


# Shape Analysis


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Princeton University  
CS526, Fall 2006




## Introduction

Images courtesy of Cyberware, ATI, & 3Dlife


3D data is becoming widely available



Cheap Scanners



Fast Graphics Cards



World Wide Web


Someday 3D models will be as common as images are today

## Motivation


Images courtesy of Stanford & Utah

When 3D data is ubiquitous, there will be a shift in research focus


Previous research has asked:  
"How do we acquire 3D data?"



Utah VW Bug



Utah Teapot




Stanford Bunny

Future research will ask:  
"How do we find 3D data?"

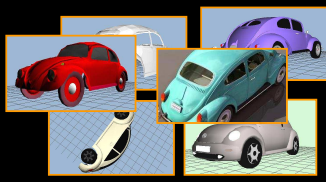
## Introduction

Images courtesy of De Espona & Utah

3D data acquired via the Web will often be void of structural and semantic information



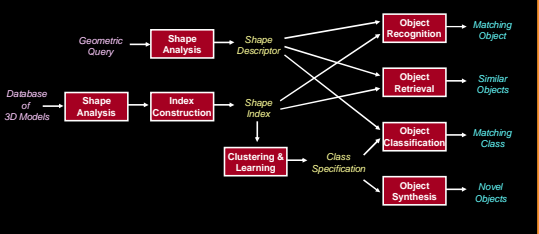
Utah VW Bug



Analysis algorithms also are needed to create "useful" 3D models from "raw" 3D data

## Introduction

Research in retrieval & analysis 3D data will follow the trends of other media types




```

    graph TD
      GQ[Geometric Query] --> SA1[Shape Analysis]
      SA1 --> SD[Shape Descriptor]
      SA1 --> OR[Object Recognition]
      SA1 --> OR2[Object Retrieval]
      SA1 --> OC[Object Classification]
      SA1 --> OS[Object Synthesis]
      DB[Database of 3D Models] --> SA2[Shape Analysis]
      SA2 --> IC[Index Construction]
      IC --> SI[Shape Index]
      SI --> OR
      SI --> OR2
      SI --> OC
      SI --> OS
      CL[Clustering & Learning] --> CS[Class Specification]
      CS --> OC
      CS --> OS
  
```


## Introduction

Images courtesy of Georgia Tech and www.dreamhorse.com

Which is harder to analyze?



3D Model



2D Image

## Lecture Outline

- Introduction
- Problems** ←
- Applications
- Shape Matching

## Shape Analysis Problems


Examples:

- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Recognition
- Classification
- Clustering
- Retrieval

## Shape Analysis Problems

Examples:

- Ø **Feature detection**
- Segmentation
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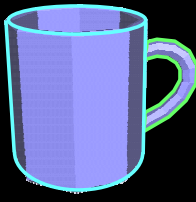


"How can we find significant geometric features robustly?"

## Shape Analysis Problems

Examples:

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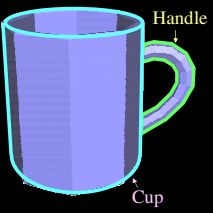


"How can we decompose a 3D model into its parts?"

## Shape Analysis Problems

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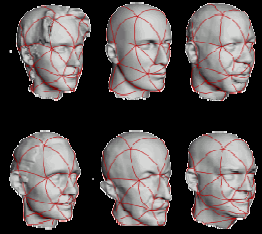


"How can we decompose a 3D model into its parts?"

## Shape Analysis Problems

Examples:

- Feature detection
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- Ø **Registration**
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"How can we align features of 3D models?"

## Shape Analysis Problems

Image courtesy of Ilya Vakser, GRAM

Examples:

- Feature detection
- Segmentation
- Labeling
- Registration
- Ø **Matching**
- Retrieval
- Recognition
- Classification
- Clustering

"How can we compute a measure of geometric similarity?"

## Shape Analysis Problems

Image courtesy of Darpa E3D Project

Examples:

- Feature detection
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- Ø **Retrieval**
- Recognition
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"How can we find 3D models best matching a query?"

## Shape Analysis Problems

Image courtesy of Florida State Univ.

Examples:

- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Retrieval
- Ø **Recognition**
- Classification
- Clustering

"How can we find a given 3D model in a large database?"

## Shape Analysis Problems

Image courtesy of Darpa E3D Project

Examples:

- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Retrieval
- Recognition
- Ø **Classification**
- Clustering

"How can we determine the class of a 3D model?"

## Shape Analysis Problems

Image courtesy of Viewpoint

Examples:

- Feature detection
- Segmentation
- Labeling
- Registration
- Matching
- Retrieval
- Recognition
- Classification
- Ø **Clustering**

"How can we learn classes of 3D models automatically?"

## Lecture Outline

- Introduction
- Problems
- Applications** ←
- Shape Matching

## Shape Analysis Applications

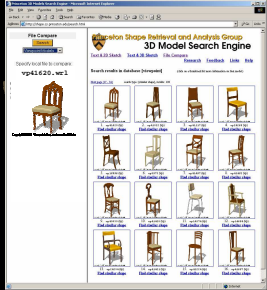
Examples:

- Virtual worlds
- Animation
- Mechanical CAD
- Chemistry
- Military
- Paleontology
- Molecular bio
- Medicine
- Forensics
- Art

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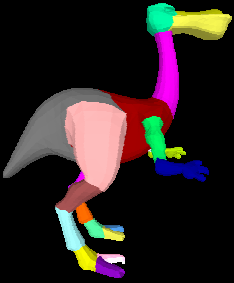


## Shape Analysis Applications

Image courtes Ayellet Tal, Technion & Princeton University

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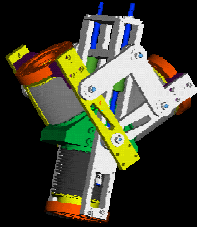


## Shape Analysis Applications

Images courtes Bill Rehg, Drexel University

Examples:

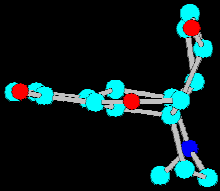
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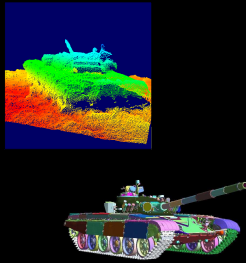
Morphine

## Shape Analysis Applications

Images courtes Darpa E3D Project

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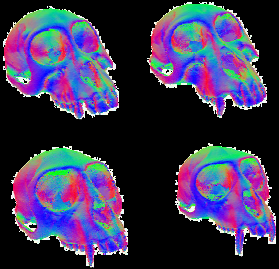


## Shape Analysis Applications

Images courtesy of Delson & Frahm

Examples:

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- **Paleontology**
- Molecular bio
- Medicine
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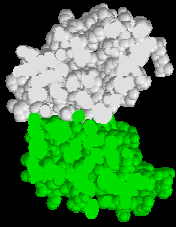


## Shape Analysis Applications

Image courtesy of Ilya Vakser, GRAMM

Examples:

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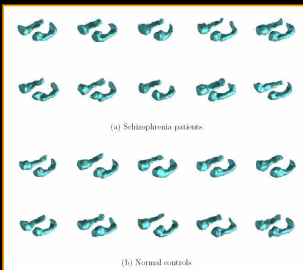


## Shape Analysis Applications

Image courtesy of Polina Golland, MIT

Examples:

- Virtual worlds
- Animation
- Mechanical CAD
- Chemistry
- Military
- Paleontology
- Molecular bio
- **Medicine**
- Forensics
- Art




Hippocampus-amygdala study in schizophrenia

## Shape Analysis Applications

Image courtesy of Boeing

Examples:

- Virtual worlds
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## Shape Analysis Applications

Image courtesy of Stanford University

Examples:

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- **Art**



## Lecture Outline

- Introduction
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- Shape Matching** ←

## Shape

Definition from Merriam-Webster's Dictionary:

- a** : the visible makeup characteristic of a particular item or kind of item
- b** : spatial form or contour

## Shape Matching

Need a shape distance function  $d(A,B)$  that:

- Matches our intuitive notion of shape similarity
- Can be evaluated by a computer

Perhaps, shape distance function should be a metric:

- Non-negative:  $d(A,B) \geq 0$  for all A and B
- Identity:  $d(A,B) = 0$  if and only if  $A=B$
- Symmetry:  $d(A,B) = d(B,A)$  for all A and B
- Triangle inequality:  $d(A,B) + d(B,C) \geq d(A,C)$

## Shape Matching Challenges

Need shape descriptor & matching method that is:

- Concise to store
- Quick to compute
- Efficient to match
- Discriminating

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3D Query → Shape Descriptor → 3D Database → Best Matches

### Shape Matching Challenges

Need shape descriptor & matching method that is:

- Concise to store
- Quick to compute
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- Discriminating
- Ø Invariant to transformations
  - Invariant to deformations
  - Insensitive to noise
  - Insensitive to topology
  - Robust to degeneracies

Different Transformations  
(translation, scale, rotation, mirror)

### Shape Matching Challenges

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Different Articulated Poses

### Shape Matching Challenges

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Scanned Surface

### Shape Matching Challenges

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Different Genus  
Different Tessellations

### Shape Matching Challenges

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No Bottom!  
&\*Q?@#A%!


## Taxonomy of 3D Matching Methods

Structural representations

- Skeletons
- Part-based methods
- Feature-based methods

Statistical representations

- Attribute feature vectors
- Volumetric methods
- Surface-based methods
- View-based methods



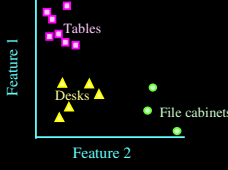
## Taxonomy of 3D Matching Methods

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## Example

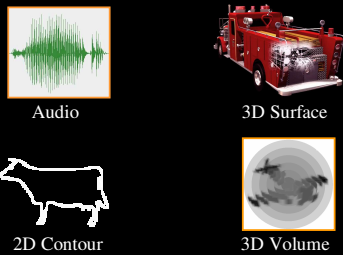
Shape distributions

- Shape representation: probability distributions
- Distance measure: difference between distributions
- Evaluation method: classification performance

We are starting with discussion of a simple method to introduce the basic ideas

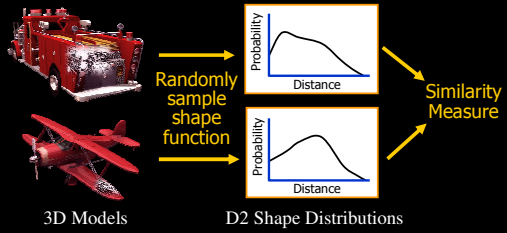
## Shape Distributions

Motivation: general approach to finding a common parameterization for matching



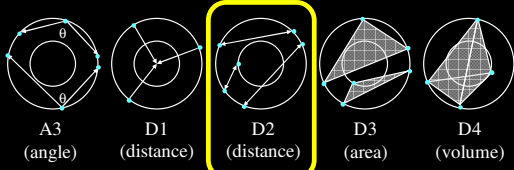
## Shape Distributions

Key idea: map 3D surfaces to common parameterization by randomly sampling shape function



## Which Shape Function?

Implementation: simple shape functions based on angles, distances, areas, and volumes



[Ankerst 99]



## D2 Shape Distribution

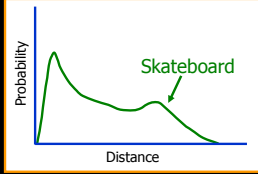
Properties

- Concise to store?
- Quick to compute?
- Invariant to transforms?
- Efficient to match?
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## D2 Shape Distribution

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512 bytes (64 values)  
0.5 seconds (10<sup>6</sup> samples)

## D2 Shape Distribution

Properties

- Concise to store
- Quick to compute
- Invariant to transforms?
  - Translation
  - Rotation
  - Mirror
  - Scale (w/ normalization)
- Efficient to match?
- Insensitive to noise?
- Insensitive to topology?
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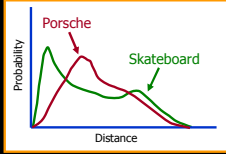


Normalized Means

## D2 Shape Distribution

Properties

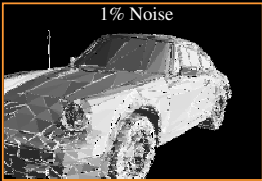
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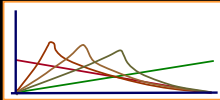


1% Noise

## D2 Shape Distribution

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Ellipsoids with Different Eccentricities

### D2 Shape Distribution

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- ✗ Invariant to deformations
- ø Discriminating?

### D2 Shape Distribution Results

Question

- How discriminating are D2 shape distributions?

Test database

- 133 polygonal models
- 25 classes

### D2 Shape Distribution Results

D2 distributions are different across classes

D2 shape distributions for 15 classes of objects

### D2 Shape Distribution Results

D2 distributions reveal gross shape of object

D2 shape distributions for 15 classes of objects

### D2 Shape Distribution Results

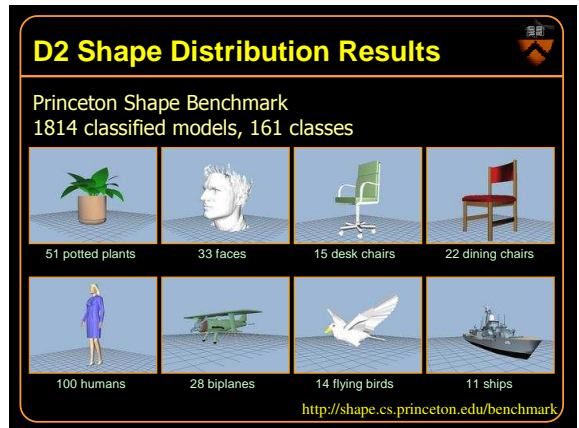
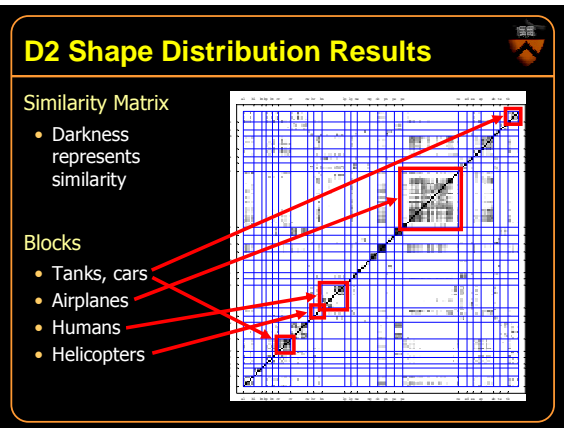
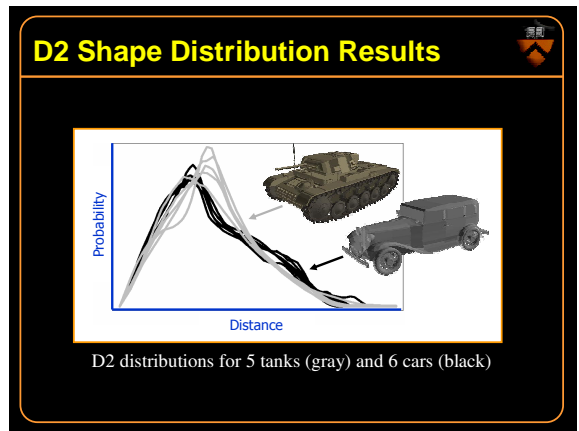
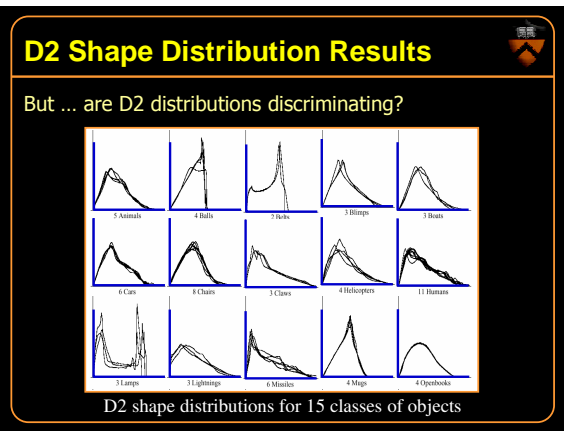
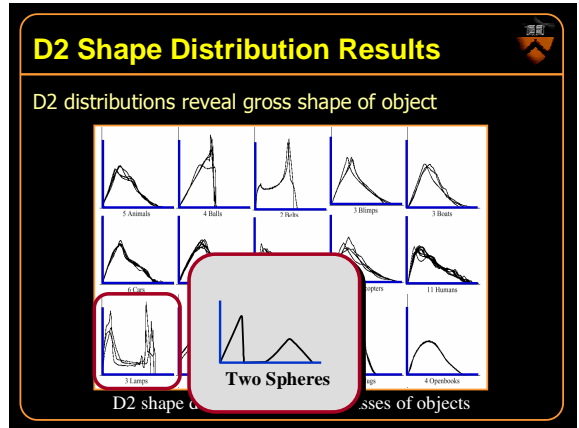
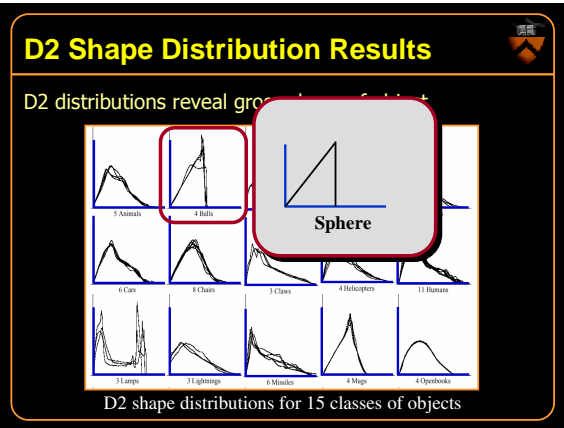
D2 distributions reveal gross shape of object

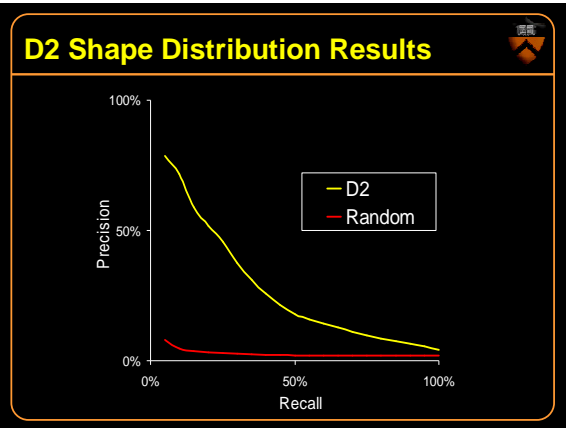
D2 shape distributions for 15 classes of objects

### D2 Shape Distribution Results

D2 distributions reveal gross shape of object

D2 shape distributions for 15 classes of objects





### D2 Shape Distribution Results

Shape Descriptor	Storage (bytes)	Compute Time (s)	Compare Time (us)	Nearest Neighbor	DCGain	Norm DCGain
LFD	3.25	4.700	1.300	66%	64%	21%
REXT	2.22	17.416	229	60%	60%	13%
SHD	1.69	2.184	27	56%	58%	10%
GEDT	1.69	32.776	450	60%	58%	10%
EXT	1.17	552	8	55%	56%	6%
SECSHEL	1.38	32.776	451	55%	55%	3%
VOXEL	1.34	32.776	450	54%	54%	2%
SECTORS	0.90	552	14	50%	53%	0%
CEGI	0.37	2.056	27	42%	48%	-10%
EGI	0.41	1.032	14	38%	47%	-11%
<b>D2</b>	<b>1.12</b>	<b>136</b>	<b>2</b>	<b>31%</b>	<b>43%</b>	<b>-18%</b>
SHELLS	0.66	136	2	23%	39%	-27%
RANDOM	0	0	0	2%	26%	-54%

### Next Times ...

Better shape representations  
More shape analysis applications