COS 226	Algorithms and Data Structures	Fall 2008

Final Solutions

1. **Depth-first search.**

   (a) `A B C F E G D H`

   (b) `E D G F H C B A`

   (c) false, true, true

2. **Minimum spanning tree.**

   (a) `1 2 3 4 5 11 12 13 17`

   (b) `4 3 2 1 5 13 12 17 11`

3. **Convex hull.**

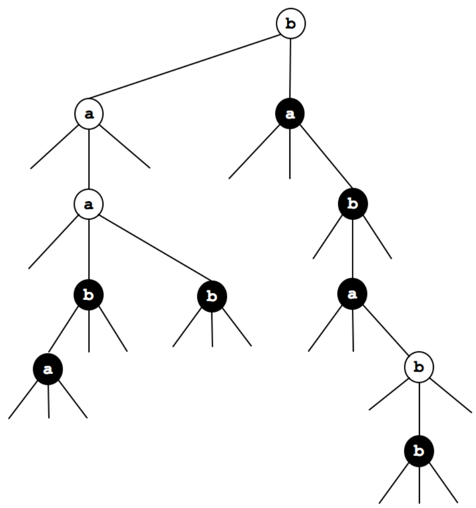
   (a) `B C D G H F J I E`

   (b)
```
1.  A -> B -> C
2.  A -> B -> C -> D
3.  A -> B -> G
4.  A -> B -> H
5.  A -> B -> H -> F
6.  A -> B -> H -> F -> J
7.  A -> B -> H -> F -> J -> I
8.  A -> B -> H -> E
```

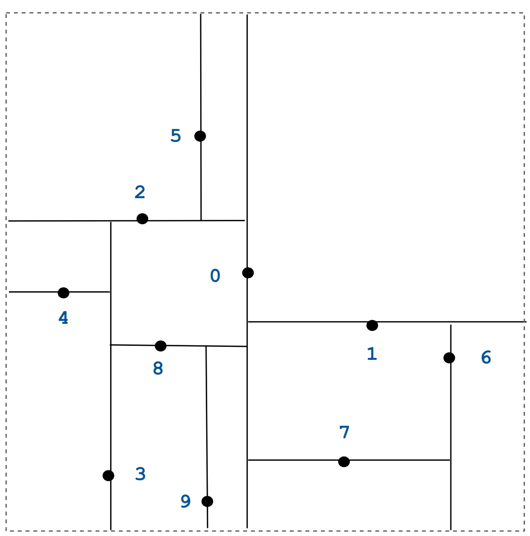
4. **TST.**

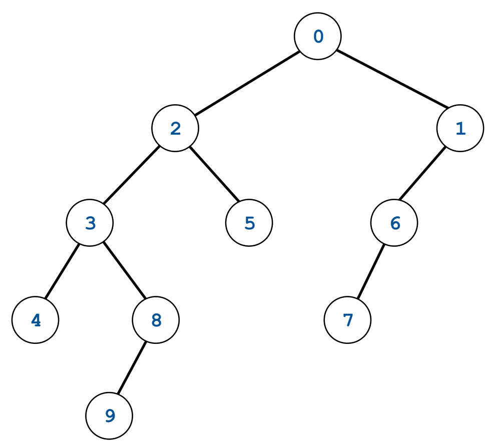
(a) `aaa` `aab` `ab` `ba` `bb` `bba` `bbbb`

(b)



5. **2D tree. (8 points)**





6. **Radix sorting.**

   Put an `X` in each box if the string sorting algorithm (the standard version considered in class) has the corresponding property.

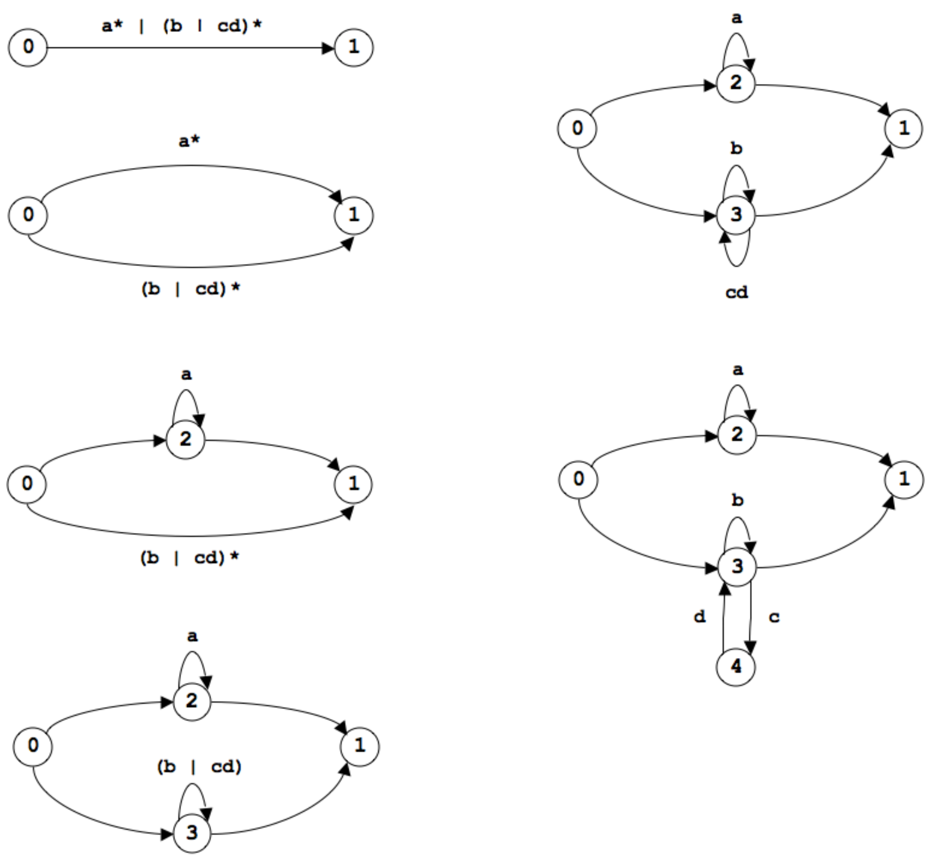
	mergesort	LSD radix sort	MSD radix sort	3-way radix quicksort
stable	X	X	X	
in-place				X
sublinear time (in best case)	X		X	X
fixed-length strings only		X		

   Note that in the best case, mergesort takes $N \log N$ string compares, but a string compare can takes constant time (if the two strings differ after a constant number of characters). Thus, the overall running time can be sublinear in the input size (total number of characters in the N strings).

7. **Data compression.**

   `a b a b b a b a b b a b b a b b`

8. **Regular expressions.**



9. **1D nearest neighbor.**

- *constructor:* create an empty red-black tree.
- *insert(x):* insert x into the red-black tree.
- *query(y):* if the data structure is empty return null; otherwise compute the floor and ceiling of y and return the (non-null) value closest to x.

10. **Prefix-free codes.**

    (a) Insert all of the codewords into a binary trie, marking the terminating nodes. The set of string is not prefix-free if when inserting a codeword (i) you pass through a marked node (an existing codeword is a prefix of the codeword you are inserting) or (ii) the node you mark is not a leaf node (the codeword you're inserting is a prefix of an existing codeword).

    (b) W (we go down one level in the tree for each bit we examine).

    (c) W (at most one trie node for each bit in the input).

    Alternate solution 1: use a TST instead of a binary trie. Since the radix size is 2, the running time will still be linear: you at most double the length of a search path for a codeword.

    Alternate solution 2: sort all of the codewords (MSD radix sort or 3-way radix quicksort) and check adjacent codewords to see if one is a prefix of the other.

11. **Shortest directed cycle.**

    (a) The critical observation is that the shortest directed cycle is a shortest path (number of edges) from s to v, plus a single edge $v \to s$.

    ```
    For each vertex s:
       * Use BFS to compute shortest path from s to each other vertex.
       * For each edge v->s entering s, consider cycle formed by
         shortest path from s to v (if the path exists) plus the edge v->s.
    Return shortest overall cycle.
    ```

    (b) The running time is $O(EV)$.
    The single-source shortest path computation from s takes $O(E+V)$ time per using BFS. Finding all edges entering s takes $O(E+V)$ time by scanning all edges (though a better way is to compute the reverse graph at once and access the adjacency lists). We must do this for each vertex s. Thus, the overall running time is $O(EV)$.

    (c) The memory usage is $O(E+V)$.
    BFS uses $O(V)$ extra memory and we only need to run one at a time. (A less efficient solution is to compute a V-by-V table containing the shortest path from v to w for every v and w. This uses $O(V^2)$ memory.)

12. **Reductions.**

    (a) $0, x_1, x_2, \ldots, x_N$.
    This is the easy direction—we just choose $b = 0$.

    (b) $3x_1 - b, 3x_2 - b, \ldots, 3x_N - b$.
    Observe that $(3x_i - b) + (3x_j - b) + (3x_k - b) = 0$ if and only if $x_i + x_j + x_k = b$. Note that we use $3x_i - b$ as the input instead of $x_i - b/3$ because the input must be integral.

    (c) I, II, and III.