

Midterm Solutions

1. Analysis of algorithms.

(a) *P* Printing the keys in a binary search tree in ascending order.

U Finding a minimum spanning tree in a weighted graph.

P Finding all vertices reachable from a given source vertex in a graph.

P Checking whether a digraph has a directed cycle.

P Building the Knuth-Morris-Pratt DFA for a given string.

P Sorting an array of strings, accessing the data solely via calls to `charAt()`.

I Sorting an array of strings, accessing the data solely via calls to `compareTo()`.

I Finding the closest pair of points among a set of points in the plane, accessing the data solely via calls to `distanceTo()`.

(b) *A* Insert into a red-black tree. A. $\log N$ worst case

C Insert into a 2d-tree. B. $\log N$ amortized

B Insert into a binary heap. C. $\log N$ average case on random inputs

(c) • The N^3 one might be much easier to correctly implement, debug, and test.

• The N^3 algorithm might be faster for the values of N of interest (e.g., because of the leading constant).

• The N^3 algorithm might use less memory.

(d) 56 bytes.

Each `Point` object consumes 32 bytes (8 bytes for each of the three `double` instance variables; 8 bytes of object overhead).

Each `Node` object consumes 56 bytes (4 bytes for each of the 3 reference instance variables; 4 bytes for the `int` instance variable; 32 bytes for the `Point3D` object; 8 bytes of object overhead).

2. **Breadth-first search.**

- (a) A B C D E G F H I
- (b) d

3. **Minimum spanning tree.**

- (a) 1 2 3 5 6 7 8 12
- (b) $w \leq 8$
- (c) 6 1 3 2 5 7 8 12
- (d) Find the unique path between x and y in T . This takes $O(V)$ time using DFS because there are only $V - 1$ edges in T . We claim the edge T remains an MST if and only if w is greater than or equal to the weight of every edge on the path.
 - If any edge on the path has weight greater than w , we can decrease the weight of T by swapping the largest weight edge on the path with $x-y$. Thus, T does not remain an MST.
 - If w is greater than or equal to the weight of every edge on the path, then the cycle property asserts that $x-y$ is not in some MST (because it is the largest weight edge on the cycle consisting of the path from x to y plus the edge $x-y$). Thus, T remains an MST.

4. **Shortest paths.**

- (a) vertex: A C D F H E B G I
distance: 0 1 12 20 25 28 34 40 53

- (b) $A \rightarrow C, C \rightarrow D, C \rightarrow B, D \rightarrow F, F \rightarrow H, H \rightarrow E, E \rightarrow G, G \rightarrow I$

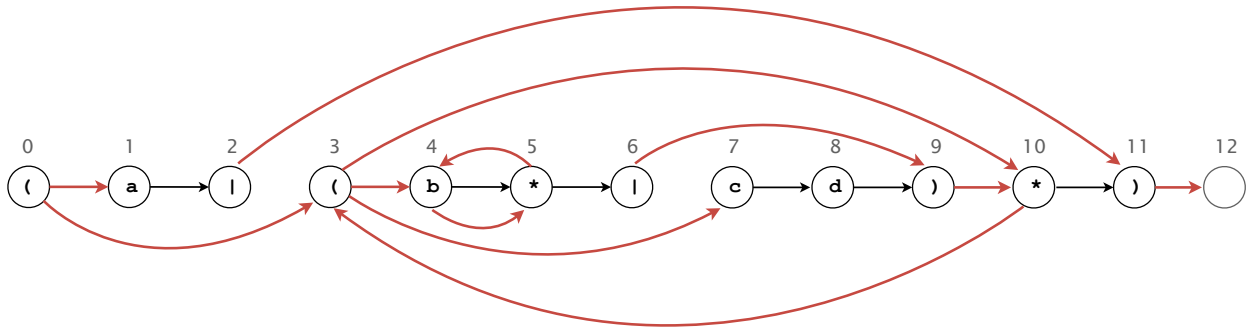
5. **Ternary search tries.**

- (a) ear fo his hitch hold holdup hotel hum humble ill
- (b)
- (c)
 - faster, especially for search miss
 - support character-based operations such as prefix match (autocomplete), longest prefix, and wildcard match

6. **Substring search.**

	0	1	2	3	4	5	6
a	1	2	2	4	5	6	2
b	0	0	0	0	0	0	7
c	0	0	3	0	0	3	3

7. Regular expressions.



8. Burrows-Wheeler transform.

(a) 5
b b a b a c a a

(b) b a b a b a a b a

9. Circular suffixes.

I only.

10. Tandem repeats.

(a) This problem is a generalization of substring search (is there at least one consecutive copy of b within s ?) so we need an algorithm that generalizes substring search. Create the Knuth-Morris-Pratt DFA for k copies of b , where $k = \lfloor N/M \rfloor$. Now, simulate DFA on input s and record the largest state that it reaches. From this, we can identify the longest repeat.

(b) $M + N$.

11. Reductions.

(a) $\{ -3M, x_1 + M, x_2 + M, \dots, x_N + M \}$

If we can force any solution to this 4SUM instance to choose $x_l = -3M$ as one of the integers, then the remaining three integers are $x_i + M$, $x_j + M$, and $x_k + M$ and we have $x_i + x_j + x_k = 0$.

We force any solution to this 4SUM instance to choose $-3M$ by choosing $M = 1 + \max\{|x_1|, |x_2|, \dots, |x_N|\}$ to be large, thereby making $-3M$ the only negative integer.

(b) None.