`: double size of `s[]` when array is full.
- `pop()`: halve size of `s[]` when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length / 2);
   return item;
}
```

Invariant. Array is always between 25% and 100% full.

Stack: dynamic array implementation trace

StdIn	StdOut	N	a.length	a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]
		0	1	*null*							
to		1	1	to							
be		2	2	to	be						
or		3	4	to	be	or	*null*				
not		4	4	to	be	or	not				
to		5	8	to	be	or	not	to	*null*	*null*	*null*
-	to	4	8	to	be	or	not	*null*	*null*	*null*	*null*
be		5	8	to	be	or	not	be	*null*	*null*	*null*
-	be	4	8	to	be	or	not	*null*	*null*	*null*	*null*
-	not	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
that		4	8	to	be	or	that	*null*	*null*	*null*	*null*
-	that	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
-	or	2	4	to	be	*null*	*null*				
-	be	1	2	to	*null*						
is		2	2	to	is						

Amortized analysis

Amortized analysis. Average running time per operation over a worst-case sequence of operations.

Proposition. Starting from empty data structure, any sequence of M push and pop ops takes time proportional to M.

running time for doubling stack with N elements

	worst	best	amortized
construct	1	1	1
push	N	1	1
pop	N	1	1

doubling or shrinking

Remark. WQUPC used amortized bound.

Stack implementations: memory usage

Linked list implementation. ~ 16N bytes.

```
private class Node
{
   String item;
   Node next;
}
```

8 bytes overhead for object
4 bytes (String item)
4 bytes (Node next)
16 bytes per item

Doubling array. Between ~ 4N (100% full) and ~ 16N (25% full).

```
public class DoublingStackOfStrings
{
   private String[] s;
   private int N = 0;
   …
}
```

4 bytes × array size (private String[] s)
4 bytes (private int N)

Remark. Our analysis doesn't include the memory for the items themselves.

Stack implementations: dynamic array vs. linked List

Tradeoffs. Can implement with either array or linked list; client can use interchangeably. Which is better?

Linked list.

- Every operation takes constant time in worst-case.
- Uses extra time and space to deal with the links.

Array.

- Every operation takes constant amortized time.
- Less wasted space.

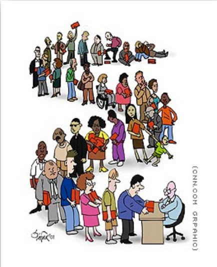
Queues

Queue operations.

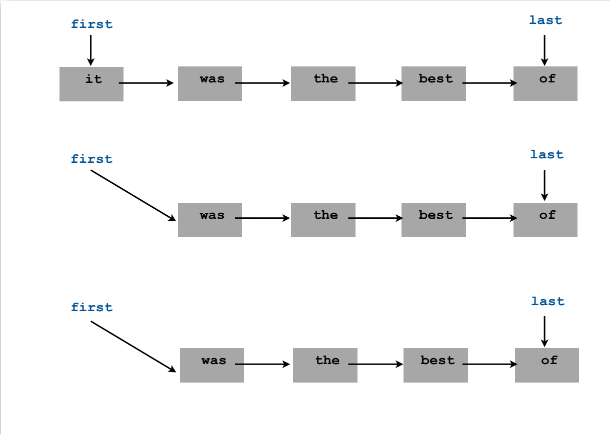
- **enqueue()** Insert a new item onto queue.
- **dequeue()** Delete and return the item least recently added.
- **isEmpty()** Is the queue empty?

```
public static void main(String[] args)
{
   QueueOfStrings q = new QueueOfStrings();
   while (!StdIn.isEmpty())
   {
      String item = StdIn.readString();
      if (item.equals("-")) StdOut.print(q.dequeue());
      else                  q.enqueue(item);
   }
}
```

```
% more tobe.txt
to be or not to - be - - that - - - is

% java QueueOfStrings < tobe.txt
to be or not to be
```



Queue dequeue: linked list implementation




```
String item = first.item;
```

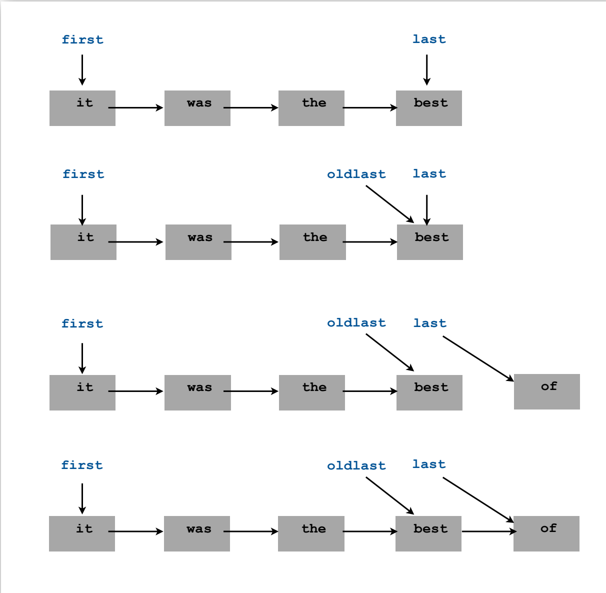
"it"

```
first = first.next;
```

```
return item;
```

"it"

Queue enqueue: linked list implementation




```
Node oldlast = last;
```

```
last = new Node();
last.item = "of";
last.next = null;
```

```
oldlast.next = last;
```

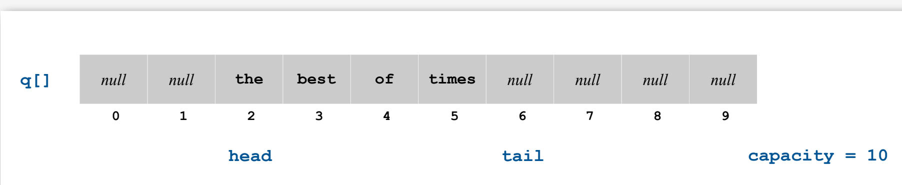

Queue: linked list implementation

```
public class QueueOfStrings
{
   private Node first, last;

   private class Node
   { String item; Node next; }

   public boolean isEmpty()
   { return first == null; }

   public void enqueue(String item)
   {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      if (isEmpty()) last = null;
      return item;
   }
}
```


Queue: dynamic array implementation

Array implementation of a queue.

- Use array `q[]` to store items in queue.
- `enqueue()`: add new item at `q[tail]`.
- `dequeue()`: remove item from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.
- Add repeated doubling and shrinking.



- stacks
- dynamic resizing
- queues
- **generics**
- iterators
- applications

Parameterized stack

We implemented: `StackOfStrings`.
We also want: `StackOfURLs`, `StackOfCustomers`, `StackOfInts`, etc?

Attempt 1. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

@#$*! most reasonable approach until Java 1.5.
[hence, used in Algorithms in Java, 3rd edition]

Parameterized stack

We implemented: `StackOfStrings`.
We also want: `StackOfURLs`, `StackOfCustomers`, `StackOfInts`, etc?

Attempt 2. Implement a stack with items of type `Object`.

- Casting is required in client.
- Casting is error-prone: run-time error if types mismatch.

```
StackOfObjects s = new StackOfObjects();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

Parameterized stack

We implemented: `StackOfStrings`.
We also want: `StackOfURLs`, `StackOfCustomers`, `StackOfInts`, etc?

Attempt 3. Java generics.

- Avoid casting in both client and implementation.
- Discover type mismatch errors at compile-time instead of run-time.

type parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = s.pop();
```

compile-time error

Guiding principles. Welcome compile-time errors; avoid run-time errors.

Generic stack: linked list implementation

```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

generic type name

Generic stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

```
public class Stack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

the way it should be

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

```
public class Stack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int capacity)
   {  s = (Item[]) new Object[capacity]; }

   public boolean isEmpty()
   { return N == 0; }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

the way it is

the ugly cast

Generic data types: autoboxing

Q. What to do about primitive types?

Wrapper type.

- Each primitive type has a wrapper object type.
- Ex: `Integer` is wrapper type for `int`.

Autoboxing. Automatic cast between a primitive type and its wrapper.

Syntactic sugar. Behind-the-scenes casting.

```
Stack<Integer> s = new Stack<Integer>();
s.push(17);        // s.push(new Integer(17));
int a = s.pop();   // int a = s.pop().intValue();
```

Bottom line. Client code can use generic stack for any type of data.

Autoboxing challenge

Q. What does the following program print?

```
public class Autoboxing {

   public static void cmp(Integer a, Integer b) {
      if      (a <  b) StdOut.printf("%d <  %d\n", a, b);
      else if (a == b) StdOut.printf("%d == %d\n", a, b);
      else             StdOut.printf("%d >  %d\n", a, b);
   }

   public static void main(String[] args) {
      cmp(new Integer(42), new Integer(42));
      cmp(43, 43);
      cmp(142, 142);
   }
}
```

```
% java Autoboxing
42 >  42
43 == 43
142 >  142
```

Best practice. Avoid using wrapper types whenever possible.

- stacks
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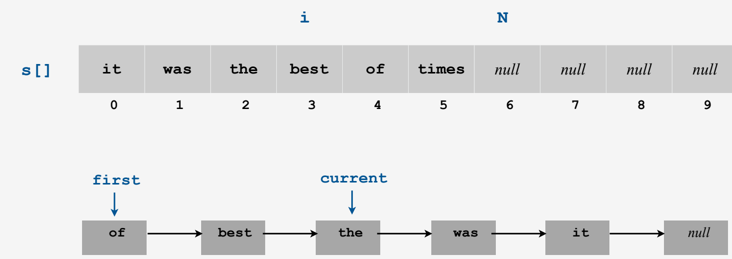
- **iterators**
- applications

Iteration

Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.



Java solution. Make stack implement the `Iterable` interface.

Iterators

Q. What is an `Iterable` ?

A. Has a method that returns an `Iterator`.

```
public interface Iterable<Item>
{
   Iterator<Item> iterator();
}
```

Q. What is an `Iterator` ?

A. Has methods `hasNext()` and `next()`.

```
public interface Iterator<Item>
{
   boolean hasNext();
   Item next();
   void remove();   ← optional; use at your own risk
}
```

Q. Why make data structures `Iterable` ?

A. Java supports elegant client code.

"foreach" statement

```
for (String s : stack)
   StdOut.println(s);
```

equivalent code

```
Iterator<String> i = stack.iterator();
while (i.hasNext())
{
   String s = i.next();
   StdOut.println(s);
}
```


Stack iterator: linked list implementation

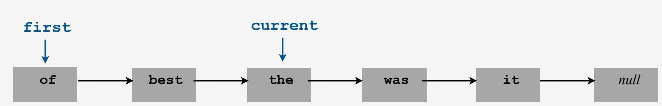
```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    ...

    public Iterator<Item> iterator() { return new ListIterator(); }

    private class ListIterator implements Iterator<Item>
    {
        private Node current = first;

        public boolean hasNext() {  return current != null;  }
        public void remove()     {  /* not supported */      }
        public Item next()
        {
            Item item = current.item;
            current   = current.next;
            return item;
        }
    }
}
```



Stack iterator: array implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
    …

    public Iterator<Item> iterator() { return new ArrayIterator(); }

    private class ArrayIterator implements Iterator<Item>
    {
        private int i = N;

        public boolean hasNext() {  return i > 0;          }
        public void remove()     {  /* not supported */  }
        public Item next()       {  return s[--i];        }
    }
}
```

				i			N			
s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

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Stack applications

Real world applications.

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

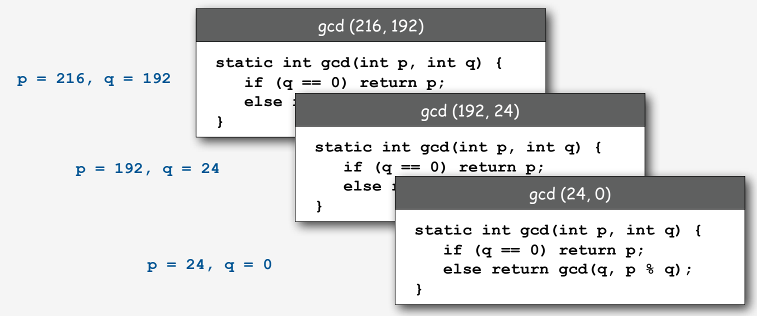
Function calls

How a compiler implements a function.

- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.

Note. Can always use an explicit stack to remove recursion.



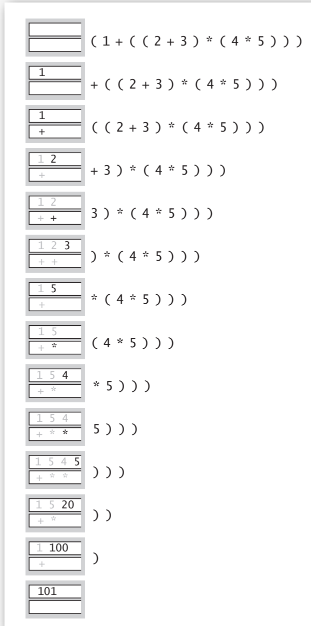
Arithmetic expression evaluation

Goal. Evaluate infix expressions.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

operand operator

value stack
operator stack



Two-stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Context. An interpreter!

Arithmetic expression evaluation

```
public class Evaluate
{
   public static void main(String[] args)
   {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")"))
         {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```


Correctness

Q. Why correct?

A. When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity.

Stack-based programming languages

Observation 1. The 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

Observation 2. All of the parentheses are redundant!

```
1 2 3 + 4 5 * * +
```

Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, …

PostScript

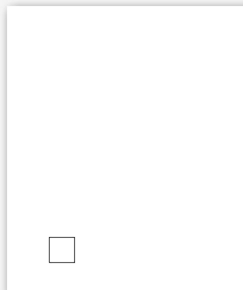
Page description language.
- Explicit stack.
- Full computational model
- Graphics engine.

Basics.
- `%!`: "I am a PostScript program."
- Literal: "push me on the stack."
- Function calls take arguments from stack.
- Turtle graphics built in.

a PostScript program

```
%!
72 72 moveto
0 72 rlineto
72 0 rlineto
0 -72 rlineto
-72 0 rlineto
2 setlinewidth
stroke
```

its output



PostScript

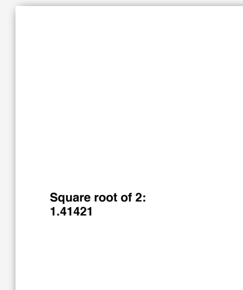
Data types.
- basic: integer, floating point, boolean, ...
- Graphics: font, path, curve, ...
- Full set of built-in operators.

Text and strings.
- Full font support.
- `show` (display a string, using current font).
- `cvs` (convert anything to a string).

System.out.print()

toString()

```
%!
/Helvetica-Bold findfont 16 scalefont setfont
72 168 moveto
(Square root of 2:) show
72 144 moveto
2 sqrt 10 string cvs show
```



PostScript

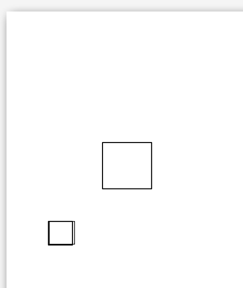
Variables (and functions).
- Identifiers start with /.
- def operator associates id with value.
- Braces.
- args on stack.

function definition

function calls

```
%!
/box
{
  /sz exch def
  0 sz rlineto
  sz 0 rlineto
  0 sz neg rlineto
  sz neg 0 rlineto
} def

72 144 moveto
72 box
288 288 moveto
144 box
2 setlinewidth
stroke
```



PostScript

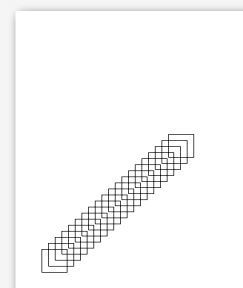
For loop.
- "from, increment, to" on stack.
- Loop body in braces.
- `for` operator.

If-else conditional.
- Boolean on stack.
- Alternatives in braces.
- `if` operator.

... (hundreds of operators)

```
%!
\box
{
  ...
}

1 1 20
{ 19 mul dup 2 add moveto 72 box }
for
stroke
```



PostScript

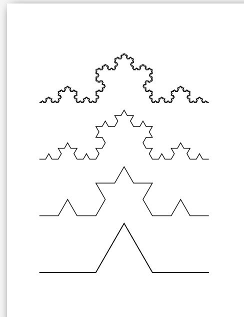
Application 1. All figures in Algorithms in Java

Application 2. Deluxe version of `StdDraw` also saves to PostScript for vector graphics.

```
%!
72 72 translate

/kochR
  {
    2 copy ge { dup 0 rlineto }
      {
        3 div
        2 copy kochR 60 rotate
        2 copy kochR -120 rotate
        2 copy kochR 60 rotate
        2 copy kochR
      } ifelse
    pop pop
  } def

0   0 moveto   81 243 kochR
0  81 moveto   27 243 kochR
0 162 moveto    9 243 kochR
0 243 moveto    1 243 kochR
stroke
```



See page 218

Queue applications

Familiar applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.

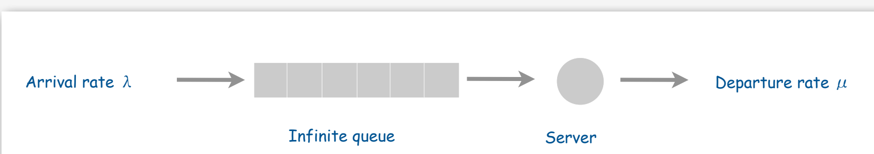
M/M/1 queuing model

M/M/1 queue.

- Customers arrive according to Poisson process at rate of λ per minute.
- Customers are serviced with rate of μ per minute.

interarrival time has exponential distribution $\Pr[X \le x] = 1 - e^{-\lambda x}$

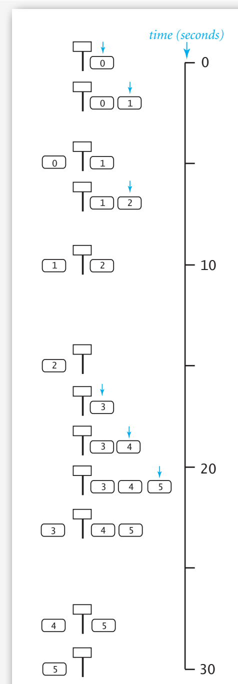
service time has exponential distribution $\Pr[X \le x] = 1 - e^{-\mu x}$



Q. What is average wait time W of a customer in system?

Q. What is average number of customers L in system?

M/M/1 queuing model: example simulation



	arrival	departure	wait
0	0	5	5
1	2	10	8
2	7	15	8
3	17	23	6
4	19	28	9
5	21	30	9

M/M/1 queuing model: event-based simulation

```
public class MM1Queue
{
    public static void main(String[] args) {
        double lambda = Double.parseDouble(args[0]);   // arrival rate
        double mu     = Double.parseDouble(args[1]);   // service rate
        double nextArrival = StdRandom.exp(lambda);
        double nextService = nextArrival + StdRandom.exp(mu);

        Queue<Double> queue = new Queue<Double>();
        Histogram hist = new Histogram("M/M/1 Queue", 60);

        while (true)
        {
            while (nextArrival < nextService)
            {
                queue.enqueue(nextArrival);
                nextArrival += StdRandom.exp(lambda);
            }

            double arrival = queue.dequeue();
            double wait = nextService - arrival;
            hist.addDataPoint(Math.min(60,  (int)  (Math.round(wait))));
            if (queue.isEmpty()) nextService = nextArrival + StdRandom.exp(mu);
            else                 nextService = nextService + StdRandom.exp(mu);
        }
    }
}
```

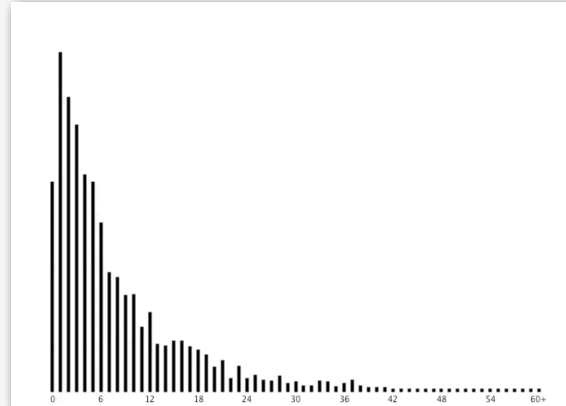
next event is an arrival

next event is a service completion

M/M/1 queuing model: experiments

Observation. If service rate μ is much larger than arrival rate λ, customers gets good service.

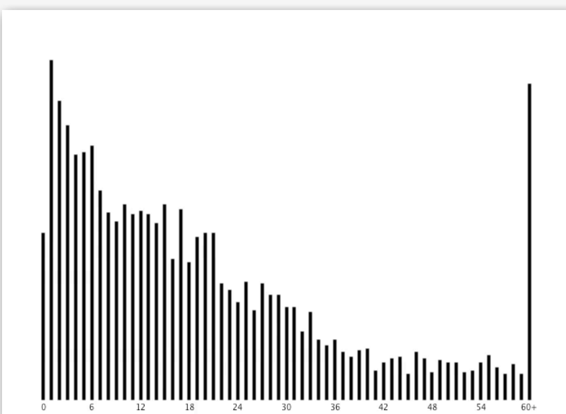
```
% java MM1Queue .2 .333
```



M/M/1 queuing model: experiments

Observation. As service rate μ approaches arrival rate λ, services goes to h***.

```
% java MM1Queue .2 .25
```



M/M/1 queuing model: experiments

Observation. As service rate μ approaches arrival rate λ, services goes to h***.

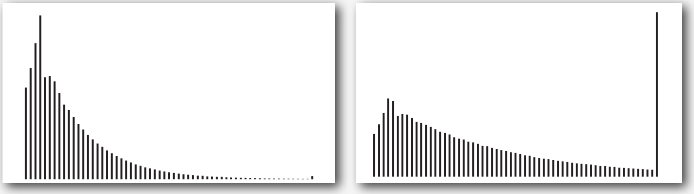
```
% java MM1Queue .2 .21
```

M/M/1 queue. Exact formulas known.

wait time W and queue length L approach infinity as service rate approaches arrival rate

Little's Law

$$W = \frac{1}{\mu - \lambda}, \quad L = \lambda W$$



More complicated queueing models. Event-based simulation essential!

Queueing theory. See ORF 309.