

# Image Alignment and Stitching

COS 429: Computer Vision



# Motivation: panorama stitching

- We have two images — how do we combine them?

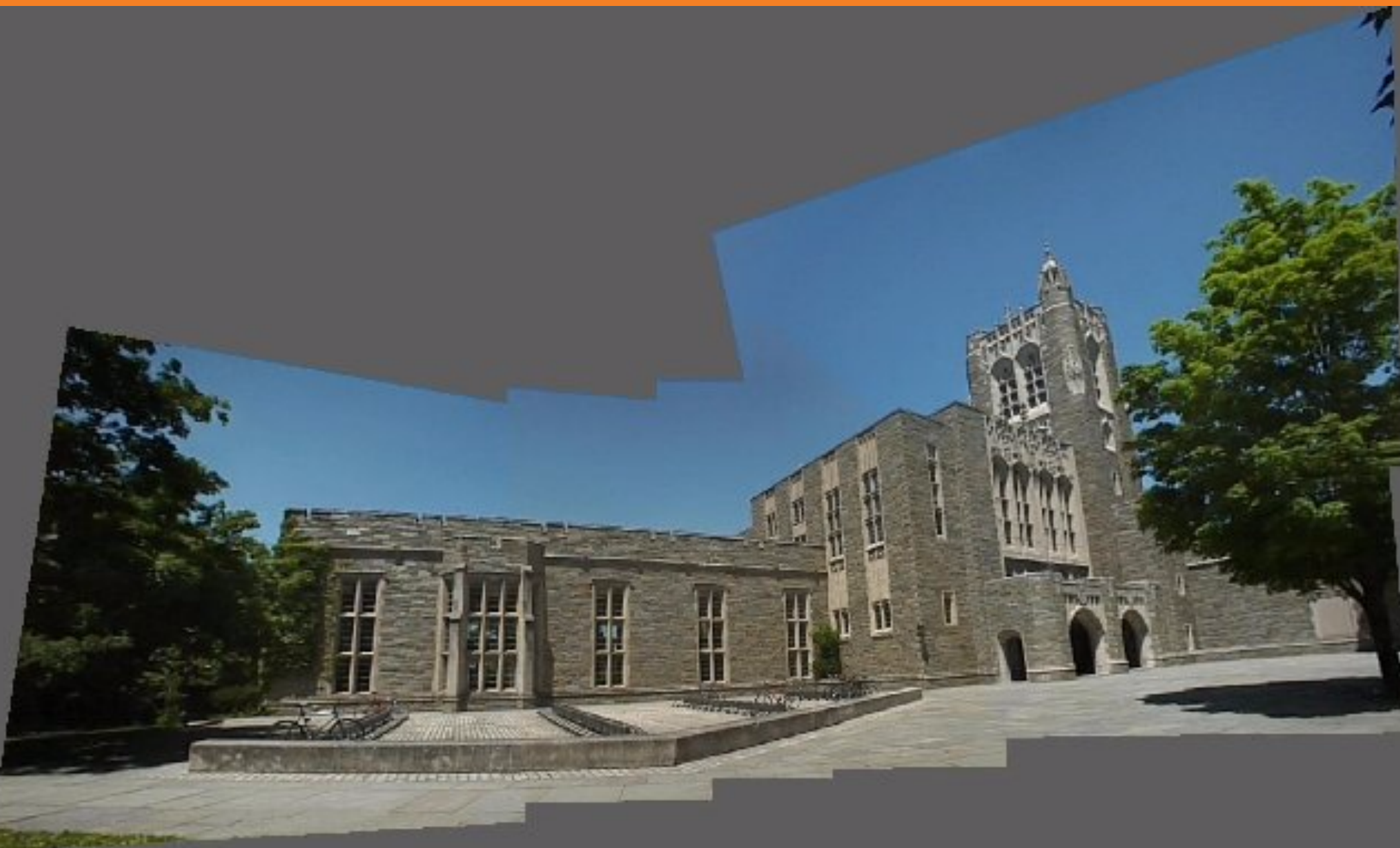


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# Panoramic Mosaics



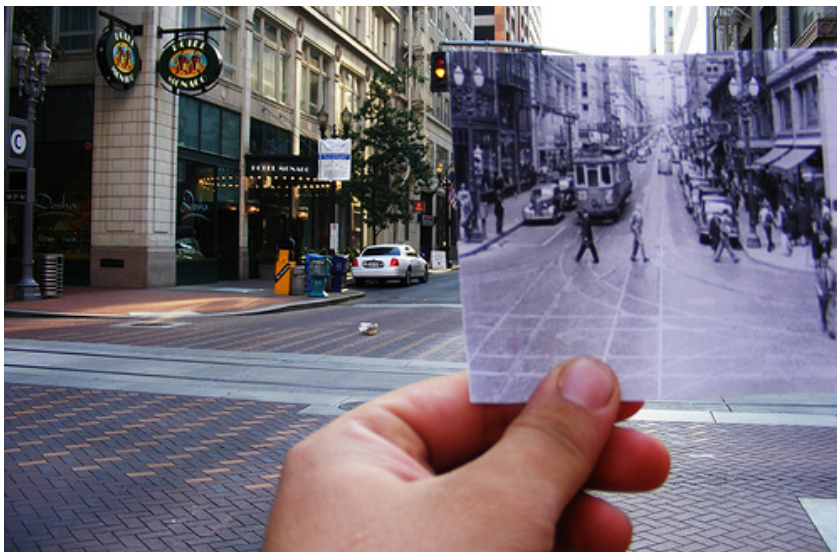


# Gigapixel Images





# Applications – Look into the Past



# Image Alignment Applications

- Local alignment:
  - Tracking
  - Stereo
- Global alignment:
  - Camera jitter elimination
  - Image enhancement
  - Panoramic mosaicing

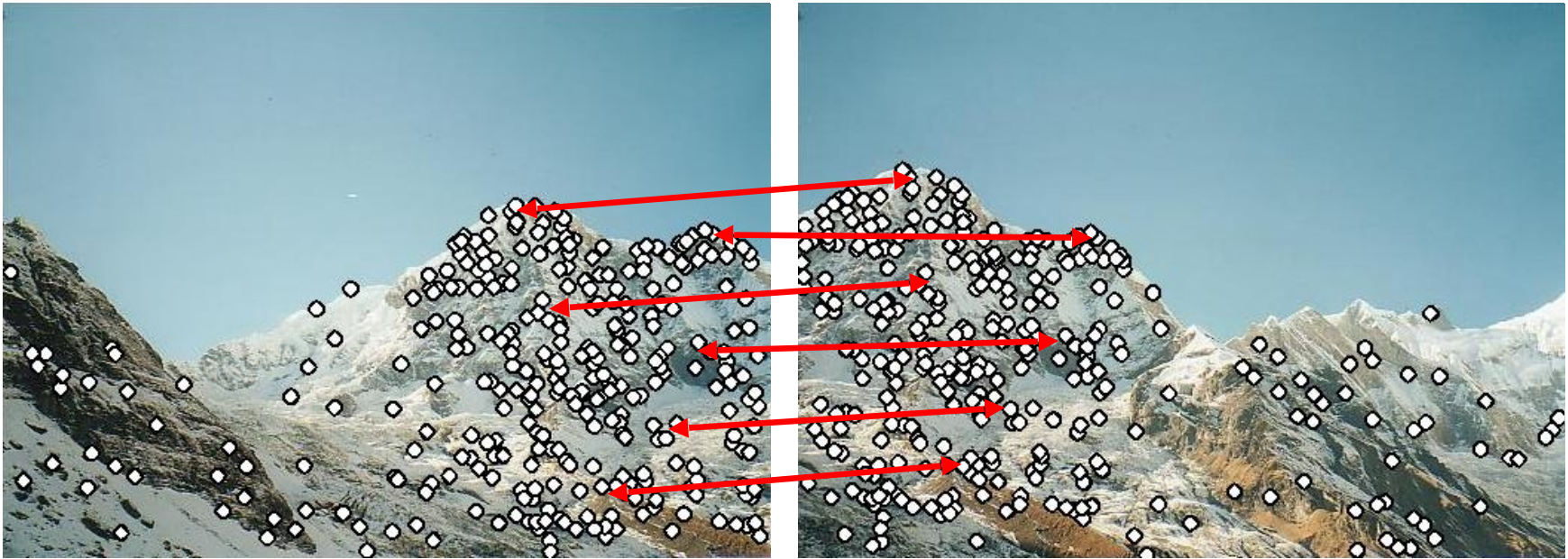
# Image Alignment Approaches

- Direct alignment: see which image transformation maximizes similarity in overlap region
  - Often performed coarse-to-fine
- **Feature-based alignment: find image transformation that matches keypoint locations**



# Panorama stitching

- We have two images — how do we combine them?



Step 1: extract keypoints

Step 2: match keypoint features

# Panorama stitching

- We have two images — how do we combine them?



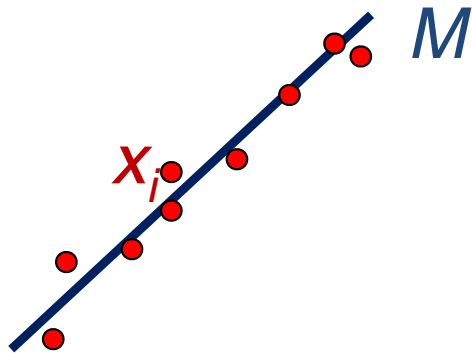
Step 1: extract keypoints

Step 2: match keypoint features

Step 3: align images

# Alignment as Fitting

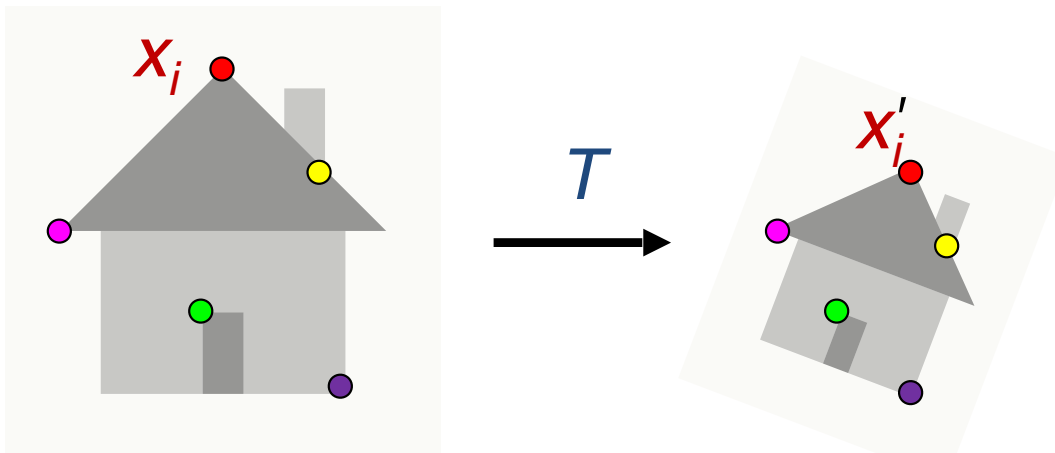
- **Previously:** fitting a model to features in one image



Find model  $M$  that minimizes

$$\sum_i L(x_i; M)$$

- **Alignment:** fitting a model to a transformation between pairs of features (matches) in two images



Find transformation  $T$  that minimizes

$$\sum_i L(T(x_i); x'_i)$$

# Feature-Based Alignment

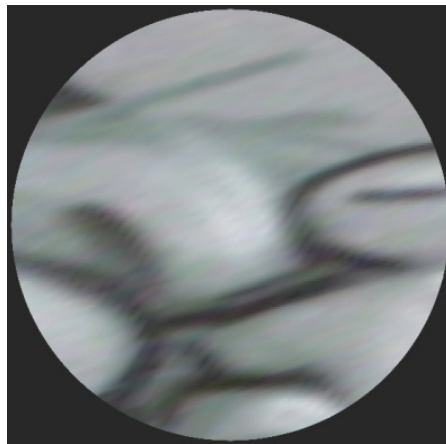
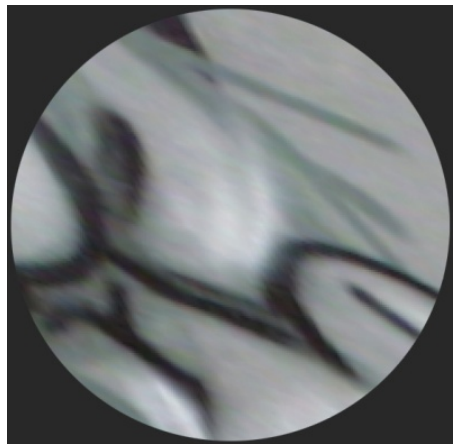
- Find keypoints; compute SIFT descriptors
- Generate candidate keypoint matches
- Use RANSAC to select a subset of matches
- **Fit to find best image transformation**
- Warp images according to transformation
- Blend images in overlapping regions



# Feature-Based Alignment

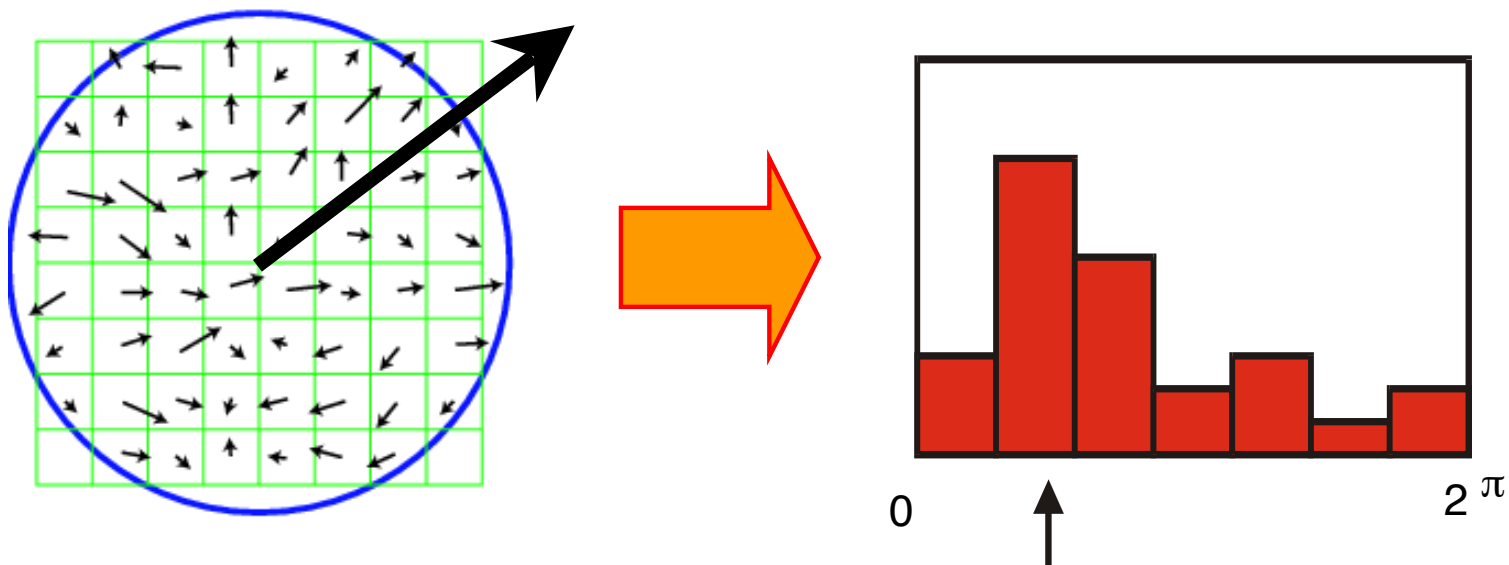
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# Recall: after blob detection and scale normalization



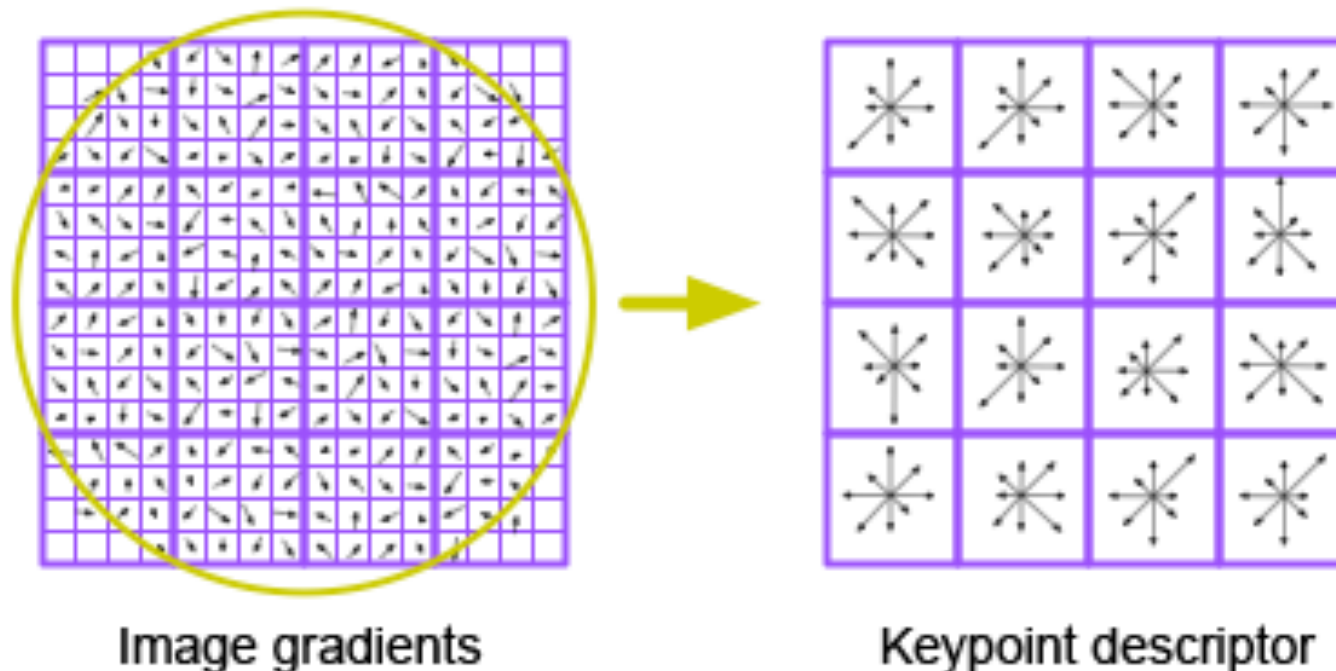
# Recall: eliminating rotation ambiguity

- To assign a unique orientation to circular image windows:
  - Create histogram of local gradient directions in the patch
  - Assign canonical orientation at peak of smoothed histogram



# Recall: SIFT Descriptor

- Divide  $16 \times 16$  window into  $4 \times 4$  grid of cells
- Compute an orientation histogram for each cell
  - 16 cells \* 8 orientations = 128-dimensional descriptor



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

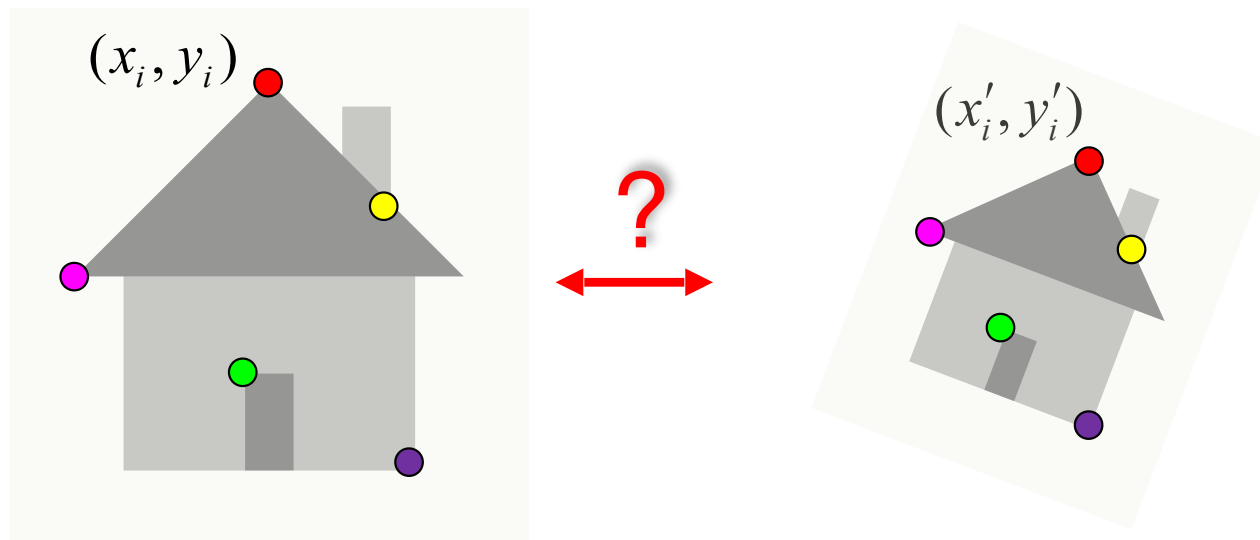


# Feature-Based Alignment

- Find keypoints; compute SIFT descriptors
- **Generate candidate keypoint matches**
- Use RANSAC to select a subset of matches
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# Candidate Matches

- For a given keypoint in image A, how to find candidate match in image B?

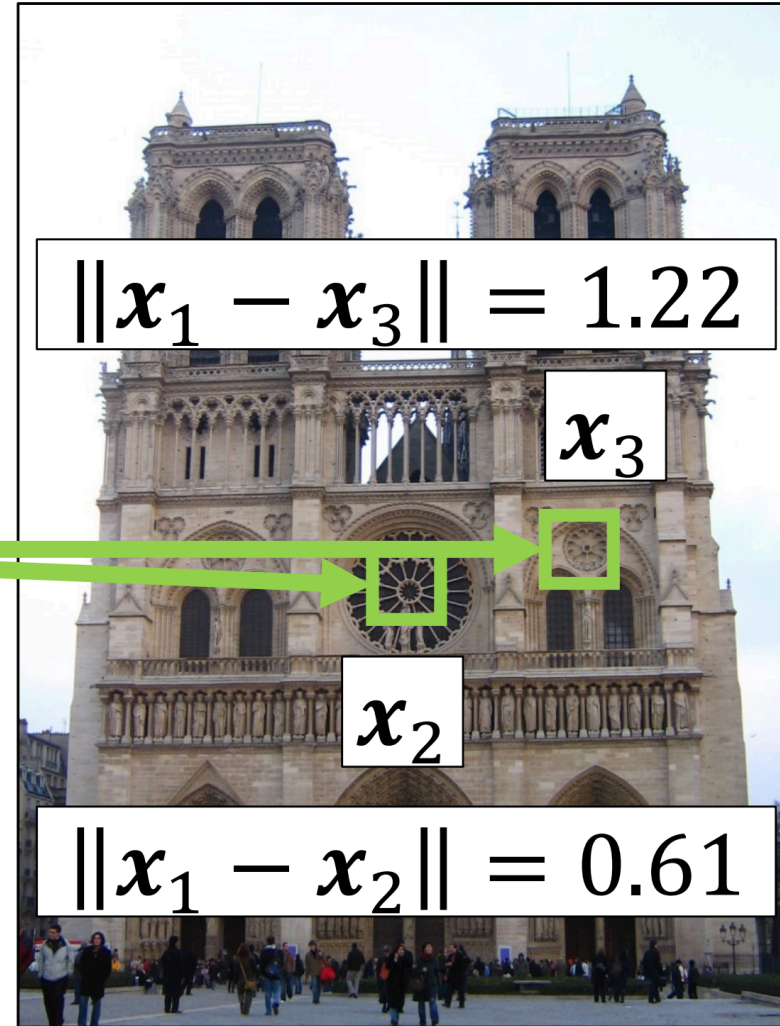
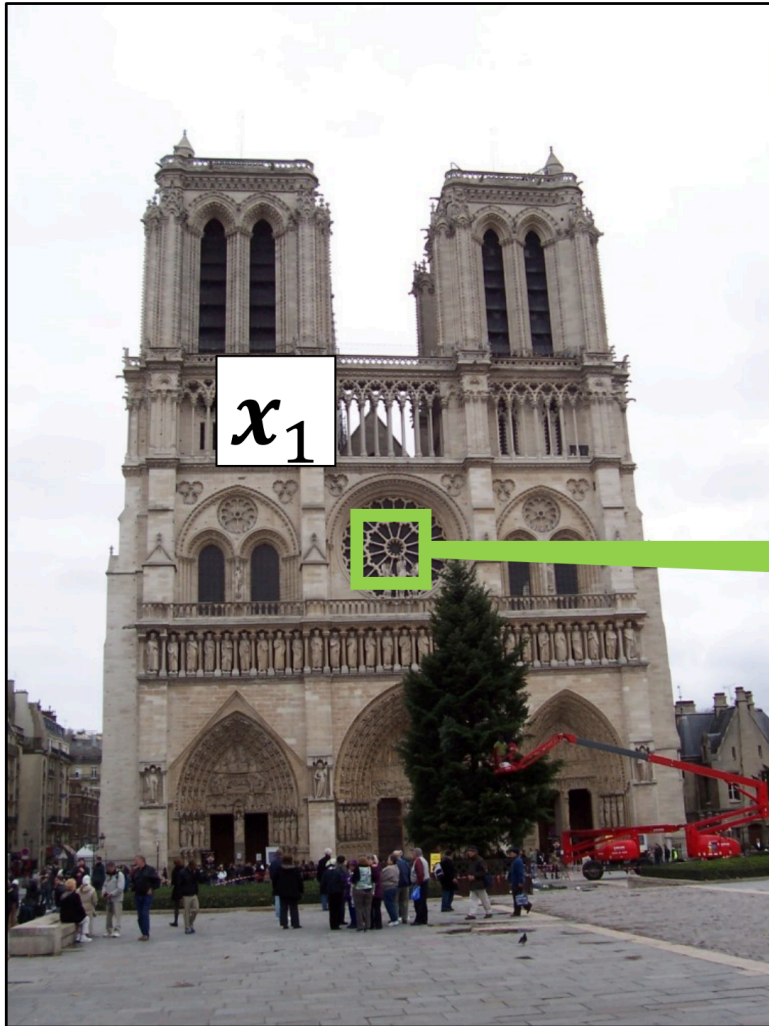


# Candidate Matches

- For each SIFT descriptor in image A, find closest (according to Euclidean distance) in image B

$$best\_match(x) = \operatorname{argmin}_{x_i'} \|x - x_i'\|^2$$

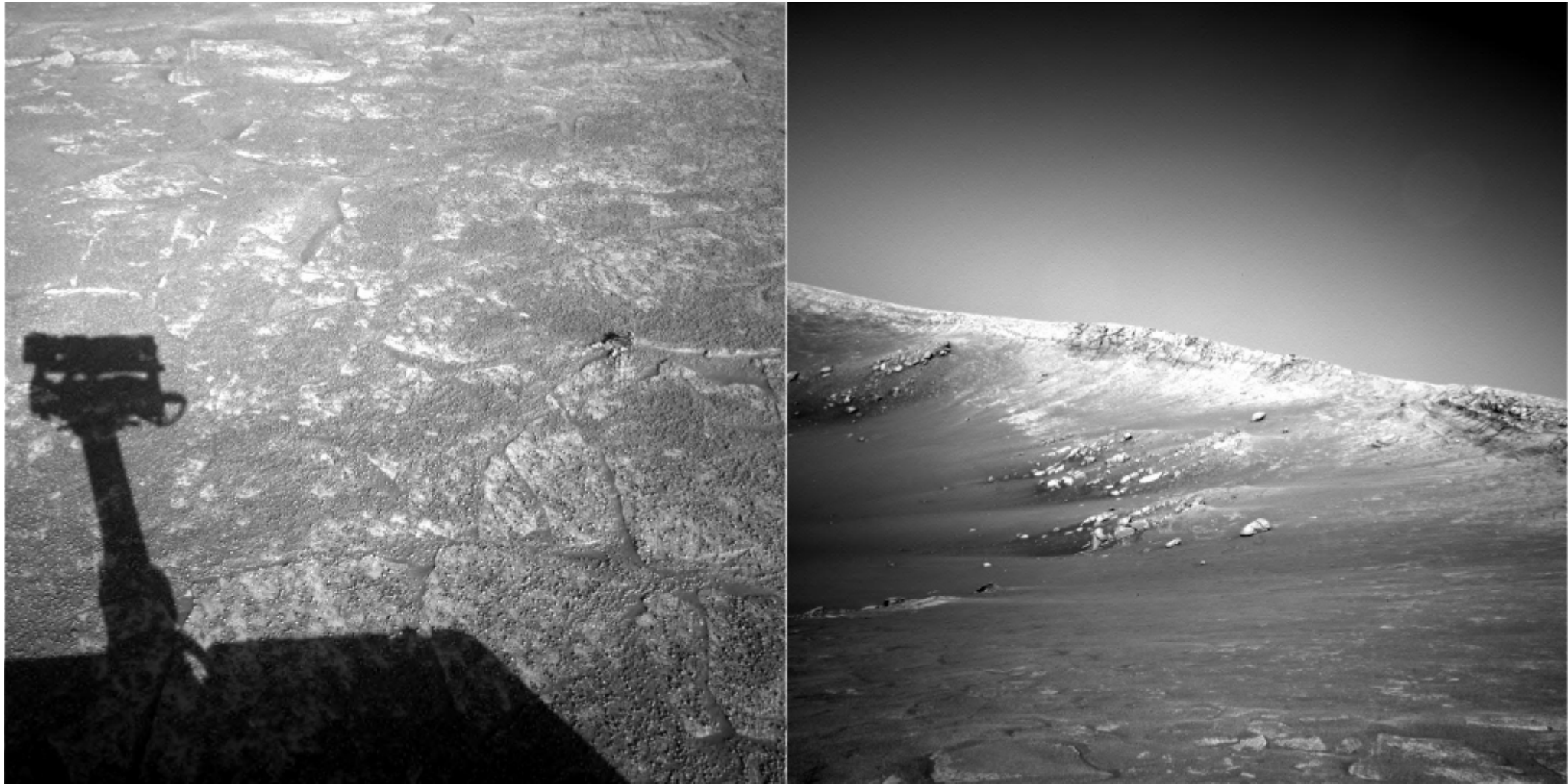
# Instance matching



$$\|x_1 - x_3\| = 1.22$$

$$\|x_1 - x_2\| = 0.61$$

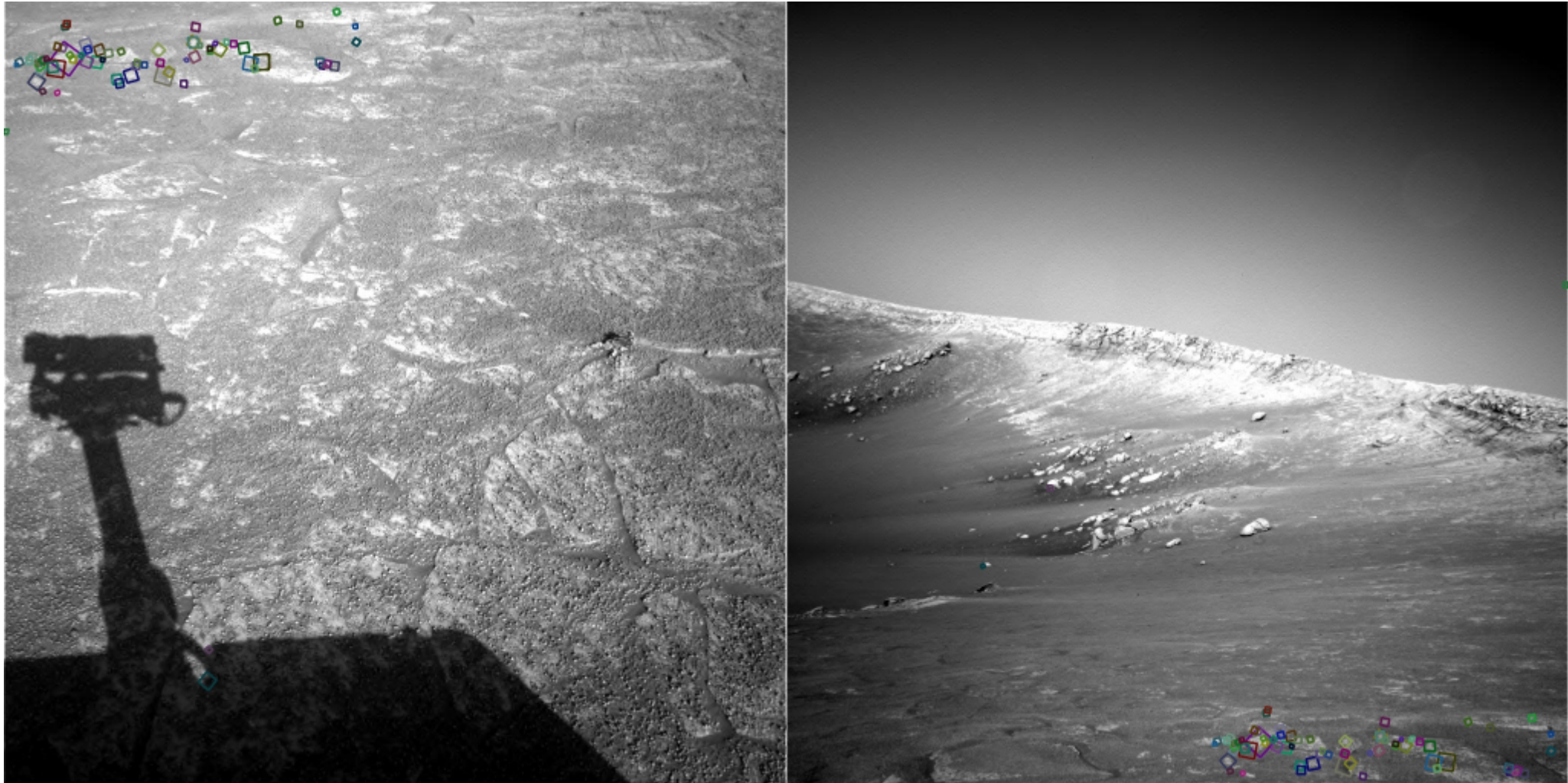
# Example: instance matching



NASA Mars Rover images



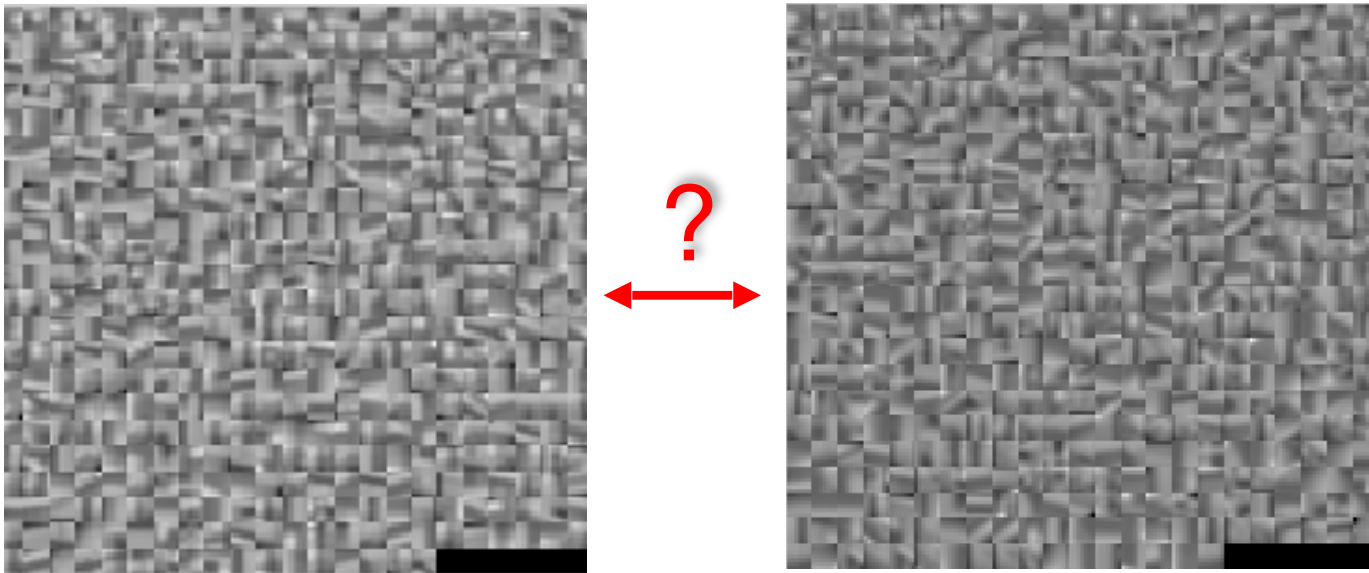
# Example: instance matching (look for tiny colored squares)



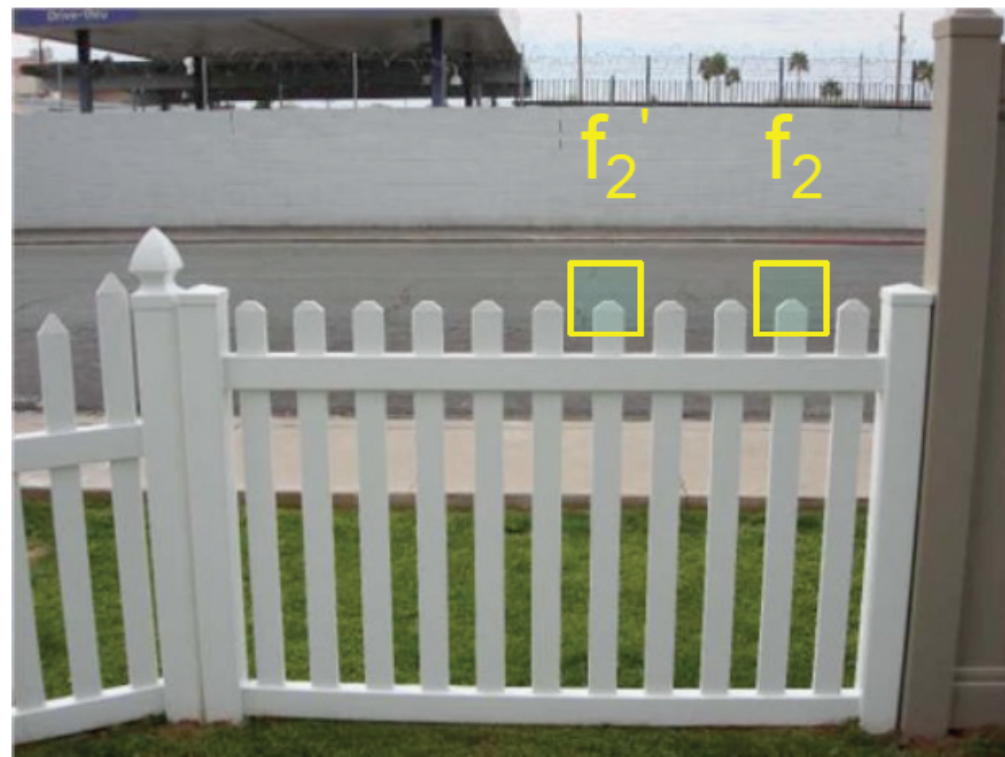
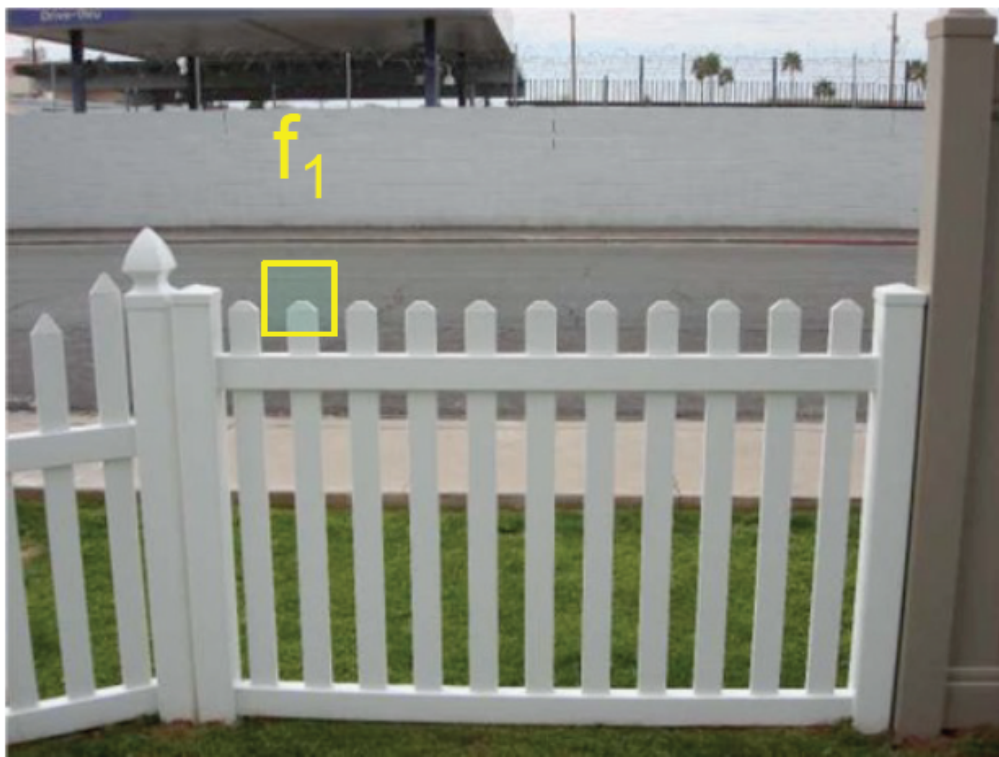
NASA Mars Rover images  
with SIFT feature matches  
Figure by Noah Snavely

# Candidate Matches

- For a given keypoint in image A, how to find candidate match in image B?
  - What if there are a lot of keypoints?



# Problem: Ambiguous Correspondences



# Candidate Matches

- For each SIFT descriptor in image A, find closest (according to Euclidean distance) in image B

$$best\_match(x) = \underset{x_i'}{\operatorname{argmin}} \|x - x_i'\|^2$$

- **Refinement:** mutual best match
  - $x'$  is most similar to  $x$  and  $x$  is most similar to  $x'$



# Candidate Matches

- For each SIFT descriptor in image A, find closest (according to Euclidean distance) in image B

$$best\_match(x) = \underset{x_i'}{\operatorname{argmin}} \|x - x_i'\|^2$$

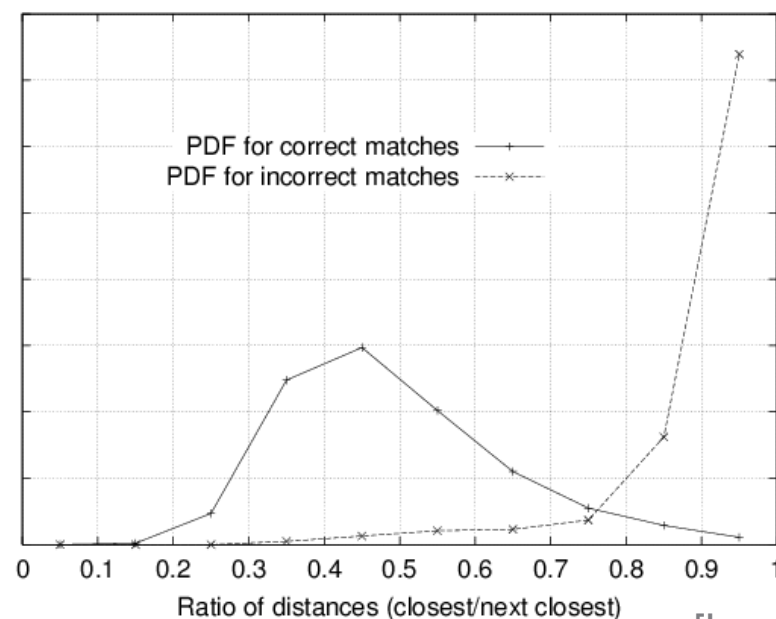
- **Refinement:** mutual best match

- $x'$  is most similar to  $x$  and  $x$  is most similar to  $x'$

- **Refinement:** best match is much better than second-best

- Ratio of closest to second-closest distance is **high** for **non-distinctive** features

- Threshold ratio of e.g. 0.8



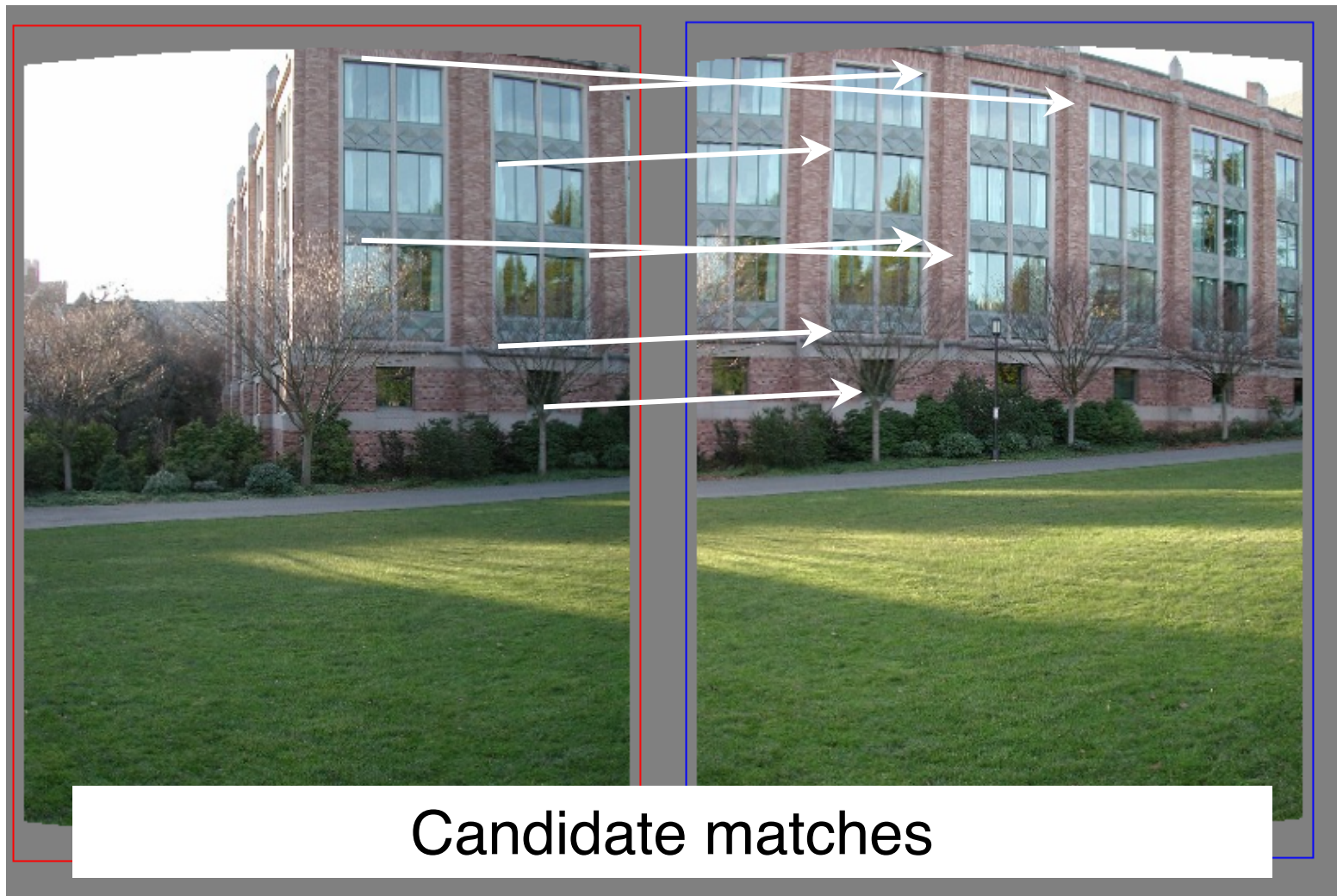
# Feature-Based Alignment

- Find keypoints; compute SIFT descriptors
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# Review: RANSAC

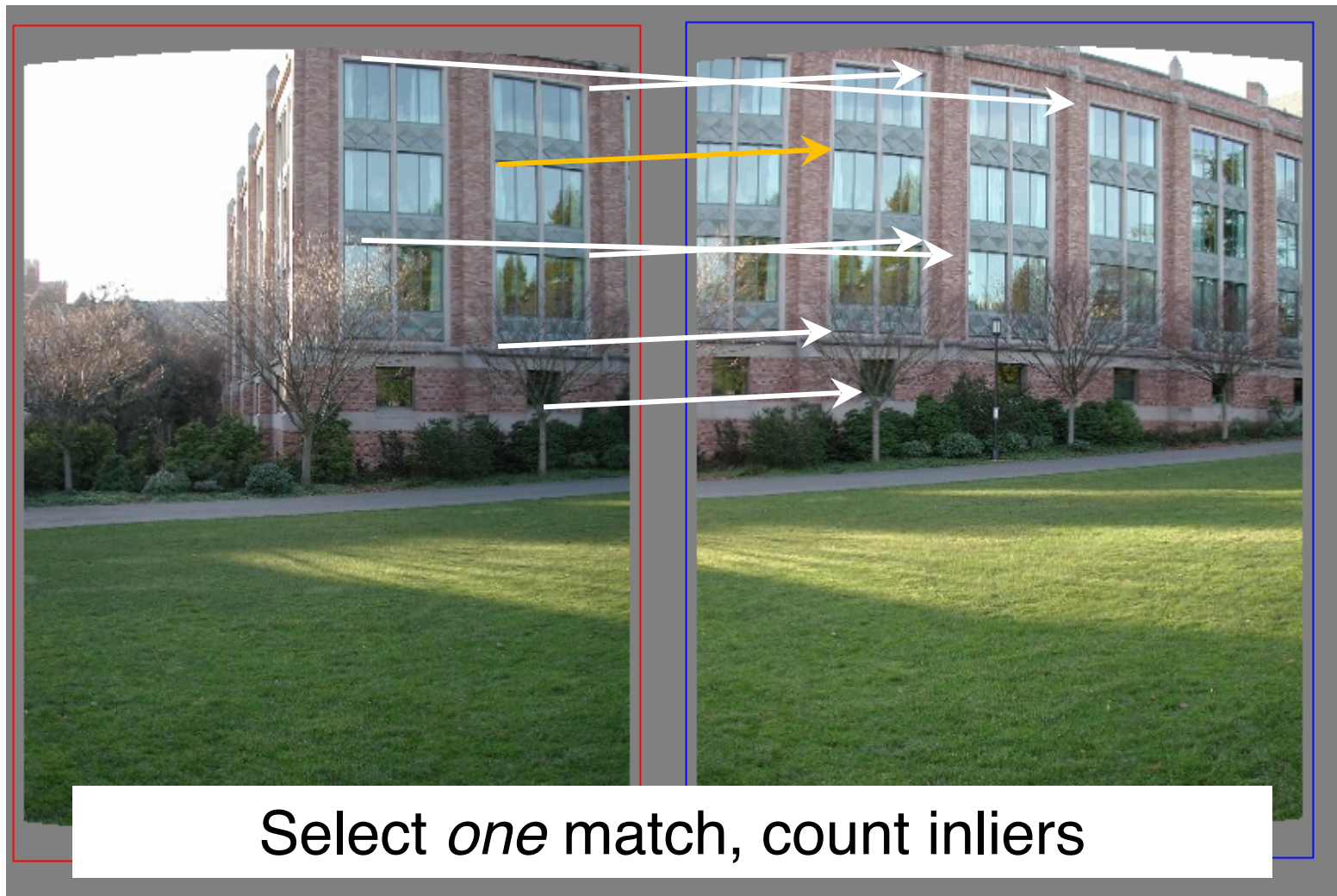
- Set of candidate matches contains many outliers
- RANSAC loop:
  - Randomly select a **minimal** set of matches
  - Compute transformation from seed group
  - Find inliers to this transformation
  - Keep the transformation with the largest number of inliers
- At end, re-estimate best transform using all inliers

# RANSAC: Translation Only

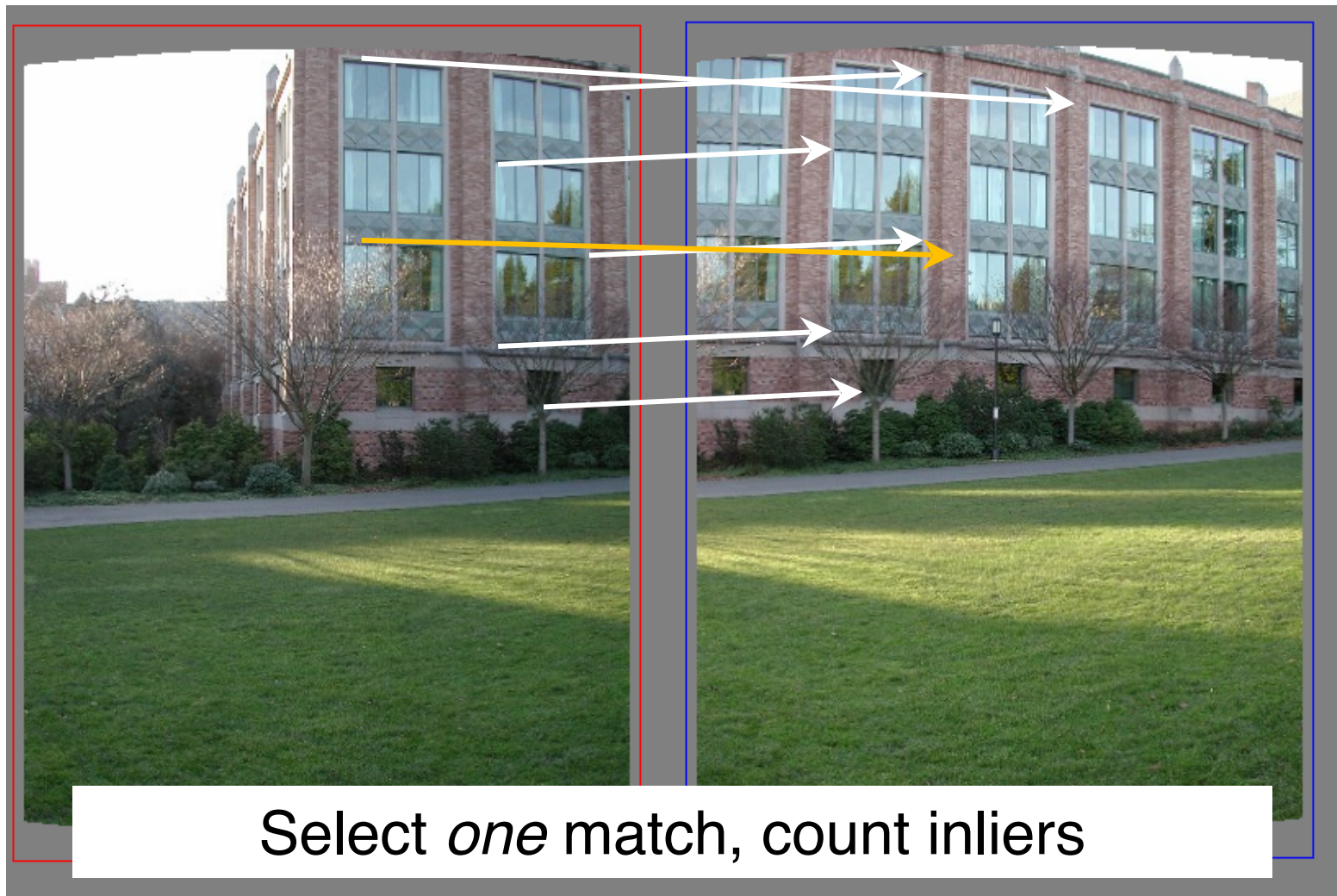




# RANSAC: Translation Only



# RANSAC: Translation Only







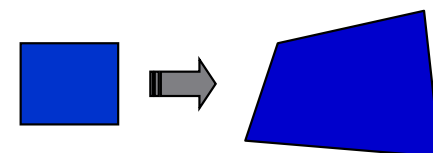
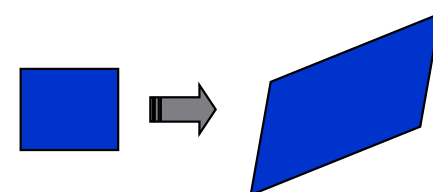
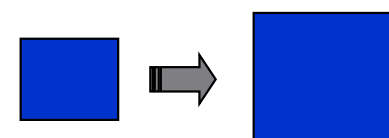
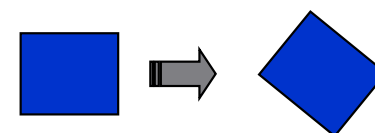
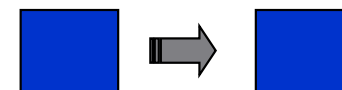
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# 2D Transformation Models

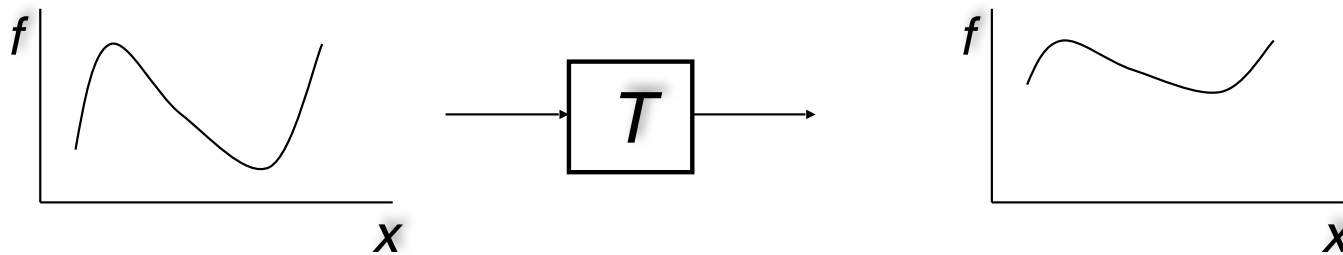
- Translation only
- Rigid body (translation+rotation)
- Similarity (translation+rotation+scale)
- Affine
- Homography (projective)



# Image Warping

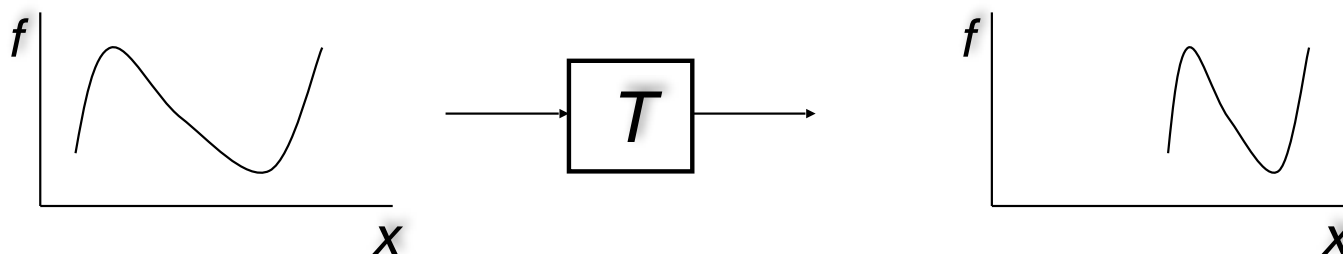
- Image filtering: change *range* of image

$$g(x) = T(f(x))$$



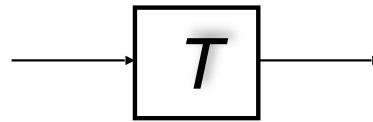
- Image warping: change *domain* of image

$$g(x) = f(T(x))$$

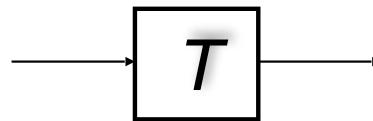


# Image Warping

- Image filtering: change *range* of image  
 $g(x) = T(f(x))$



- Image warping: change *domain* of image  
 $g(x) = f(T(x))$



# Parametric (Global) Warping

- Examples of parametric warps:



translation



rotation



aspect



affine



perspective



cylindrical



# Parametric (Global) Warping

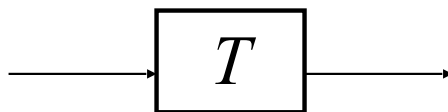
T is a coordinate changing machine

$$\mathbf{p}' = T(\mathbf{p})$$

Note: T is the same for all points, has relatively few parameters, and does **not** depend on image content



$$\mathbf{p} = (x, y)$$



$$\mathbf{p}' = (x', y')$$

# Parametric (Global) Warping

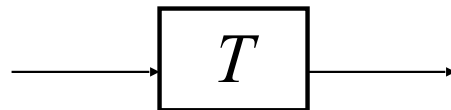
Today we'll deal with linear warps

$$\mathbf{p}' \equiv T\mathbf{p}$$

$T$ : matrix;  $\mathbf{p}$ ,  $\mathbf{p}'$ : 2D points. Start with normal points and  $\equiv$ , then do homogeneous coords and  $\equiv$



$$\mathbf{p} = (x, y)$$

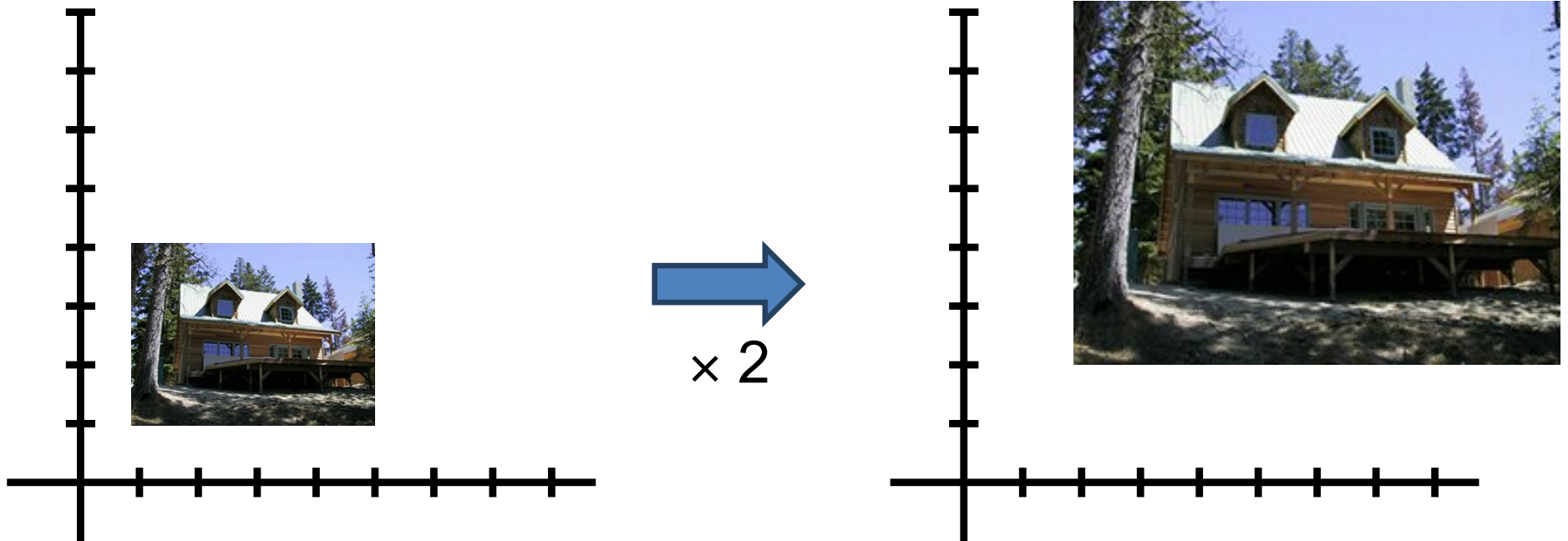


$$\mathbf{p}' = (x', y')$$

# Scaling

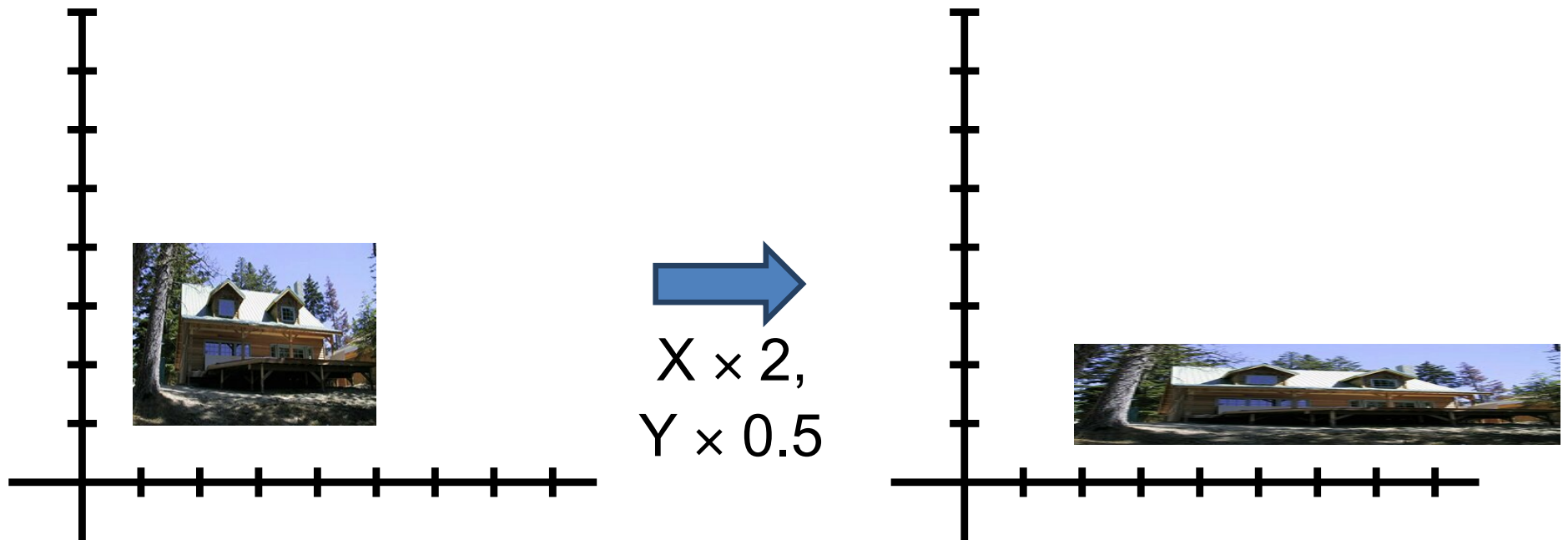
**Scaling** multiplies each component  $(x,y)$  by a scalar.  
**Uniform** scaling is the same for all components.

*Note the corner goes from  $(1,1)$  to  $(2,2)$*



# Scaling

**Non-uniform scaling** multiplies each component by a different scalar.





# Scaling

**What does T look like?**

$$x' = ax$$

$$y' = by$$

Let's convert to a matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

*scaling matrix S*

**What's the inverse of S?**

# 2D Rotation



## Rotation Matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

But wait! Aren't sin/cos non-linear?

$x'$  is a linear combination/function of  $x, y$   
 $x'$  is not a linear function of  $\theta$

What's the inverse of  $R_\theta$ ?  $I = R_\theta^T R_\theta$

# Things You Can Do With 2x2

## Identity / No Transformation



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

## Shear



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Things You Can Do With 2x2

Before



After



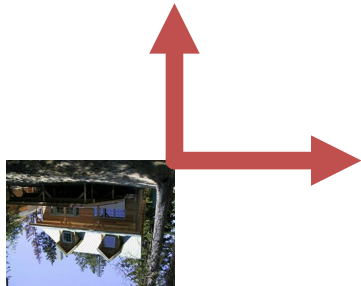
## 2D Mirror About Y-Axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Before



After



## 2D Mirror About X,Y

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



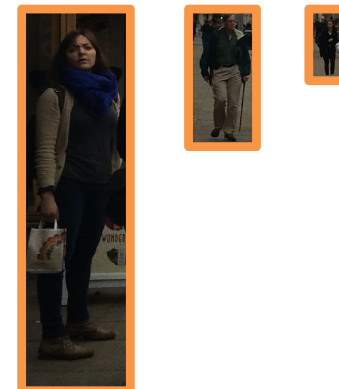
# Recall: what's preserved in images?



3D lines project to 2D lines  
so lines are preserved

Projections of parallel 3D  
lines are not necessarily  
parallel, so not parallelism

Distant objects are smaller  
so size is not preserved



# What's Preserved With a 2x2

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix}$$

After multiplication by T (irrespective of T)

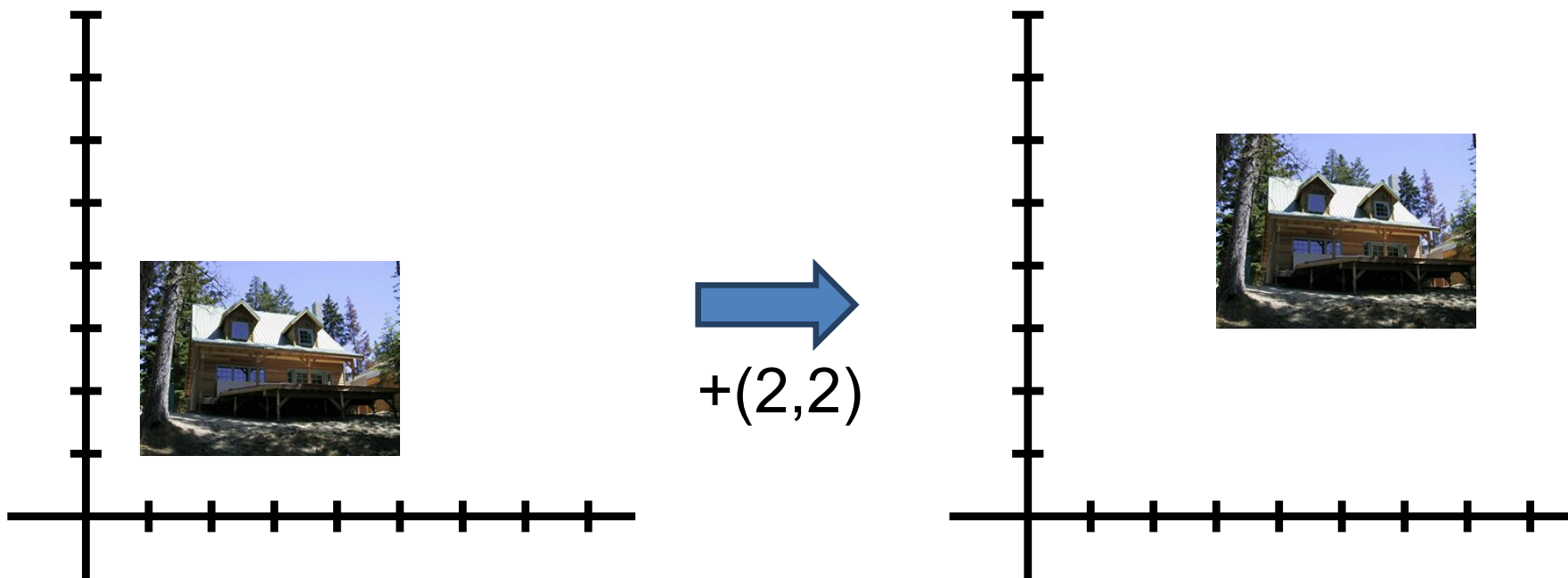
- Origin is origin:  $\mathbf{0} = T\mathbf{0}$
- Lines are lines  $\begin{bmatrix} (ax + by) + \lambda(adir_x + bdir_y) \\ (cx + dy) + \lambda(cdir_x + ddir_y) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x + \lambda dir_x \\ y + \lambda dir_y \end{bmatrix}$
- Parallel lines are parallel
- Ratios between distances the same *if scaling is uniform (otherwise no)*

# Things You Can't Do With 2x2

What about translation?

$$x' = x + t_x, y' = y + t_y$$

**How do we fix it?**

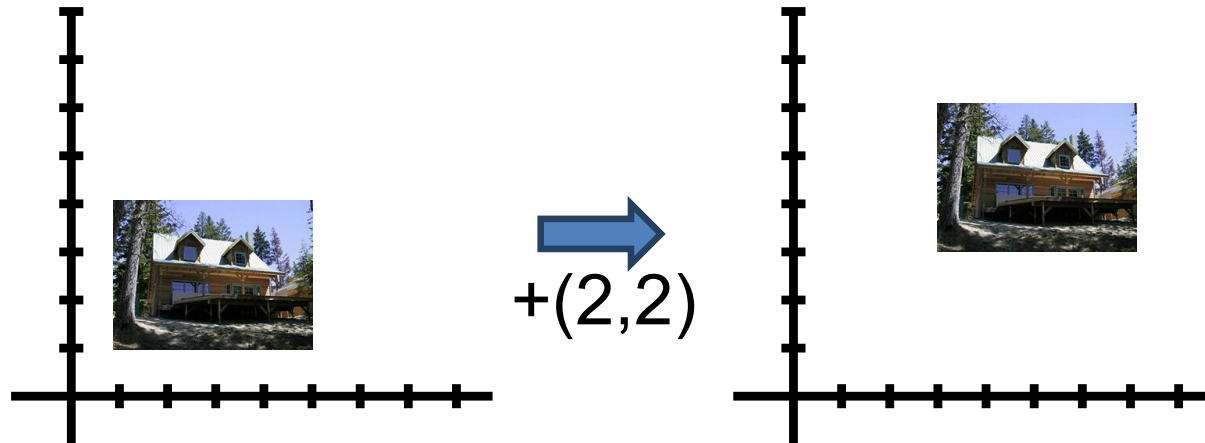


# Homogeneous Coordinates

What about translation?

$$x' = x + t_x, y' = y + t_y$$

$$\begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} \equiv \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



# Representing 2D Transformations

How do we represent a 2D transformation?

Let's pick scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \equiv \begin{bmatrix} s_x & 0 & a \\ 0 & s_y & b \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

What's **a** **b** **d** **e** **f**

**0** **0** **0** **0** **1**



# Affine Transformations

Affine: *linear transformation plus translation*



$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

**Will the last coordinate always be 1?**

In general (without homogeneous coordinates)

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$$

# Matrix Composition

We can combine transformations via matrix multiplication.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \equiv \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{T(t_x, t_y)} \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R(\theta)} \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S(s_x, s_y)} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

**Does order matter?**

# What's Preserved With Affine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

After multiplication by T (irrespective of T)

- ~~Origin is origin:  $0 = T0$~~
- Lines are lines  $\begin{bmatrix} (ax + by + c) + \lambda(adir_x + bdir_y) \\ (dx + ey + f) + \lambda(ddir_x + edir_y) \\ 1 \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x + \lambda dir_x \\ y + \lambda dir_y \\ 1 \end{bmatrix}$
- Parallel lines are parallel
- Ratios between distances? if scaling is uniform yes, otherwise no

# Perspective Transformations

Set bottom row to not  $[0,0,1]$

Called a perspective/projective transformation or a *homography*



$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

**How many degrees of freedom?**

# How Many Degrees of Freedom?

Recall: can always scale by non-zero value

$$\text{Perspective} \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \equiv \frac{1}{i} \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \equiv \frac{1}{i} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} \equiv \begin{bmatrix} a/i & b/i & c/i \\ d/i & e/i & f/i \\ g/i & h/i & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Homography can always be re-scaled by  $\lambda \neq 0$



# What's Preserved With Perspective

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

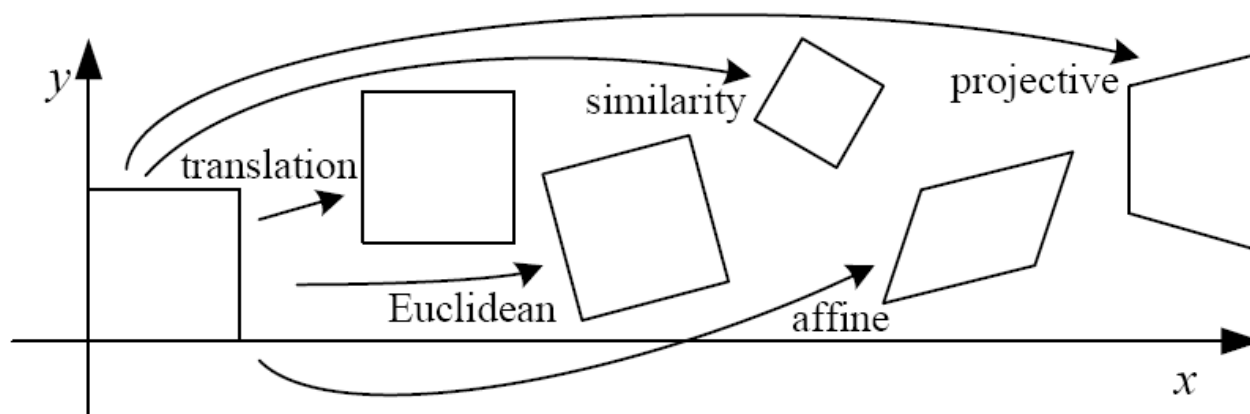
After multiplication by T (irrespective of T)






- ~~Origin is origin:  $0 = T0$~~
- Lines are lines
- ~~Parallel lines are parallel~~
- ~~Ratios between distances~~



# Transformation Families

In general: transformations are a nested set of groups



| Name              | Matrix  | # D.O.F. | Preserves:        | Icon  |
|-------------------|---|----------|-------------------|---|
| translation       | $\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$  | 2        | orientation + ... |  |
| rigid (Euclidean) | $\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$  | 3        | lengths + ...     |  |
| similarity        | $\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$ | 4        | angles + ...      |  |
| affine            | $\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$               | 6        | parallelism + ... |  |
| projective        | $\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$       | 8        | straight lines    |  |

# What Can Homographies Do?

Homography example 1: any two views of a *planar* surface



# What Can Homographies Do?

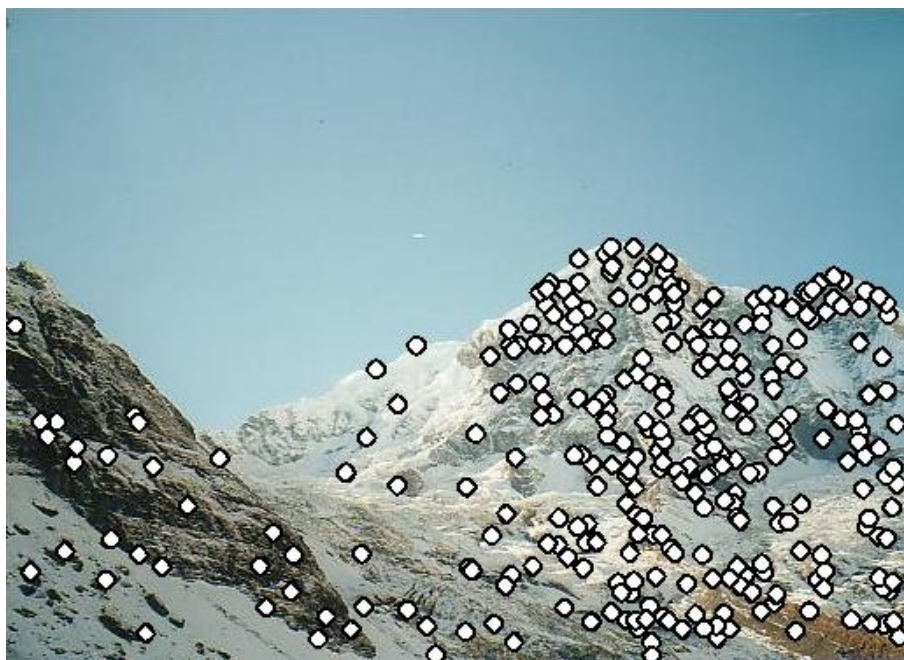
Homography example 2: any images from two cameras sharing a camera center





# What Can Homographies Do?

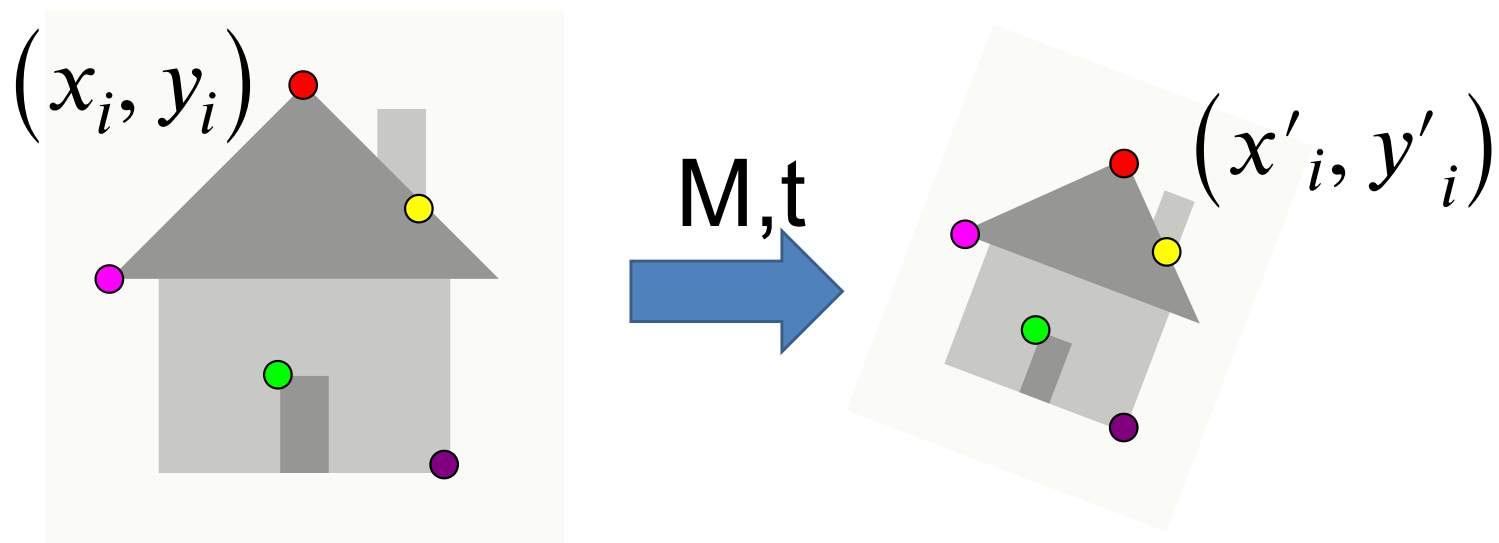
Homography sort of example “3”: far away scene that can be approximated by a plane





# Fitting Transformations

Setup: have pairs of correspondences



$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \mathbf{M} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + t$$

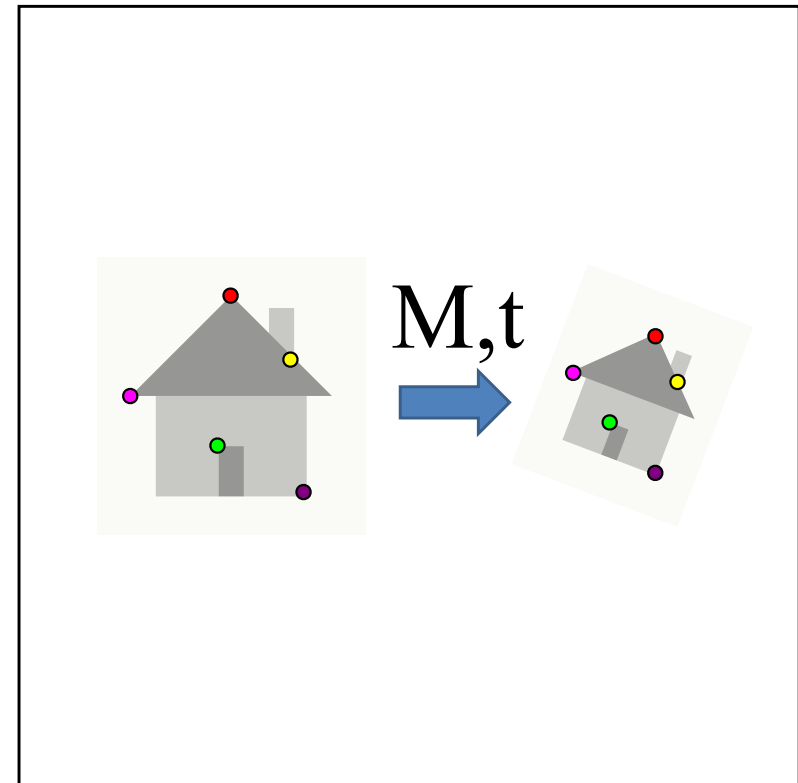
# Fitting Transformation

## Affine Transformation: $M, t$

Data:  $(x_i, y_i, x'_i, y'_i)$  for  
 $i=1, \dots, k$

Model:  
 $[x'_i, y'_i] = \mathbf{M}[x_i, y_i] + \mathbf{t}$

Objective function:  
 $\| [x'_i, y'_i] - \mathbf{M}[x_i, y_i] + \mathbf{t} \|^2$



# Fitting Transformations

Given correspondences:  $\mathbf{p}' = [x'_i, y'_i]$ ,  $\mathbf{p} = [x_i, y_i]$

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Set up two equations per point

$$\begin{bmatrix} \vdots \\ x'_i \\ y'_i \\ \vdots \end{bmatrix} = \begin{bmatrix} & & \dots & & & & \\ x_i & y_i & 0 & 0 & 1 & 0 & \\ 0 & 0 & x_i & y_i & 0 & 1 & \\ & & \dots & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_x \\ t_y \end{bmatrix}$$

# Fitting Transformations

$$\begin{bmatrix} \vdots \\ x'_i \\ y'_i \\ \vdots \end{bmatrix} = \begin{bmatrix} & & \dots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \dots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_x \\ t_y \end{bmatrix}$$

2 equations per point, 6 unknowns

**How many points do we need?**

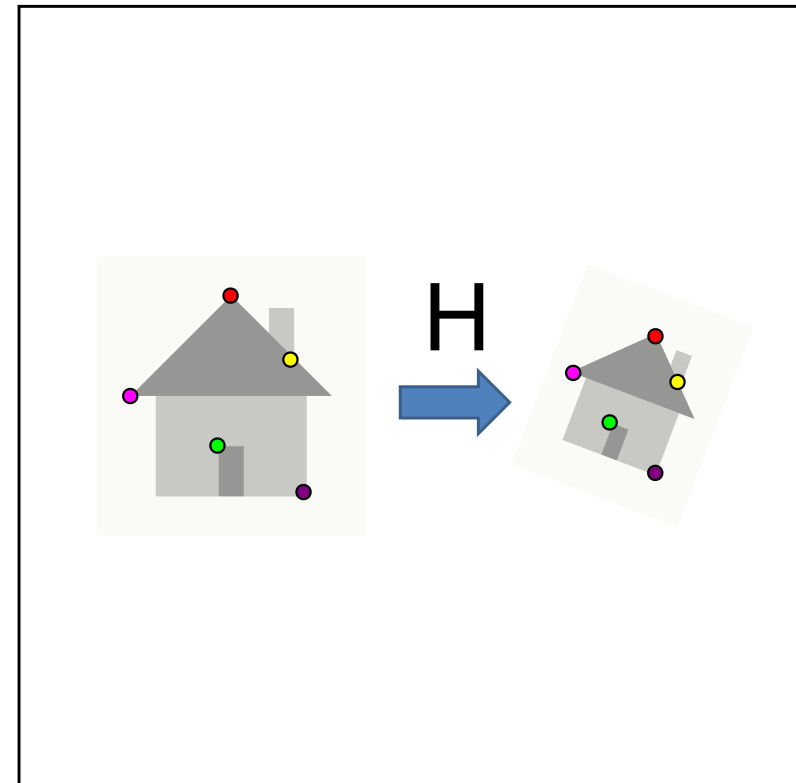
# Fitting Transformation

## Homography: H

Data:  $(x_i, y_i, x'_i, y'_i)$  for  $i=1, \dots, k$

Model:  
 $[x'_i, y'_i, 1] \equiv \mathbf{H}[x_i, y_i, 1]$

Objective function:  
It's complicated



*(Chapters 6.1, 6.2 in the book)*

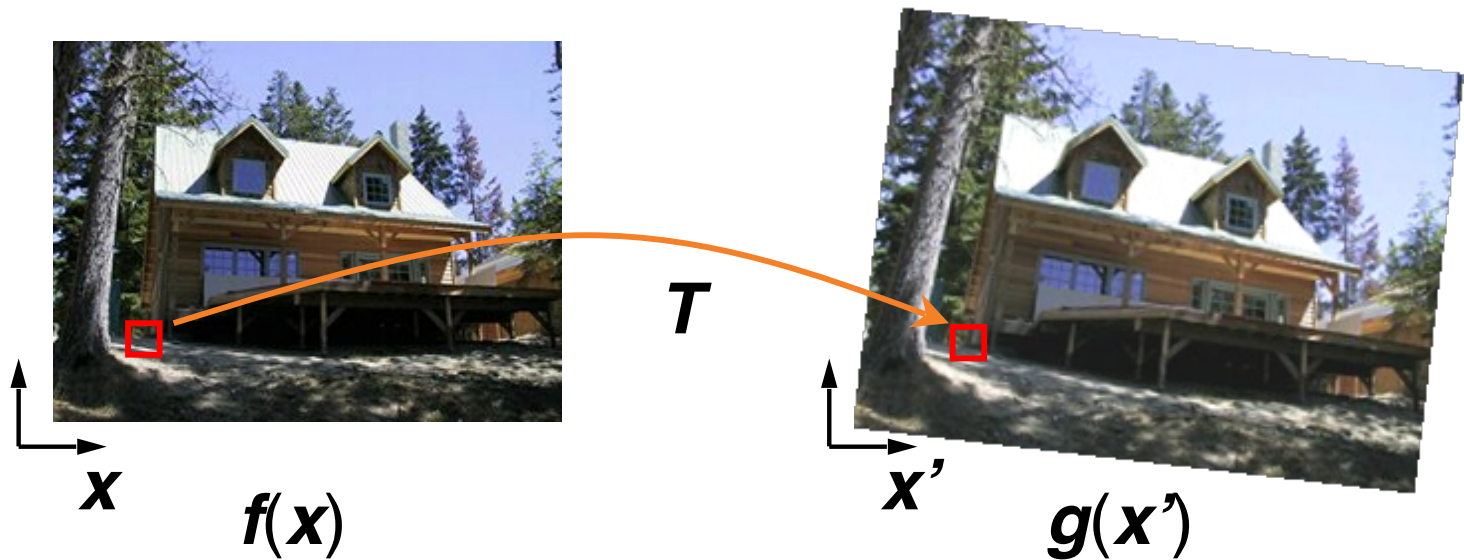


# Feature-Based Alignment

- Find keypoints; compute SIFT descriptors
- Generate candidate keypoint matches
- Use RANSAC to select a subset of matches
- Fit to find best image transformation
- **Warp images according to transformation**
- Blend images in overlapping regions

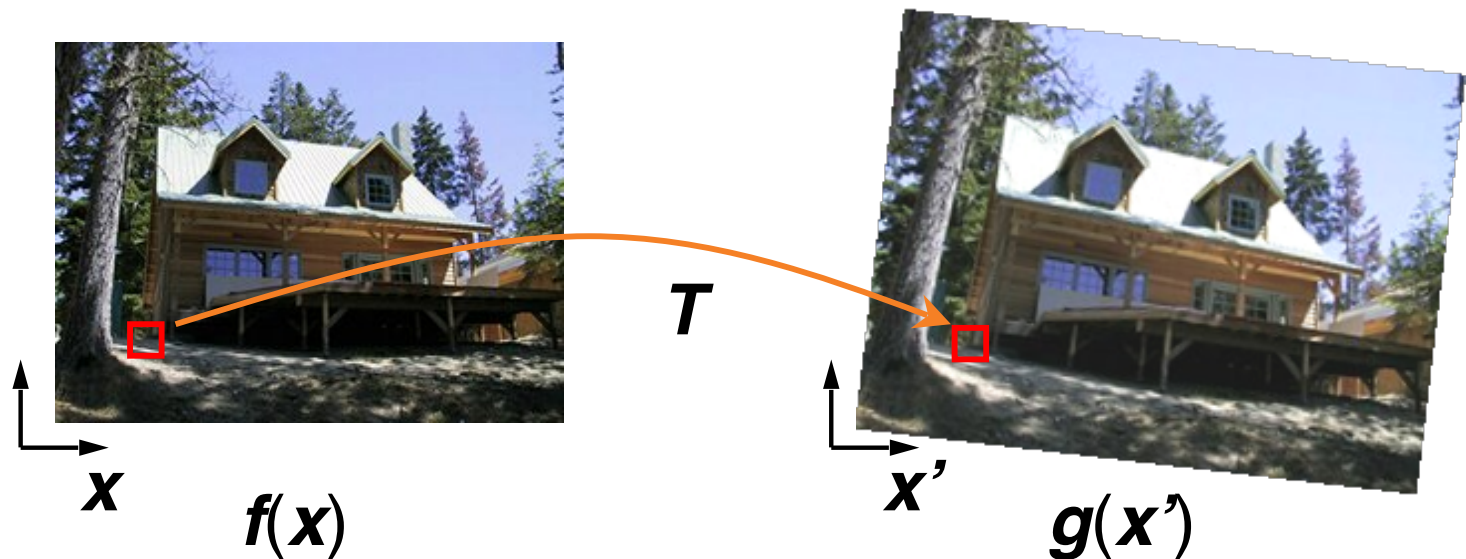
# Image Warping

- Given a coordinate transform  $\mathbf{x}' = T(\mathbf{x})$  and a source image  $f(\mathbf{x})$ , how do we compute a transformed image  $g(\mathbf{x}') = f(T(\mathbf{x}'))$ ?



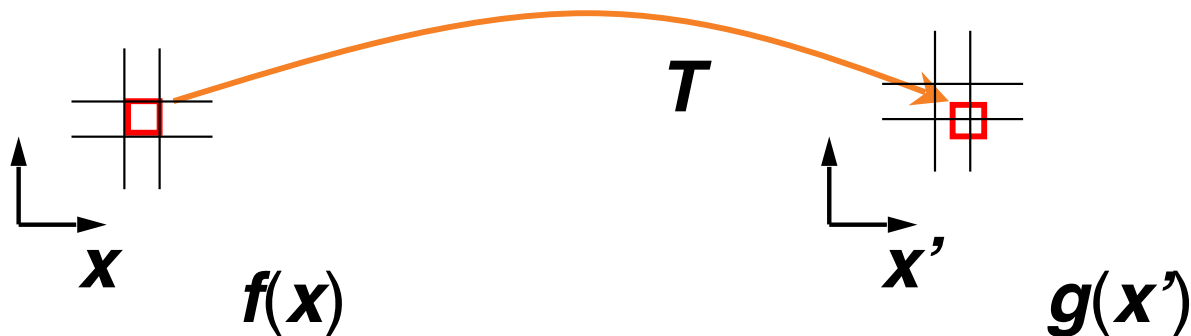
# Forward Warping

- Send each pixel  $f(\mathbf{x})$  to its corresponding location  $\mathbf{x}' = T(\mathbf{x})$  in  $g(\mathbf{x}')$
- What if pixel lands “between” two pixels?



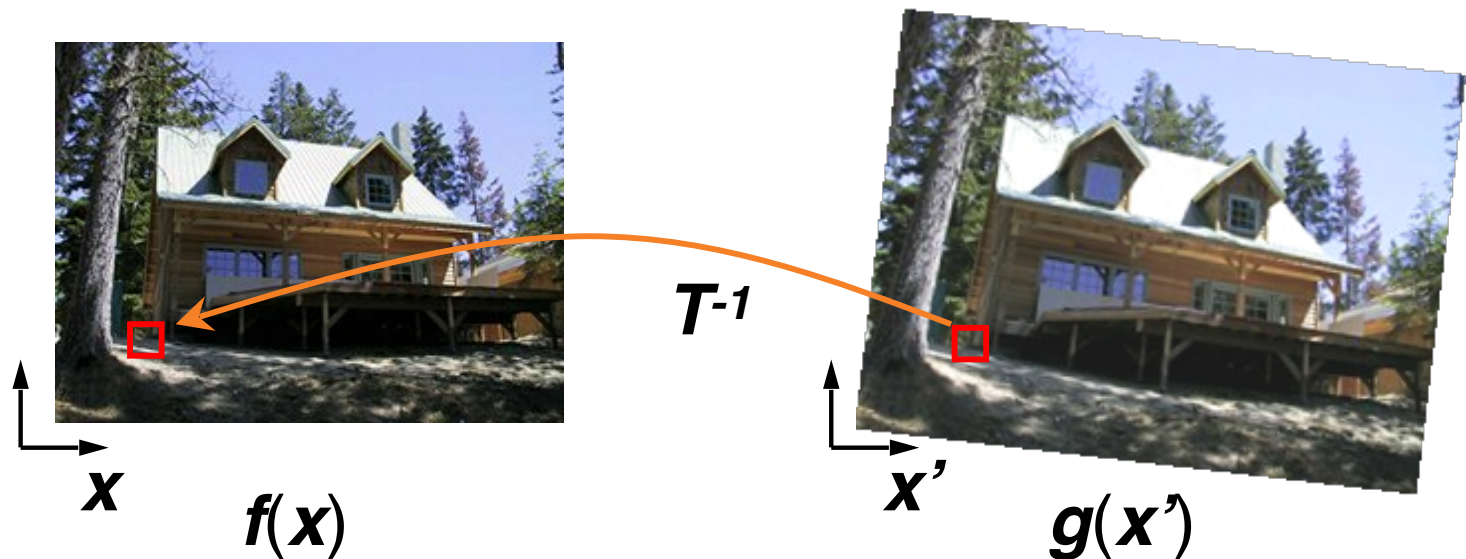
# Forward Warping

- Send each pixel  $f(\mathbf{x})$  to its corresponding location  $\mathbf{x}' = T(\mathbf{x})$  in  $g(\mathbf{x}')$
- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels, normalize later (*splatting*)



# Inverse Warping

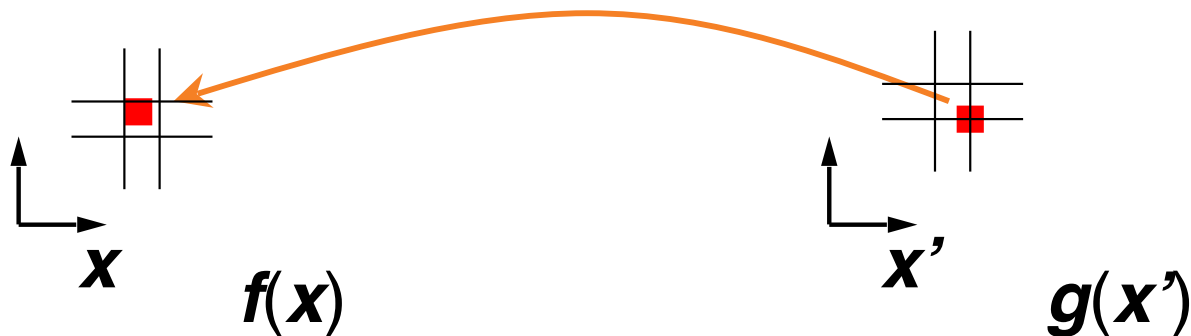
- Get each pixel  $g(\mathbf{x}')$  from its corresponding location  $\mathbf{x} = T^{-1}(\mathbf{x}')$  in  $f(\mathbf{x})$
- What if pixel comes from “between” two pixels?





# Inverse Warping

- Get each pixel  $g(\mathbf{x}')$  from its corresponding location  $\mathbf{x} = T^{-1}(\mathbf{x}')$  in  $f(\mathbf{x})$
- What if pixel comes from “between” two pixels?
- Answer: *resample* color value from *interpolated (prefiltered)* source image



# Interpolation

- Possible interpolation filters:
  - nearest neighbor
  - bilinear
  - bicubic (interpolating)
  - sinc / FIR
- See COS 426 for details on how to avoid “jaggies”



# Feature-Based Alignment

- Find keypoints; compute SIFT descriptors
- Generate candidate keypoint matches
- Use RANSAC to select a subset of matches
- Fit to find best image transformation
- Warp images according to transformation
- **Blend images in overlapping regions**

# Blending

- Blend over too small a region: seams
- Blend over too large a region: ghosting



- COS 426 for details

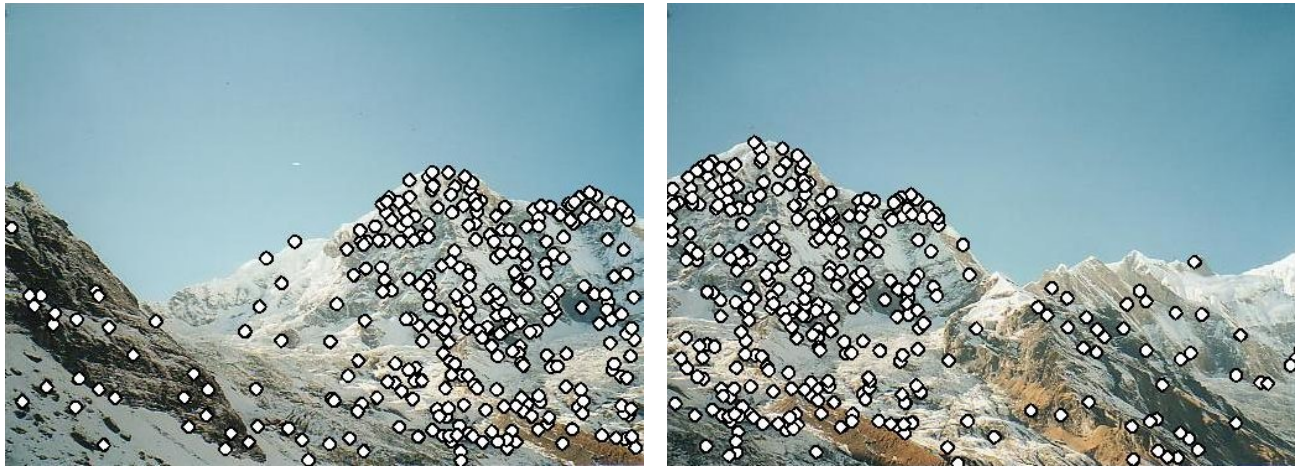
Putting it all together: making a panorama?

---



# Step 1

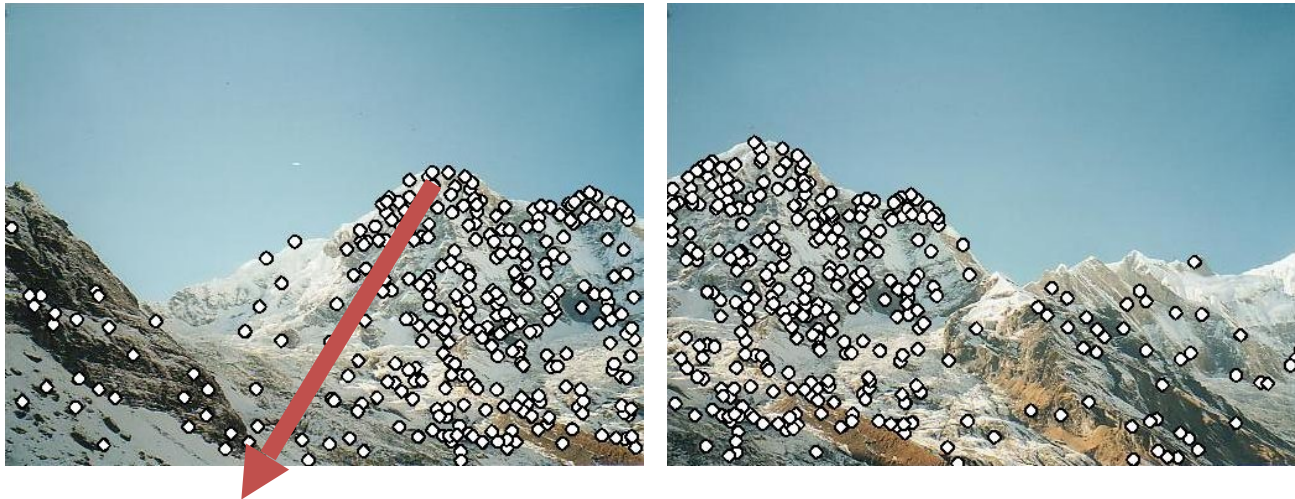
## Find corners/blobs



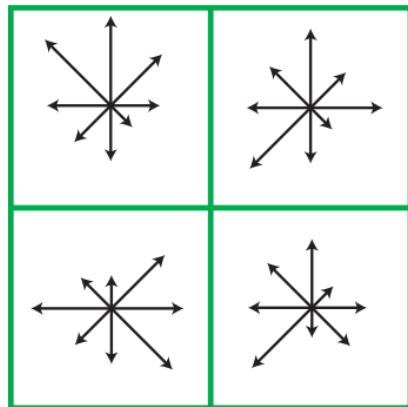
- (Multi-scale) Harris; or
- Laplacian of Gaussian

# Step 2

## Describe Regions Near Features



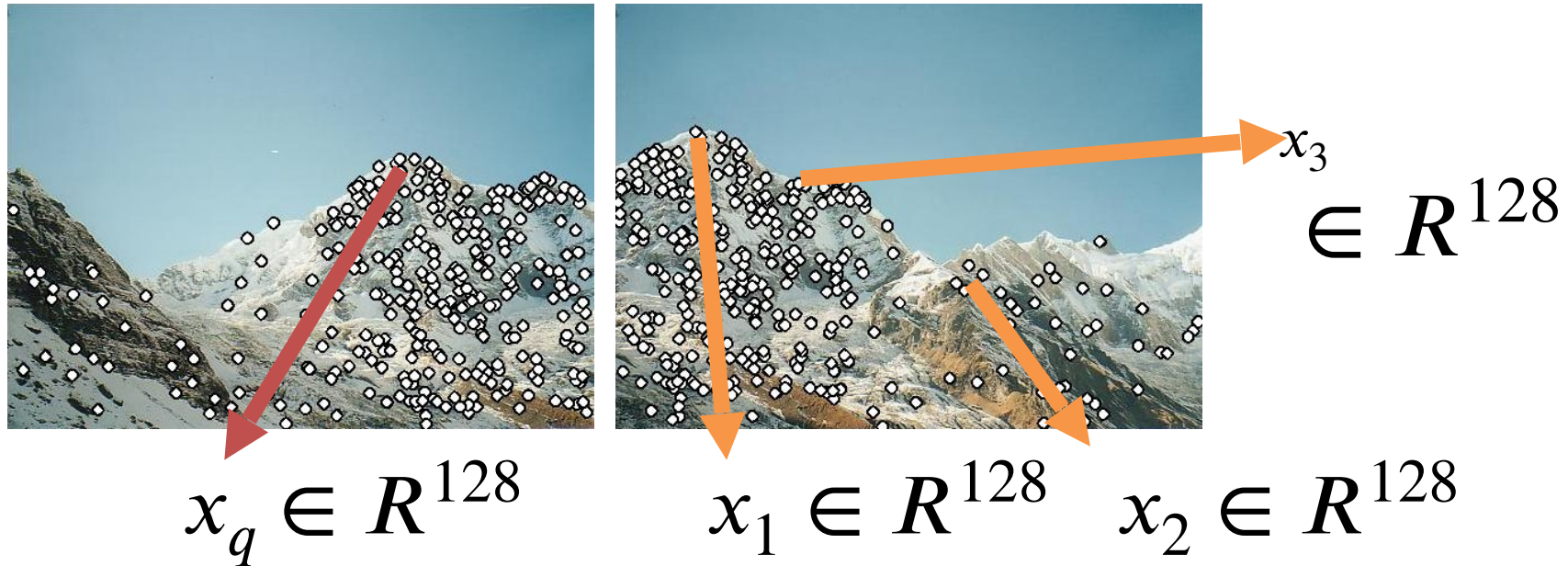
$$x_q \in \mathcal{R}^{128}$$



Build histogram of  
gradient orientations  
(SIFT)

# Step 3

## Match Features Based On Region



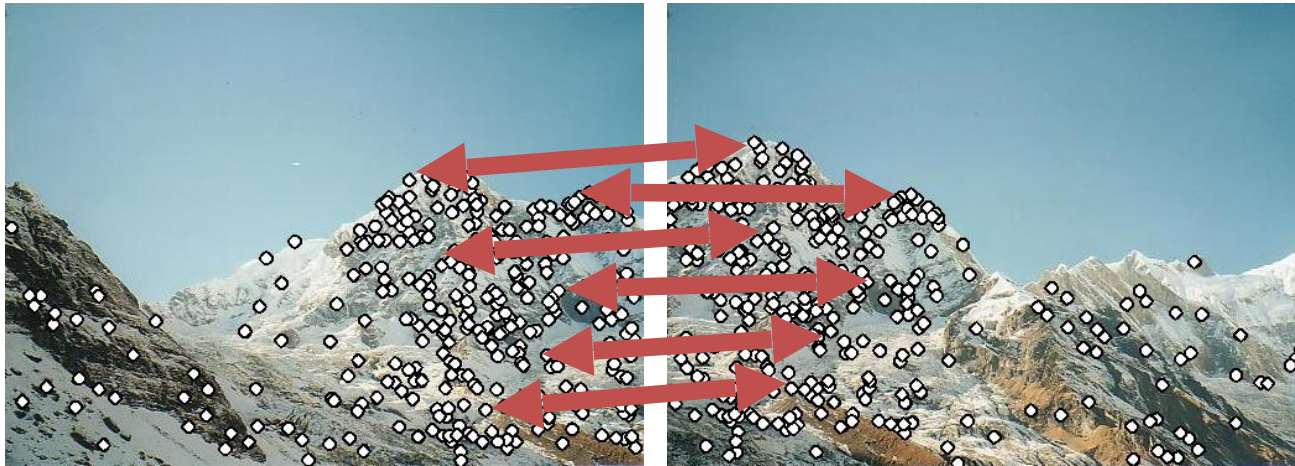
Sort by distance to:  $x_q$   $\|x_q - x_1\| < \|x_q - x_2\| < \|x_q - x_3\|$

Accept match if:  $\|x_q - x_1\| / \|x_q - x_2\|$

Nearest neighbor is far closer than 2<sup>nd</sup> nearest neighbor

# Step 4

## Fit transformation $H$ via RANSAC



for trial in range(Ntrials):  
    Pick sample  
    Fit model  
    Check if more inliers  
    Re-fit model with most inliers



# Step 5

Warp images together



Resample images with inverse warping  
and blend



# Next class: intro to recognition + basics of ML

