COS 487: Theory of Computation	**Fall 1998**
Mid-Term Exam	
Due: November 13, 5pm	*Sanjeev Arora*

Instructions: Give yourself 4 hours to do the test. Write answers legibly in the space provided. If you need extra space for an answer, you may attach extra sheets, *which the instructor will read at his discretion.* You can consult any notes/handouts from this class as well as the text. Feel free to quote, without proof, any results from class or the text. You cannot consult any other source or person in any way.

DO NOT READ THE TEST BEFORE YOU ARE READY TO WORK ON IT.

Your Name:

Write and sign the honor code pledge[1] here:

Prof. Arora will be available to answer emailed questions starting 11pm or so on Wed., Nov 11. He will hold office hours on Thurs. Nov 12, from 1pm to 4pm. You can also call him between 10am and 10pm at 683-1257 on Thurs. and Friday.

[1] The pledge is "I pledge my honor that I have not violated the honor code during this exam and followed all instruct ions."

Question 1 (20 points) For a language A let us define $\mathrm{FLIP}(A) = \{ww^R : w \in A\}$. Say whether the following statements are true or false, and give a brief (1-2 line) reason. No credit wil be given if the reason is incorrect.

(a). If A is regular, then FLIP(A) is regular.

(b). If A is context-free then FLIP(A) is context-free.

(c). If A is decidable then FLIP(A) is decidable.

(d). If A is enumerable then FLIP(A) is enumerable.

Question 2 (20 points). Let FLIP(A) be as in Question 1. Show that if FLIP(A) is regular then A is regular.

Question 3 (25 points) Give a context-free grammar generating the following language:

$$\{x \in \{0,1\}^* : x \text{ has twice as many 1's as 0s.}\}.$$

Question 4 (25 points) In class we defined a mapping reduction $\leq_m$ using a computable function f. Suppose we restrict f to be 1-1 and onto.

(a)(10 points) Suppose A is a decidable language such that both A and $\overline{A}$ are infinite. Then show that $A \leq_m 0^*1^*$ under this new definition.

(b)(15 points) Suppose A, B are languages such that $A \leq_m B$ under this new definition. Then show that $B \leq_m A$.

THAT'S ALL FOLKS. GOOD LUCK. NEXT QUESTION IS OPTIONAL.

Question 5 (Optional) (25 points; you can work on this question after the alloted 4 hours)

Two languages A, B are *decisively different* if there exists a decidable language C such that $C \cap A = \phi$ and $B \subseteq C$. Describe two disjoint languages that are enumerable but not decisively different. Give a brief proof.