

# Lecture 15

## 3D and Stereo

### COS 429: Computer Vision



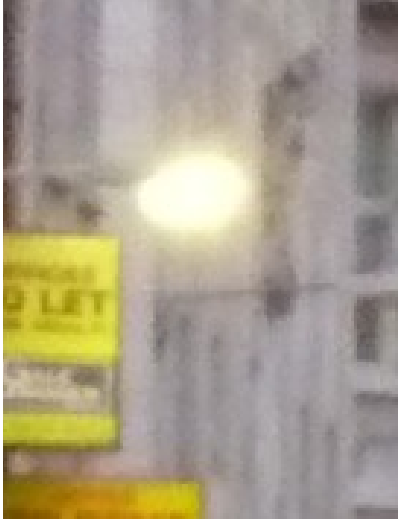
Slides credit:

Many slides adapted from James Hays, Derek Hoiem, Lana Lazebnik, Silvio Saverse, who in turn adapted slides from Steve Seitz, Rick Szeliski, Martial Hebert, Mark Pollefeys, and others



*Meanwhile, in Plato's Cave*

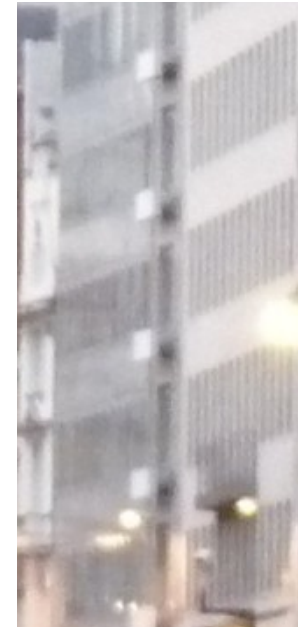
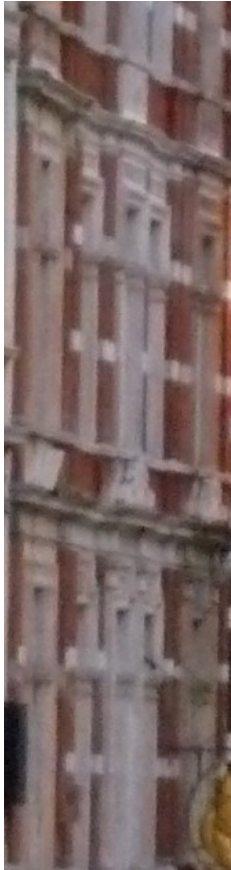
# Which is closer?



# Which is closer?



# The world is 3D



# The world is 3D



# Computer Vision: infer 3D from 2D



# Why bother with 3D?

The world is 3D

Compact representation of relationships

Ability to navigate & manipulate

Some 2D vision problems are easier in 3D

Occlusion

Variation with lighting

Variation with viewpoint

Segmentation

Recognition



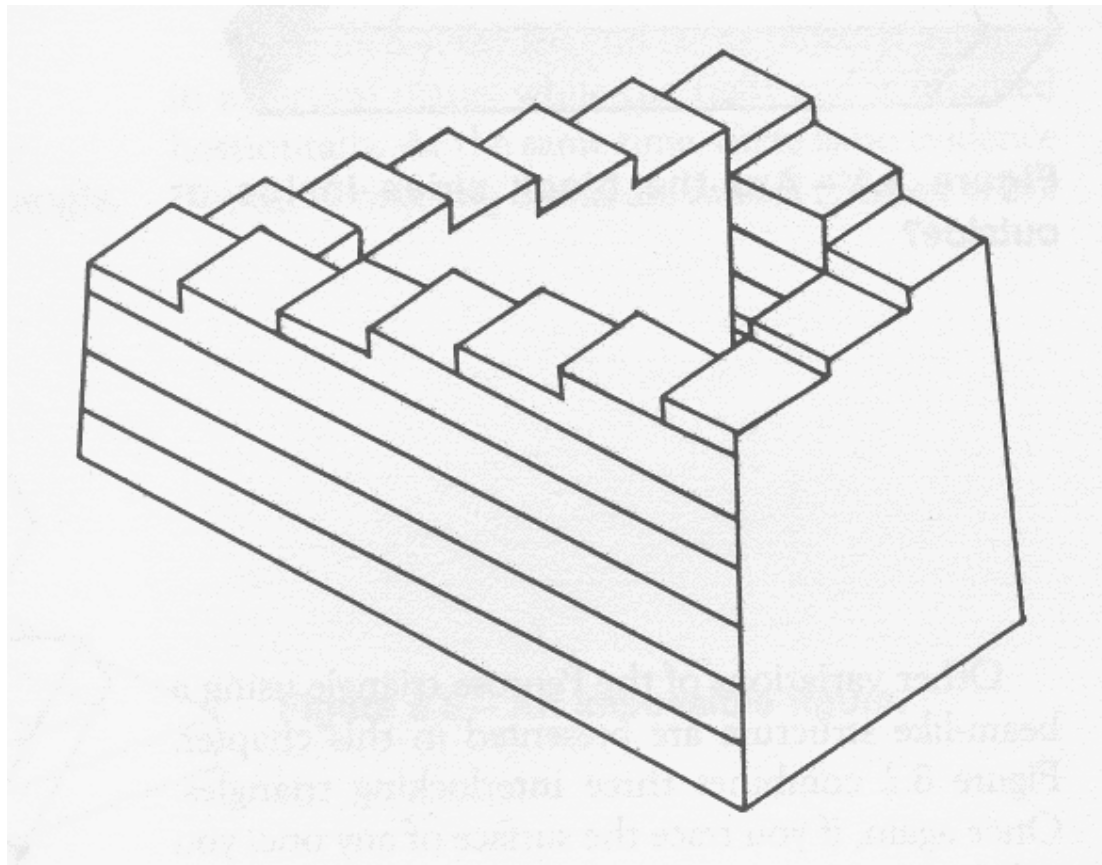
# Computer Vision: infer 3D from 2D



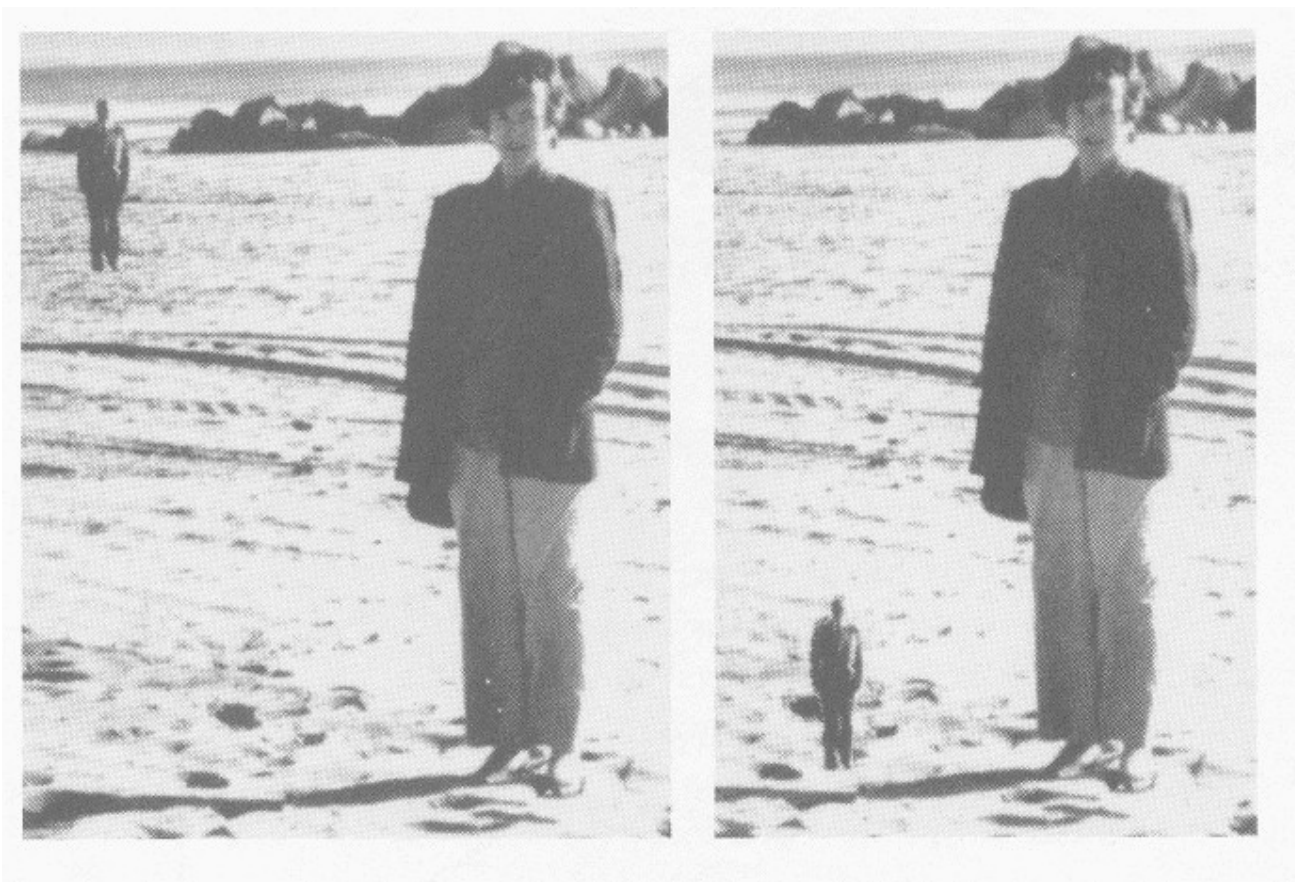
# Monocular Image Depth Cues

- Occlusion (Interposition)
  - Near surfaces overlapping far ones
- Perspective
  - Parallel lines converging in the distance
- Texture gradient
  - Statistics of texture change (more details nearby)
- Size (relative, familiar, absolute)
  - Smaller objects, especially when known, appear farther
- Relative Position (Elevation)
  - Higher object tend to be farther
- Focus
  - Some depths are less in focus (could be near or far)
- In-Scattering (haze)
  - Far object have lower contrast (look hazy)
- ...

# 3D Perception: Illusions



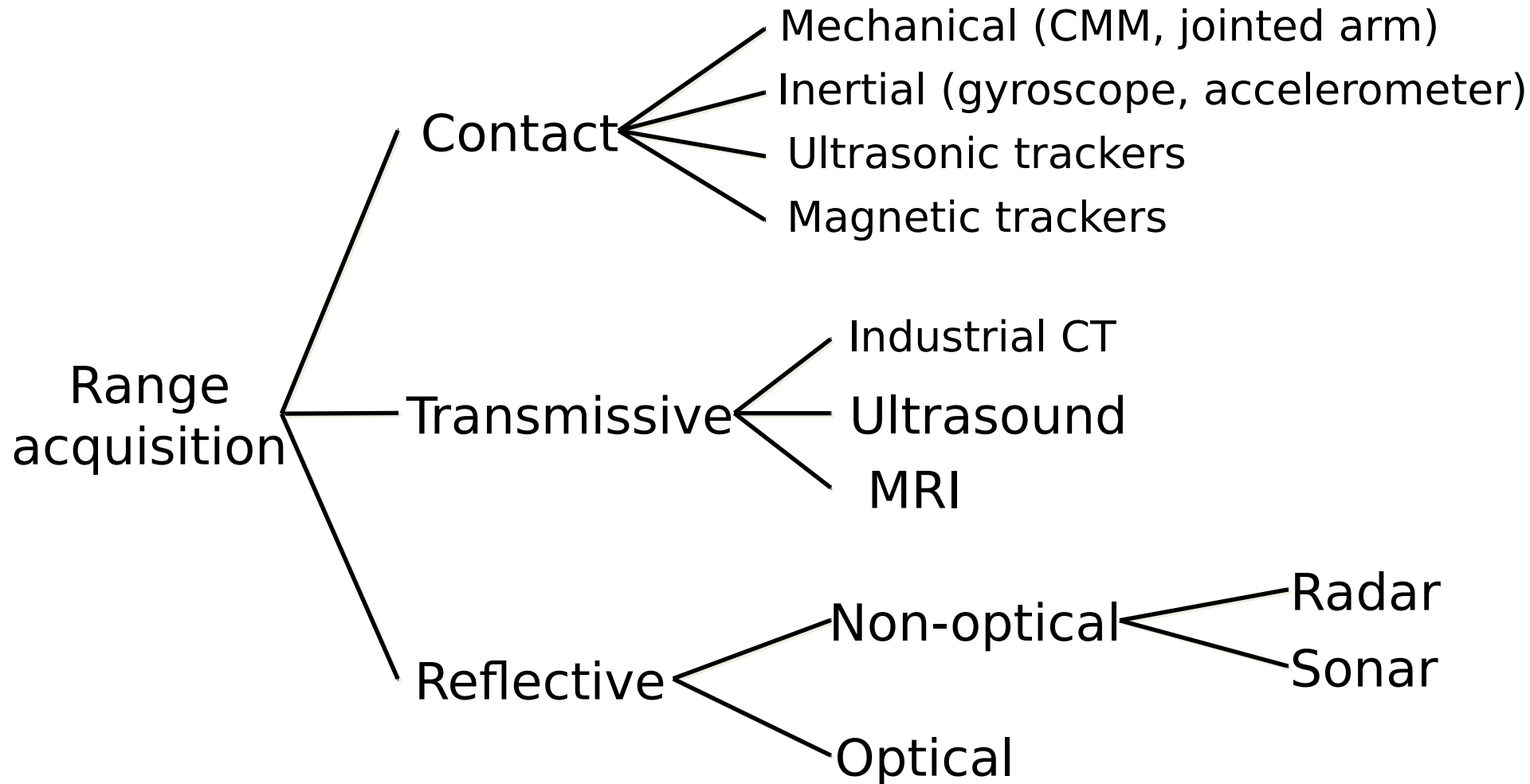
# 3D Perception: Illusions



# 3D Perception: Illusions



# Range Acquisition Taxonomy



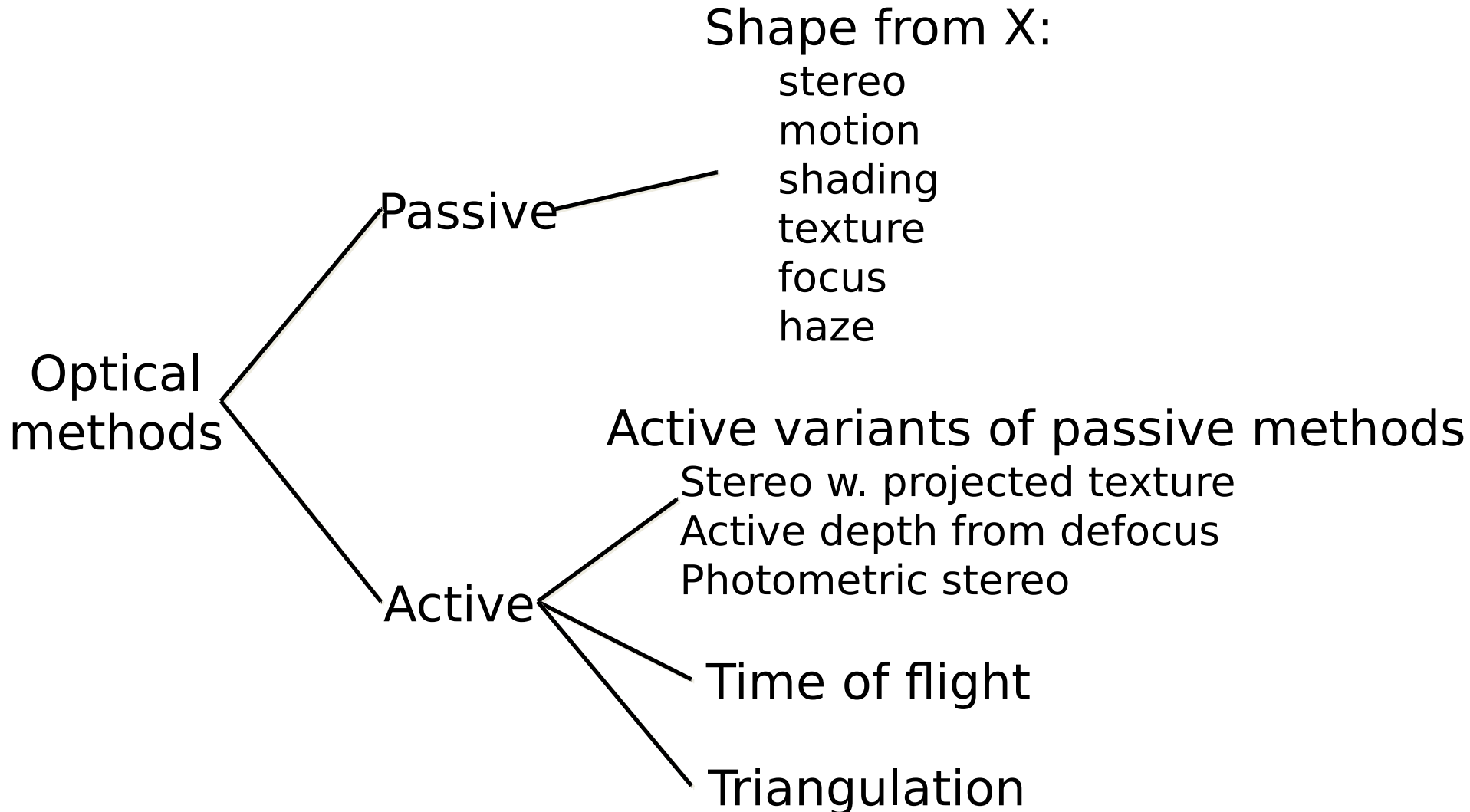
# Touch Probes

- Jointed arms with angular encoders
- Return position, orientation of tip



Faro Arm – Faro Technologies, Inc.

# Range Acquisition Taxonomy

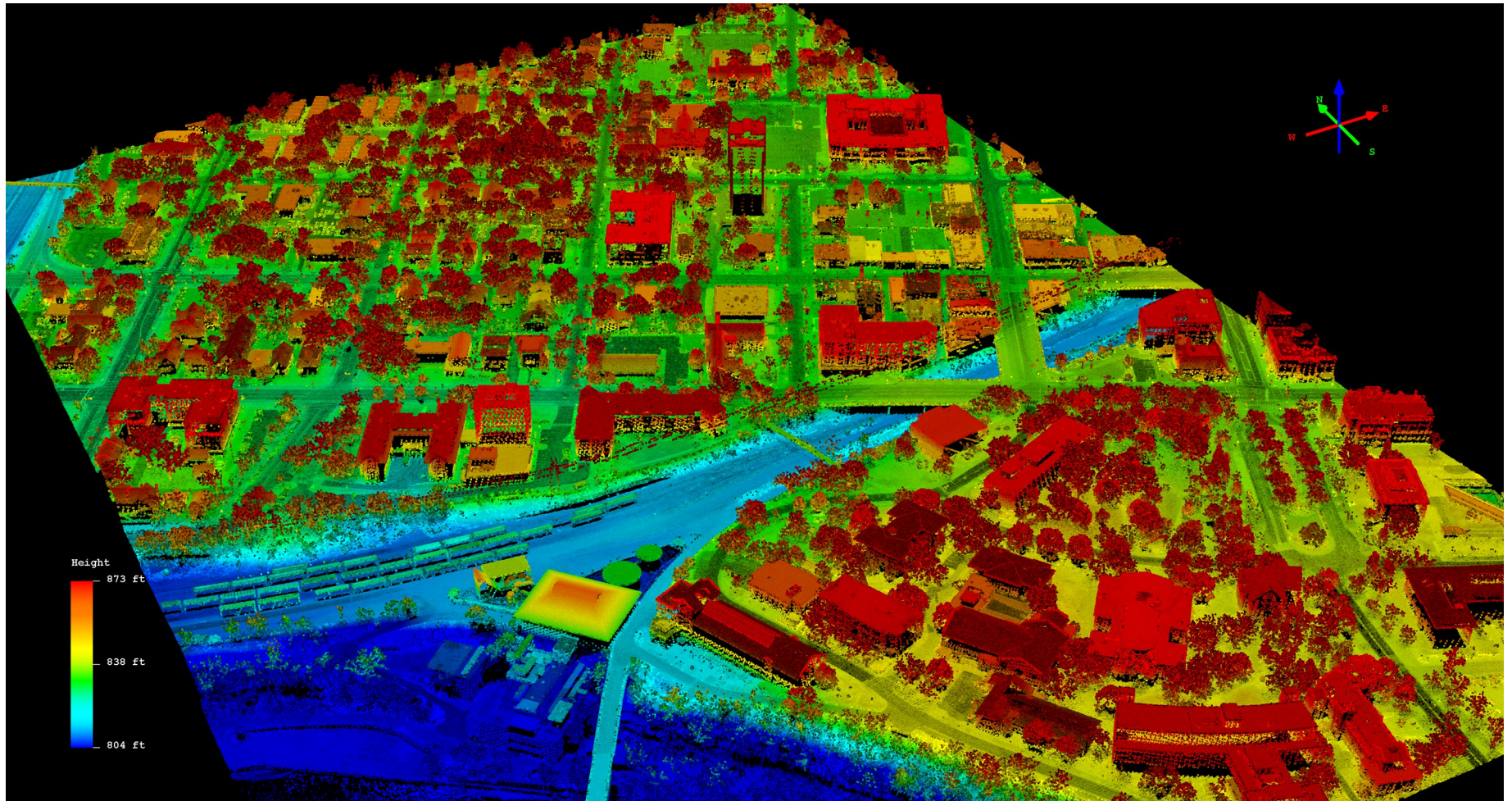




# Lidar



# Lidar



# Optical Range Acquisition

- Advantages:
  - Non-contact
  - Safe
  - Usually fast
- Disadvantages:
  - Sensitive to transparency
  - Confused by specularities and interreflections
  - Texture (helps some methods, hurts others)

# Passive Optical Range Acquisition

- Advantages:
  - Very Dense (high resolution)
  - Does not interfere with environment
  - Inexpensive
- Disadvantages:
  - Heavy Processing (CPU time)
  - Only works on textured regions
  - Depth accuracy depends on baseline

# 3D Data Types

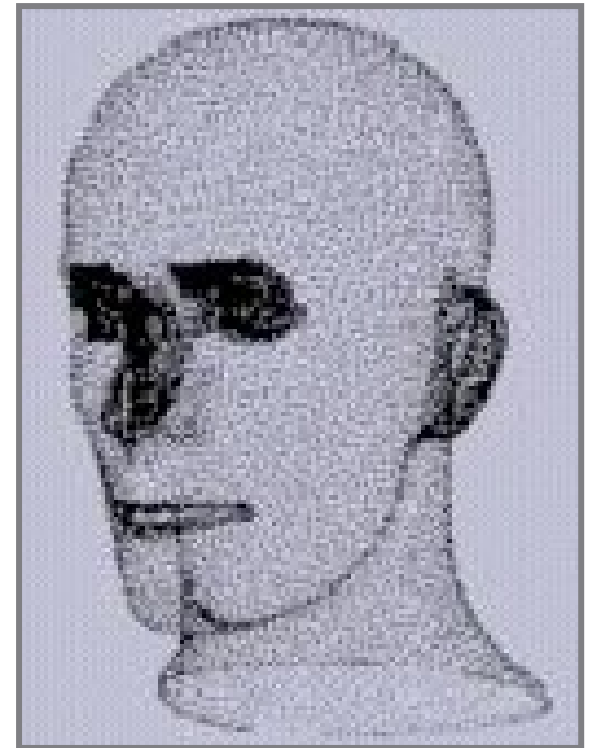
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How do we represent the 3D world?

- Point Data
- Volumetric Data
- Surface Data

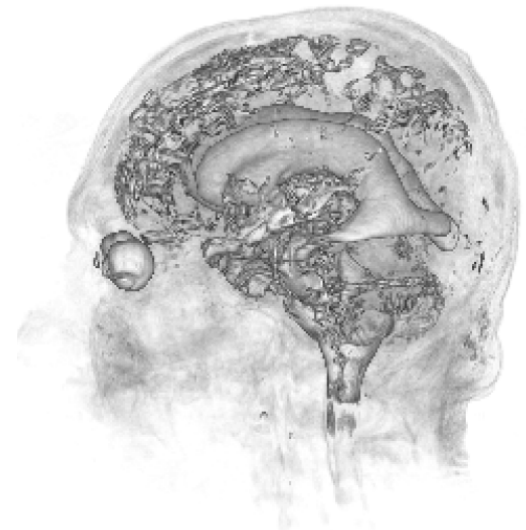
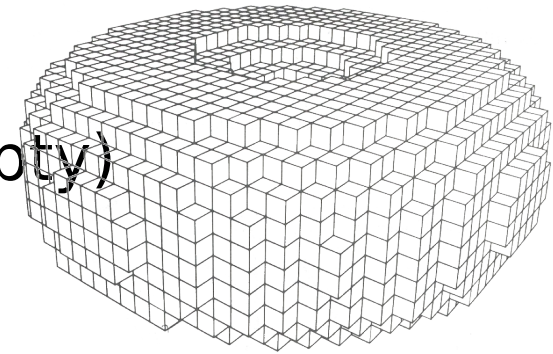
# 3D Data Types: Point Data

- “Point clouds”
- Advantage: simplest data type
- Disadvantage: no information adjacency / connectivity



# 3D Data Types: Volumetric Data

- Regularly-spaced grid in  $(x,y,z)$ : “voxels”
- For each grid cell, store
  - Occupancy (binary: occupied / empty)
  - Density
  - Other properties
- Popular in medical imaging
  - CAT scans
  - MRI



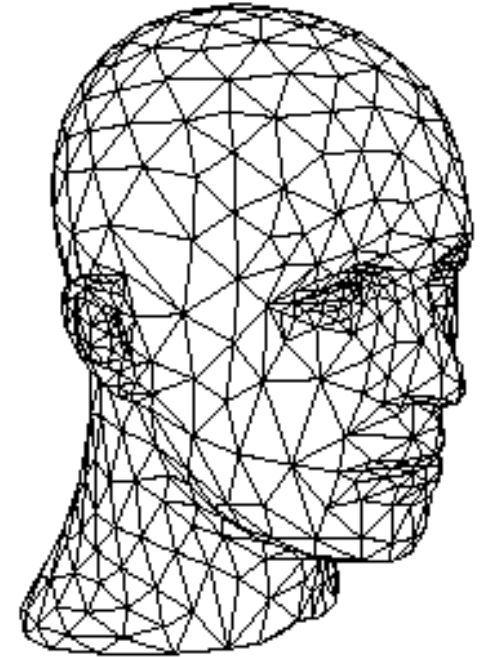
# 3D Data Types: Volumetric Data

- Advantages:
  - Can represent inside of object
  - Uniform sampling: simpler algorithms
- Disadvantages:
  - Lots of data
  - Wastes space if only storing a surface
  - Most “vision” sensors / algorithms return point or surface data



# 3D Data Types: Surface Data

- Polyhedral
  - Piecewise planar
  - Polygons connected together
  - Most popular: “triangle meshes”
- Smooth
  - Higher-order (quadratic, cubic, etc.) curves
  - Bézier patches, splines, NURBS, subdivision surfaces, etc.
  - See COS 426 for details...



# 3D Data Types: Surface Data

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- Advantages:
  - Usually corresponds to what we see
  - Usually returned by vision sensors / algorithms
- Disadvantages:
  - How to find “surface” for translucent objects?
  - Parameterization often non-uniform
  - Non-topology-preserving algorithms difficult

# 2½-D Data

- Image: stores an intensity / color along each of a set of regularly-spaced rays in space
- **Range image:** stores a **depth** along each of a set of regularly-spaced rays in space
- Not a complete 3D description: does not store objects occluded (from some viewpoint)
- View-dependent scene description

# 2<sup>1</sup>/<sub>2</sub>-D Data

- This is what most sensors / algorithms really return
- Advantages
  - Uniform parameterization
  - Adjacency / connectivity information
- Disadvantages
  - Does not represent entire object
  - View dependent

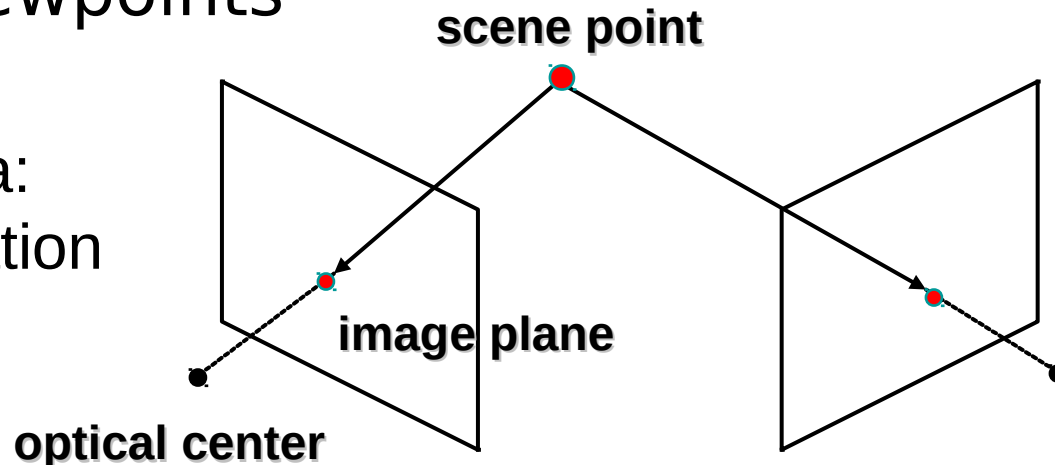
# 2½-D Data

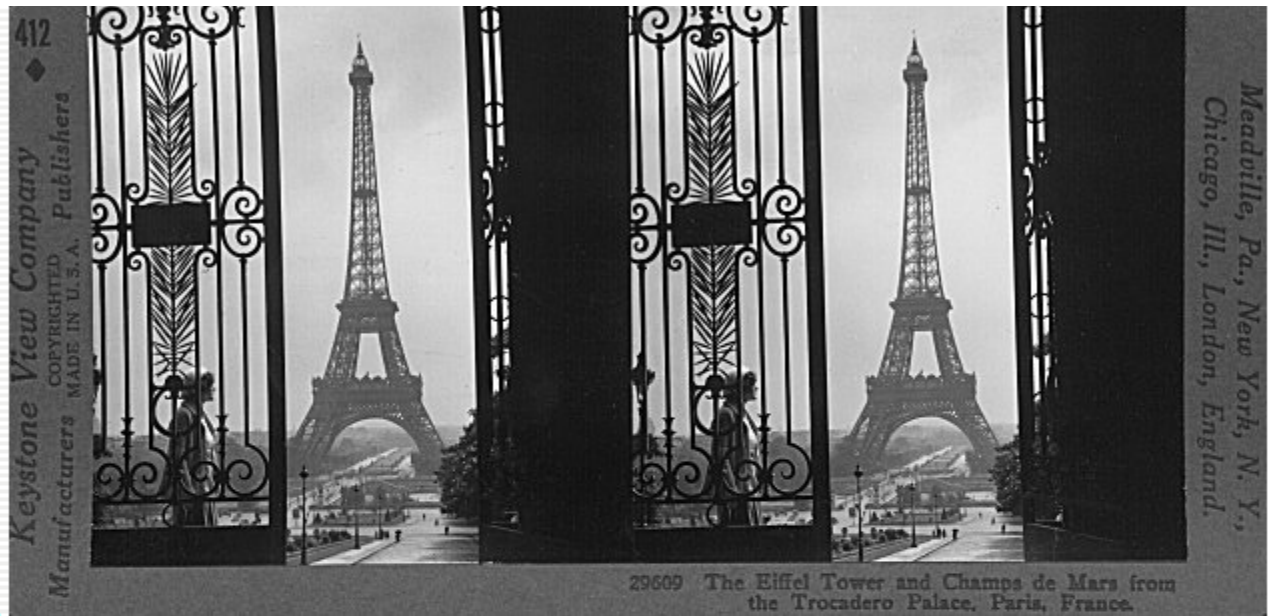
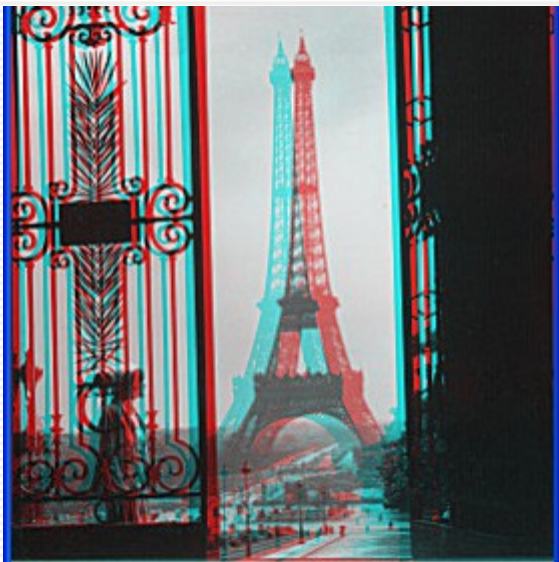
- RGBD
- Range images
- Range surfaces
- Depth images
- Depth maps
- Height fields
- 2½-D images
- Surface profiles
- xyz maps
- ...

# Stereo Image Matching

- Passive Optical Depth Methods (aka. “Shape from X”): Shading, Texture, Focus, Motion...
- **Stereo:**
  - shape from “motion” between two views
  - infer 3d shape of scene from two (multiple) images from different viewpoints

Main idea:  
Triangulation





© Copyright 2001 Johnson-Shaw Stereoscopic Museum

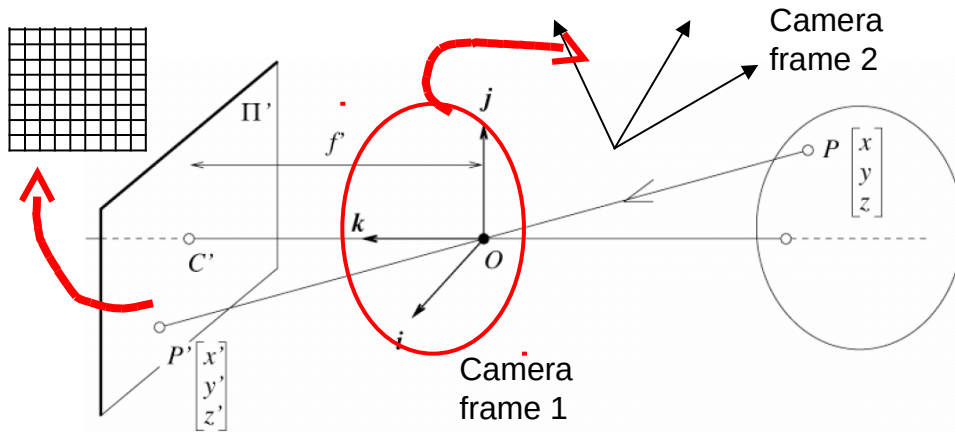
<http://www.johnsonshawmuseum.org>

Public Library, Stereoscopic Looking Room,  
Chicago, by Phillips, 1923





# Camera parameters



**Extrinsic** parameters:

Camera frame 1  $\leftrightarrow$  Camera frame 2

**Intrinsic** parameters:

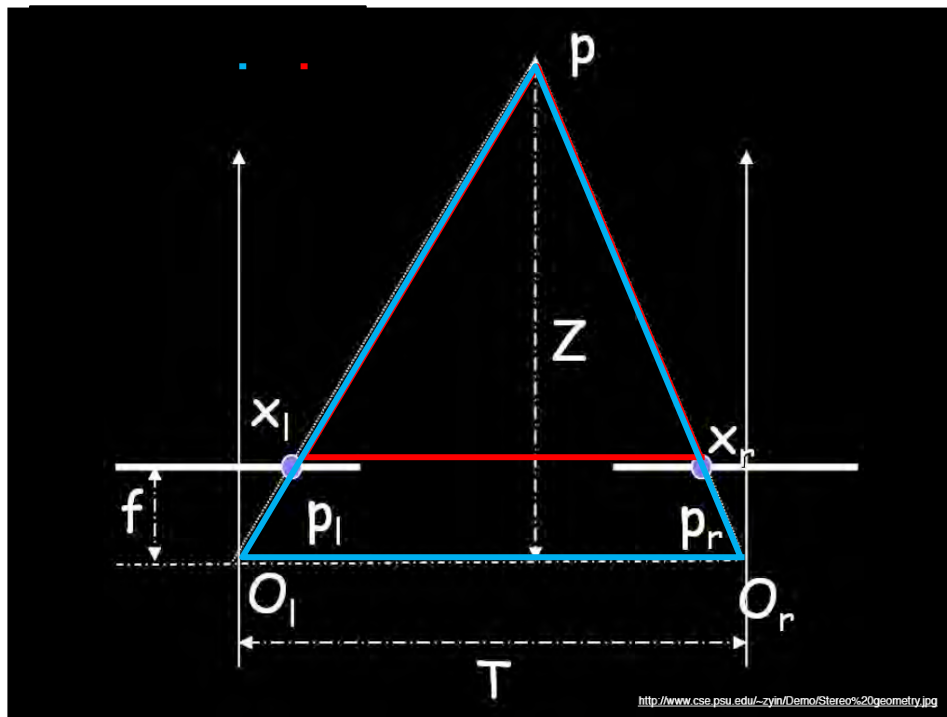
Image coordinates relative to camera  $\leftrightarrow$  Pixel coordinates

- *Extrinsic* params: rotation matrix and translation vector
- *Intrinsic* params: focal length, pixel sizes (mm), image center point, radial distortion parameters

*We'll assume for now that these parameters are given and fixed.*

# Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is**



Similar triangles  $(p_l, P, p_r)$  and  $(O_l, P, O_r)$ :

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f \frac{T}{x_r - x_l}$$

disparity  $\rightarrow$   $x_r - x_l$

# Depth from disparity

image  $I(x,y)$



Disparity map  $D(x,y)$

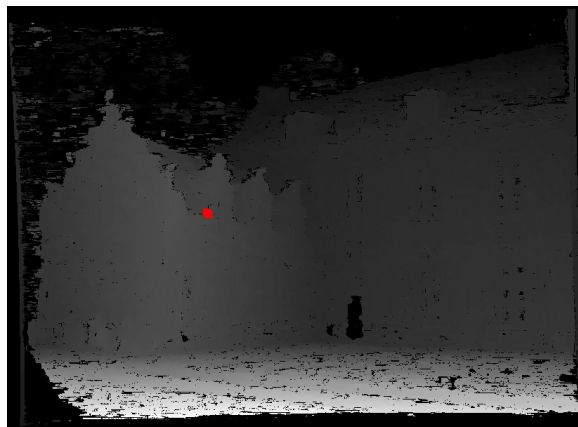


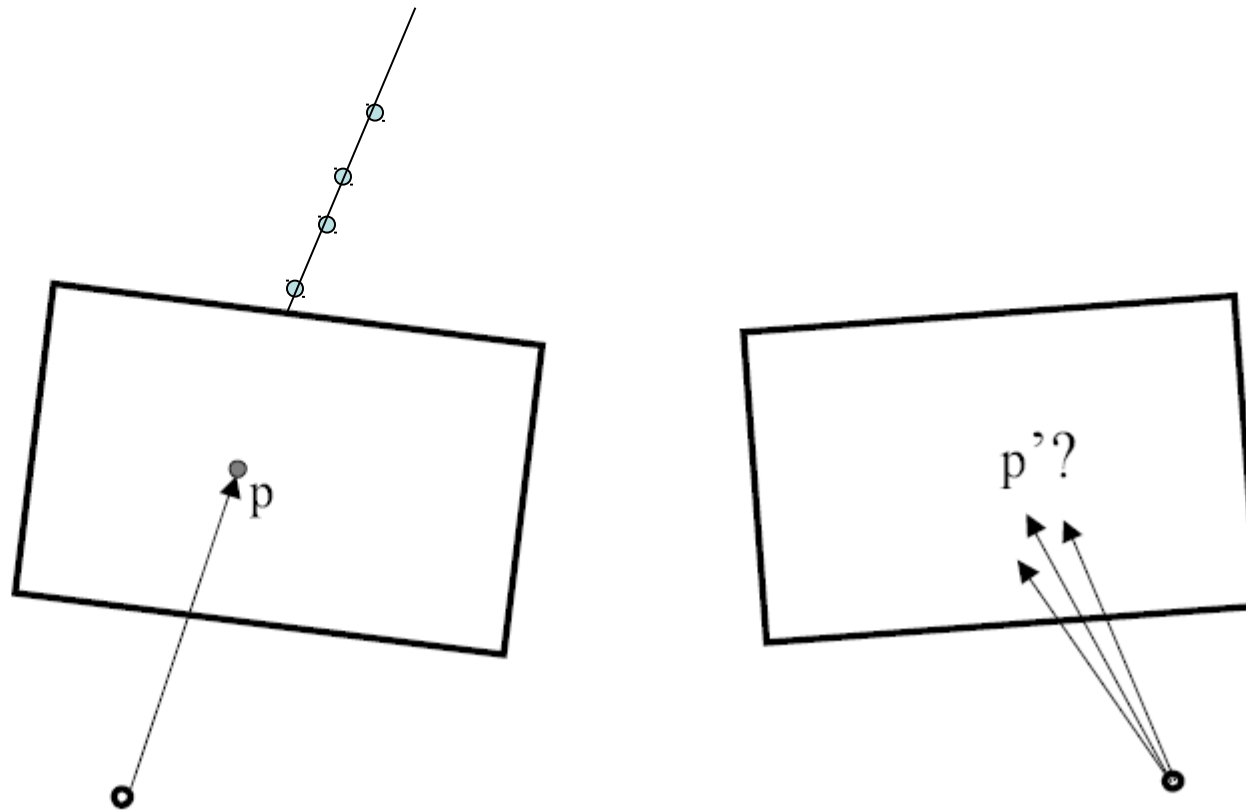
image  $I'(x',y')$



$$(x',y')=(x+D(x,y), y)$$

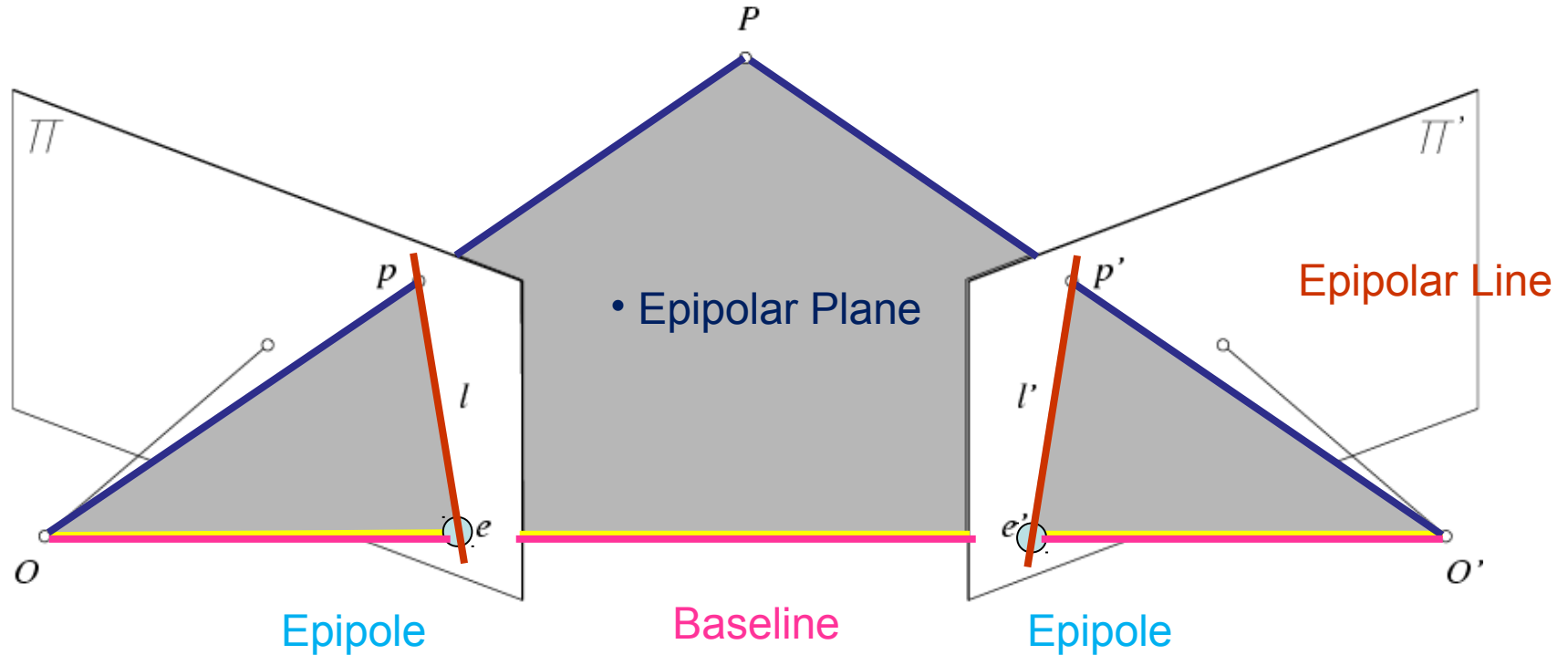
So if we could find the **corresponding points** in two images, we could **estimate relative depth**...

# Stereo correspondence constraints



- Given  $p$  in left image, where can corresponding point  $p'$  be?

# Epipolar geometry

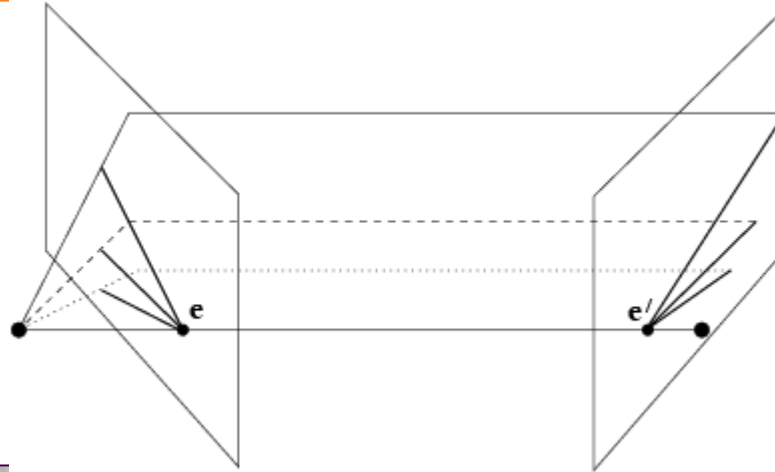


<http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html>

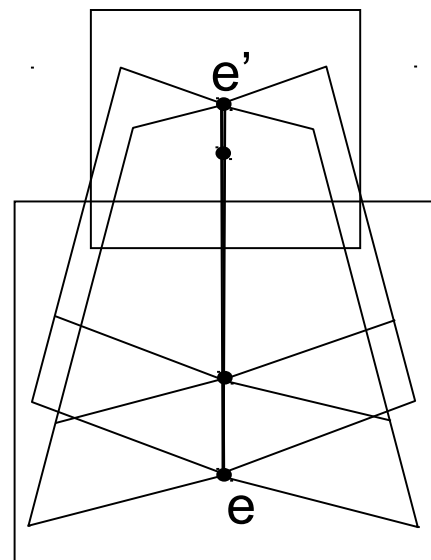
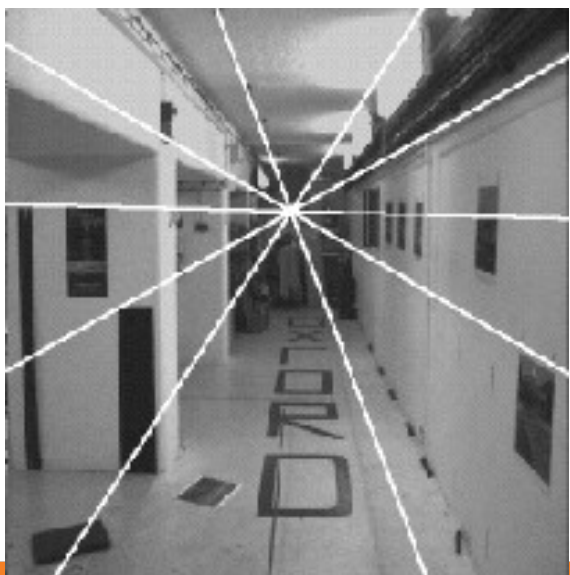
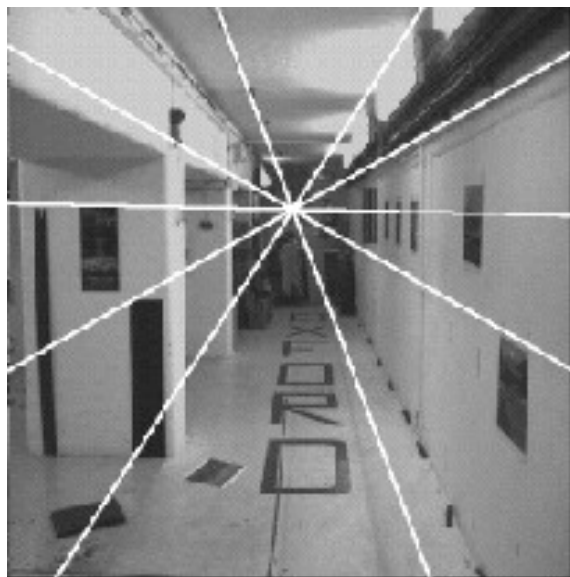
# Example



# Example: converging cameras



# Example: Forward motion



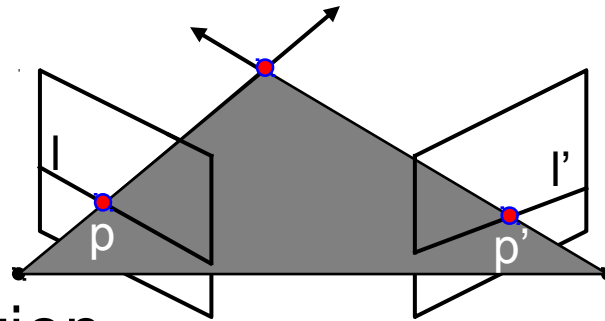
Epipole has same coordinates in both images.

Points move along lines radiating from  $e$ :  
“Focus of expansion”



# Fundamental matrix

Let  $p$  be a point in left image,  $p'$  in right image



Epipolar relation

- $p$  maps to epipolar line  $l'$
- $p'$  maps to epipolar line  $l$

Epipolar mapping described by a 3x3 matrix  $F$

$$l' = Fp$$

$$l = p'F$$

It follows that

$$p'Fp = 0$$

# Fundamental matrix

This matrix  $F$  is called

- the “Essential Matrix”
  - when image intrinsic parameters are known
- the “Fundamental Matrix”
  - more generally (uncalibrated case)

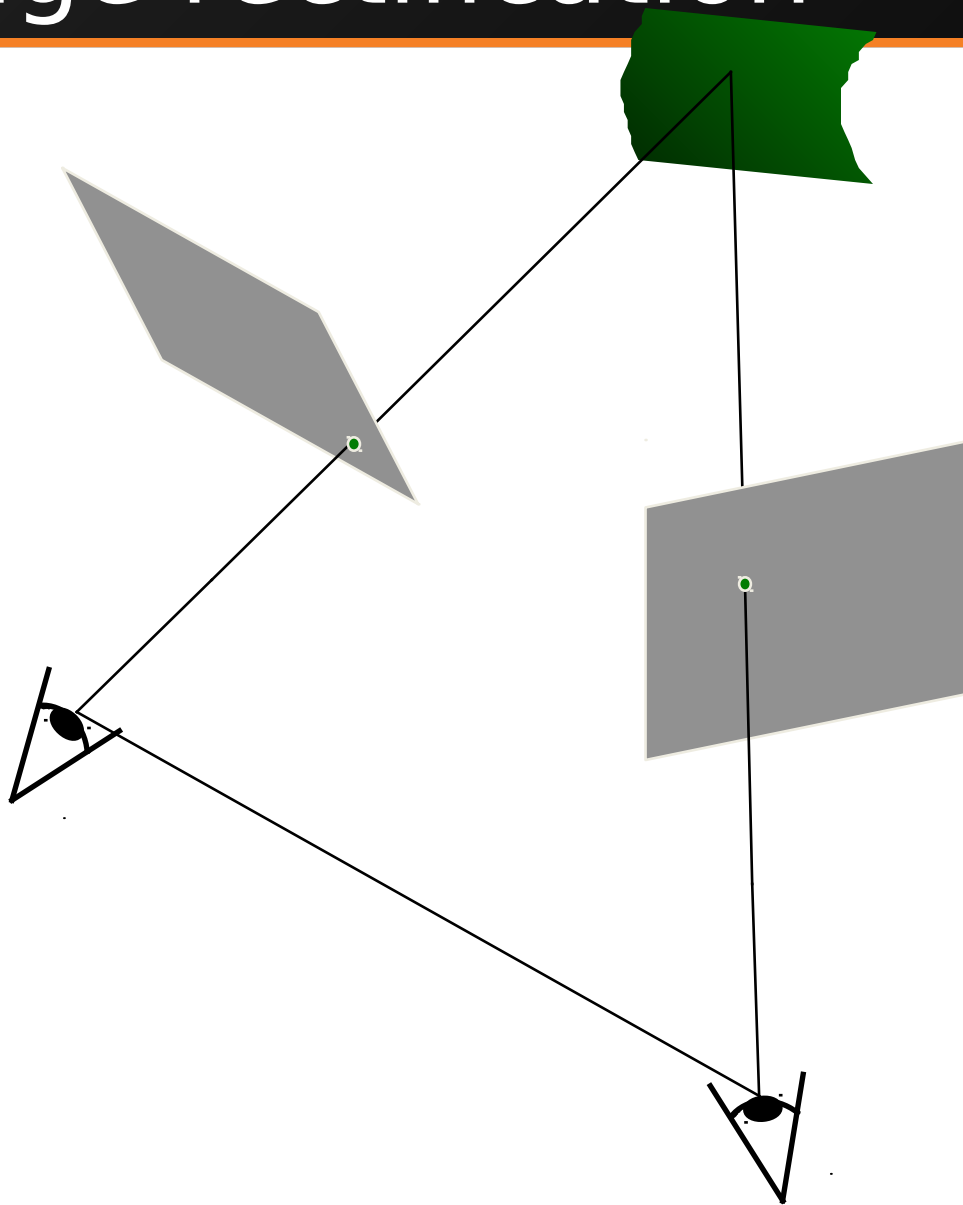
Can solve for  $F$  from point correspondences

- Each  $(p, p')$  pair gives one linear equation in entries of  $F$

$$p' F p = 0$$

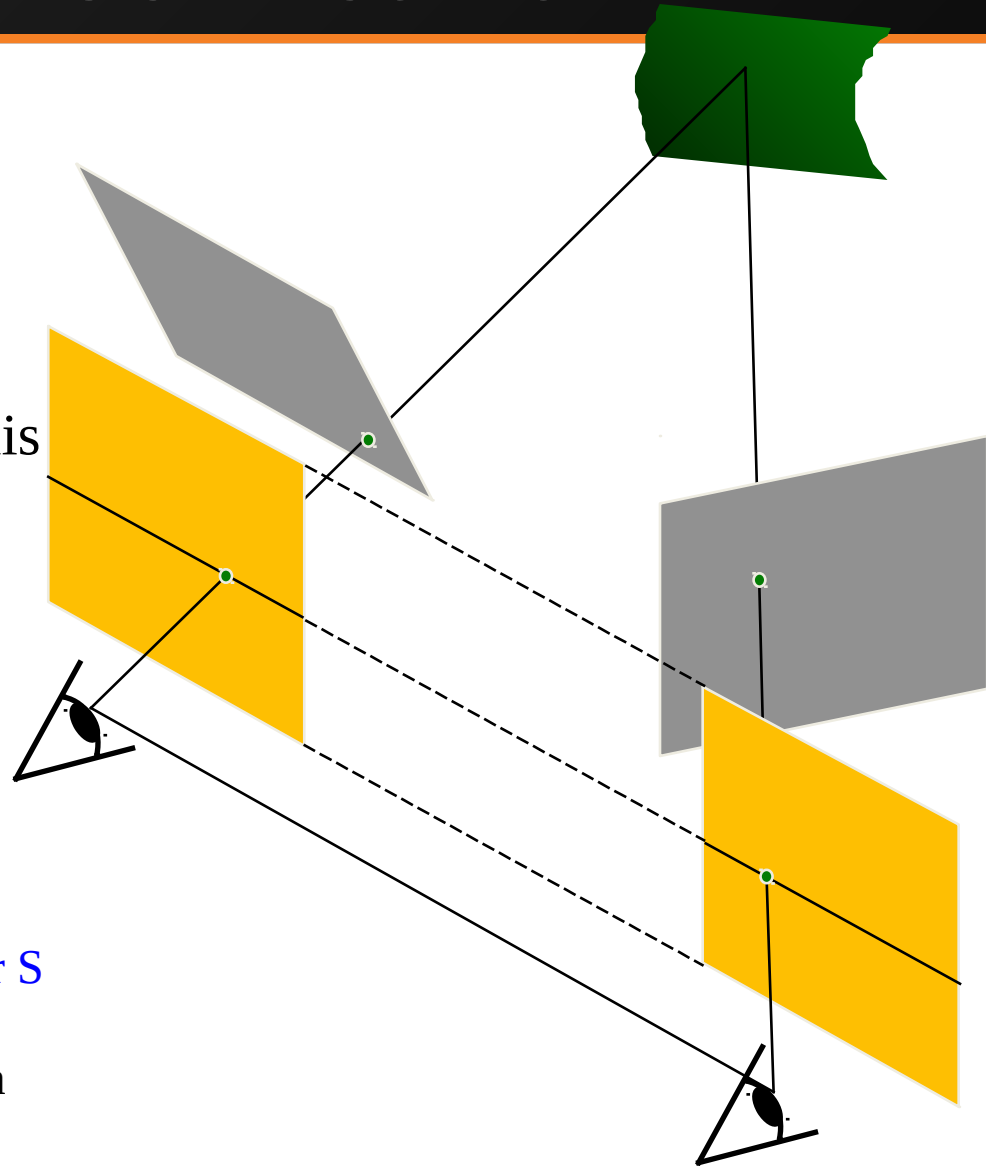
- 8 points give enough to solve for  $F$  (8-point algorithm)
- see [Marc Pollefeys's notes](#) for a nice tutorial

# Stereo image rectification

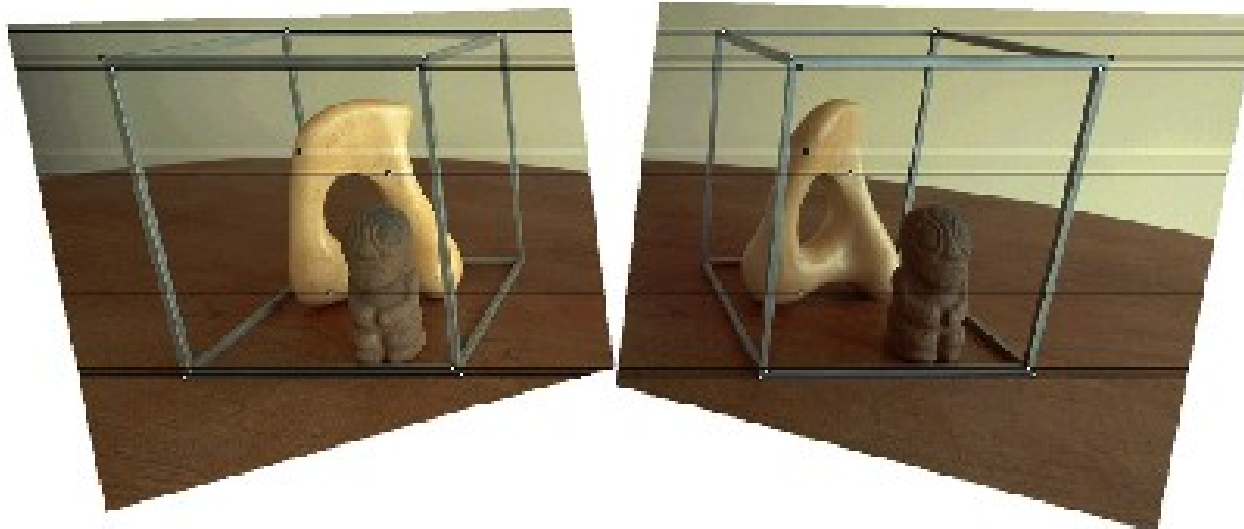
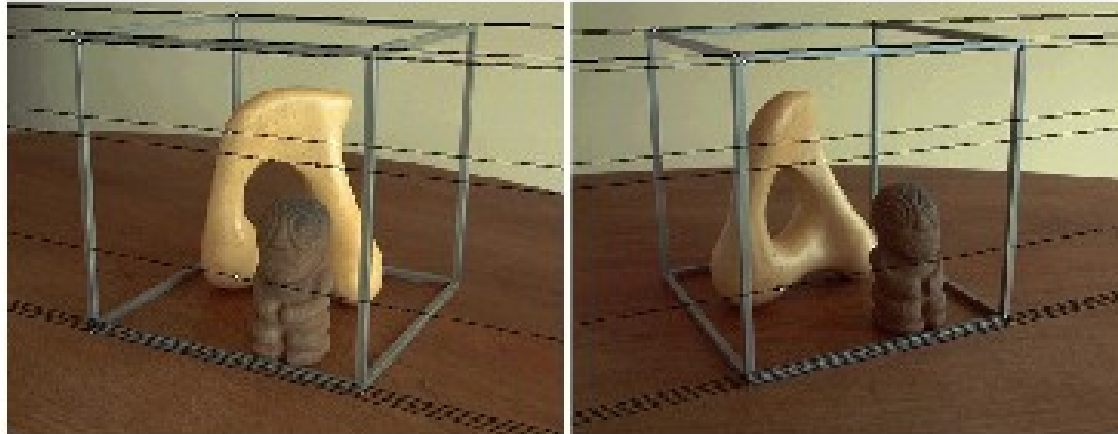


# Stereo image rectification

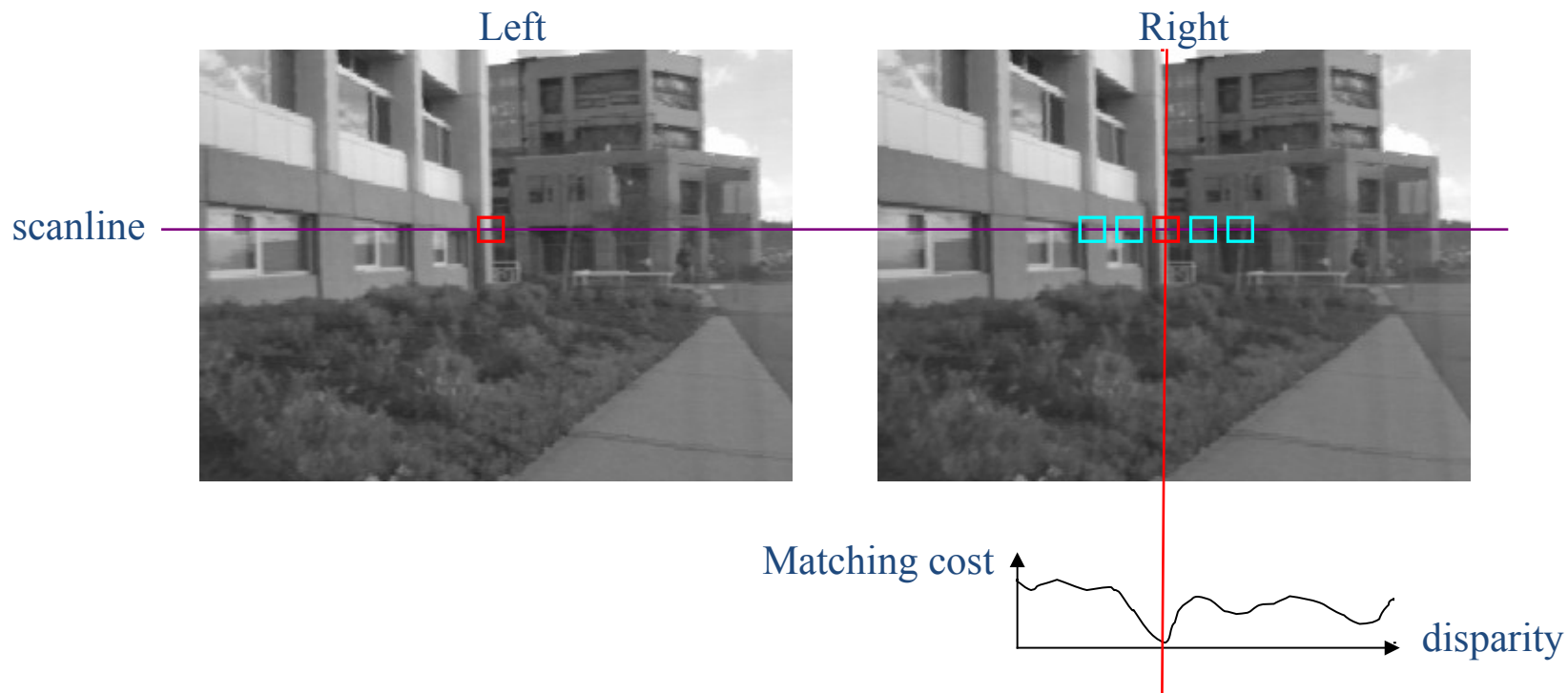
- Reproject image planes onto a common plane parallel to the line between camera centers
  - Pixel motion is horizontal after this transformation
  - Two homographies (3x3 transform), one for each input image reprojection
- C. Loop and Z. Zhang.  
[Computing Rectifying Homographies for Stereo Vision](#)  
IEEE Conf. Computer Vision and Pattern Recognition, 1999.



# Rectification



# Correspondence search

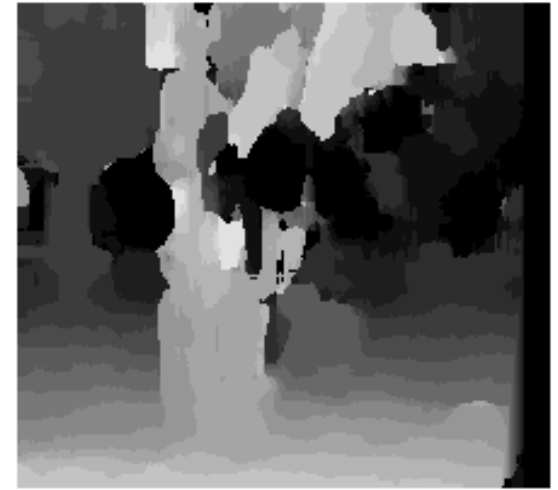


- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

# Effect of window size



$W = 3$



$W = 20$

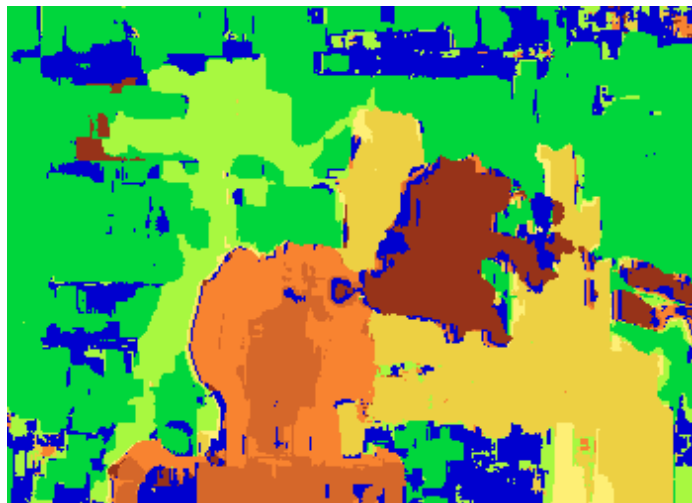
- Smaller window
  - + More detail
  - More noise
- Larger window
  - + Smoother disparity maps
  - Less detail

# Results with window search

Data



Window-based matching



Ground truth



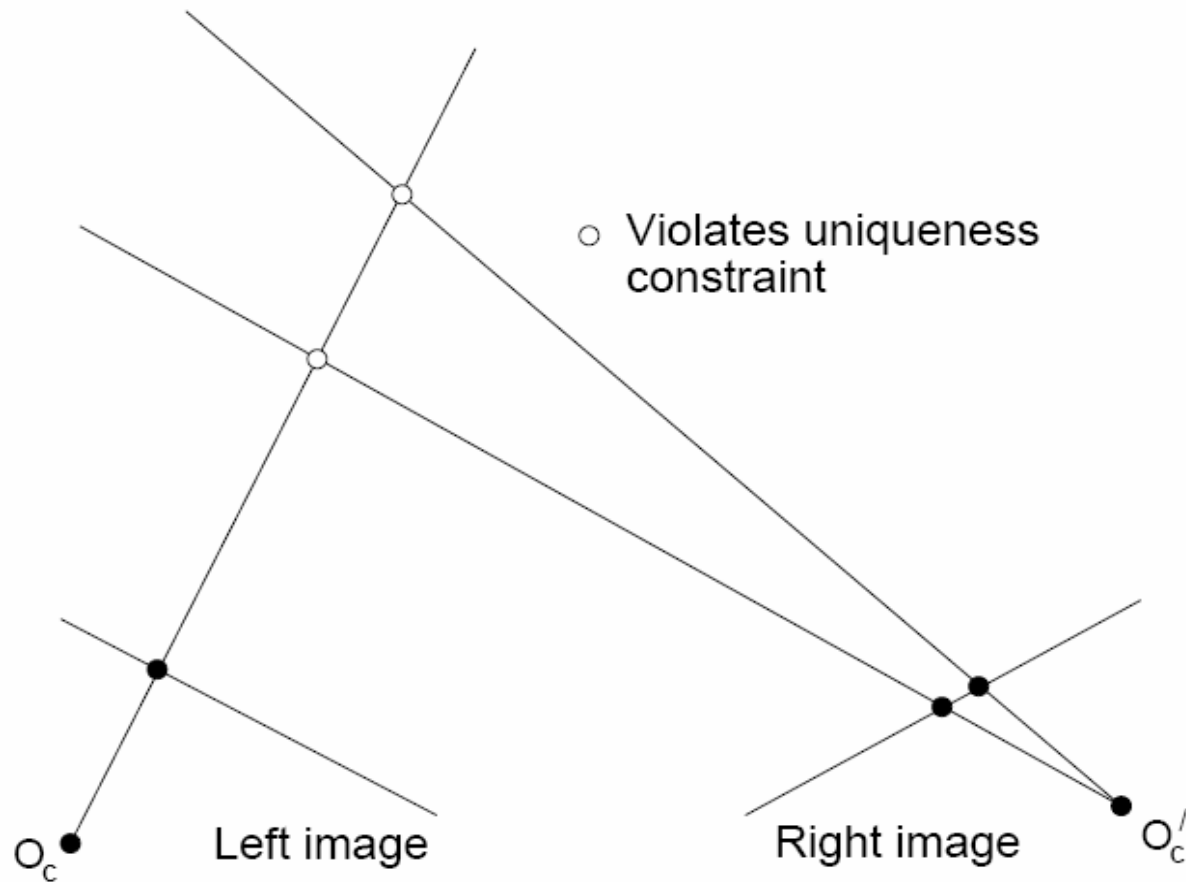


# How can we improve window-based matching?

- So far, matches are independent for each point
- What constraints or priors can we add?

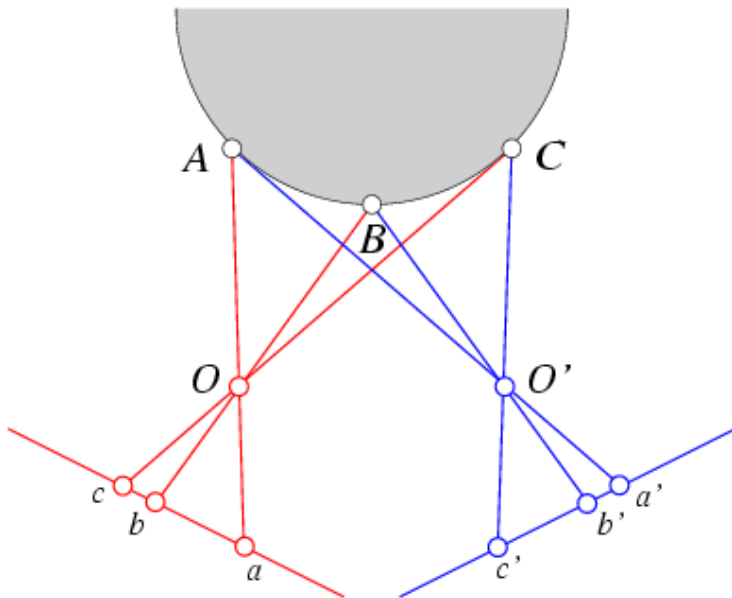
# Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image



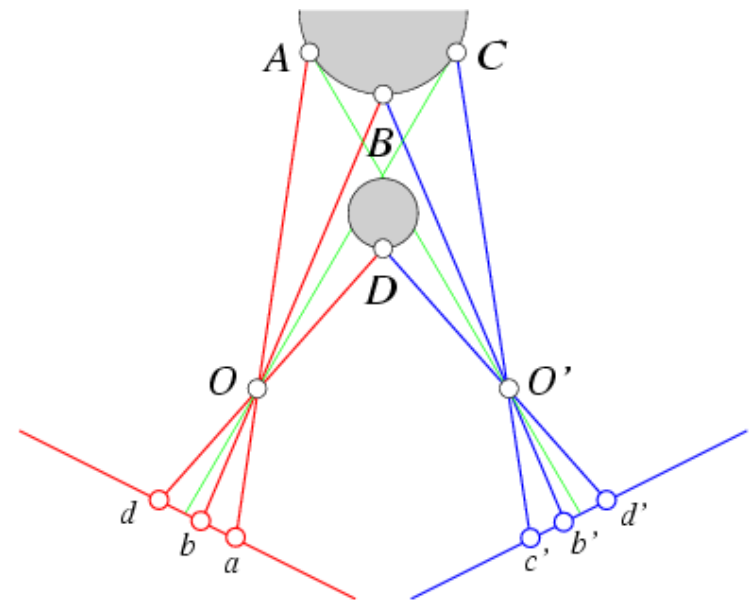
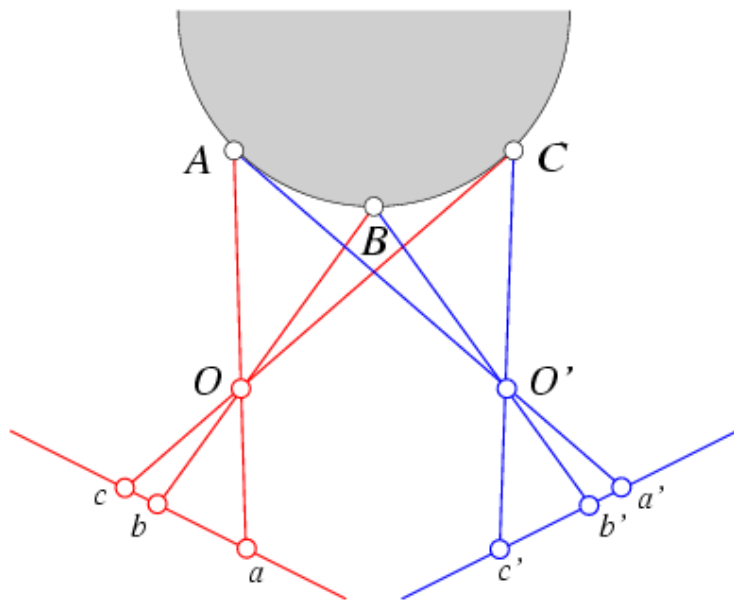
# Stereo constraints/priors

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views



# Stereo constraints/priors

- Uniqueness
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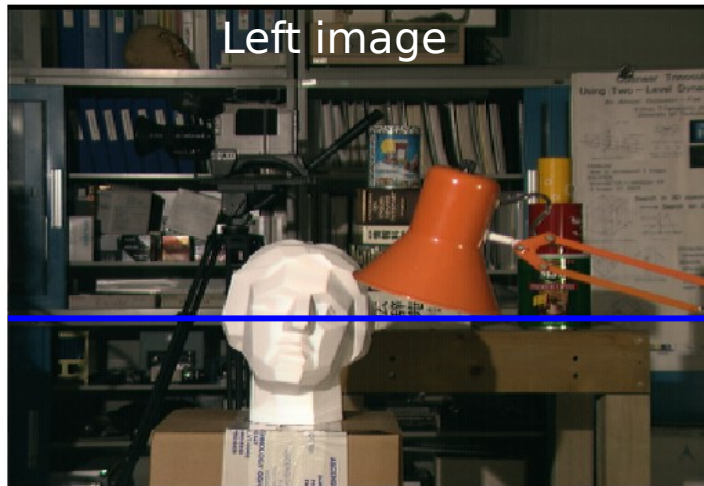
Ordering constraint doesn't hold

# Priors and constraints

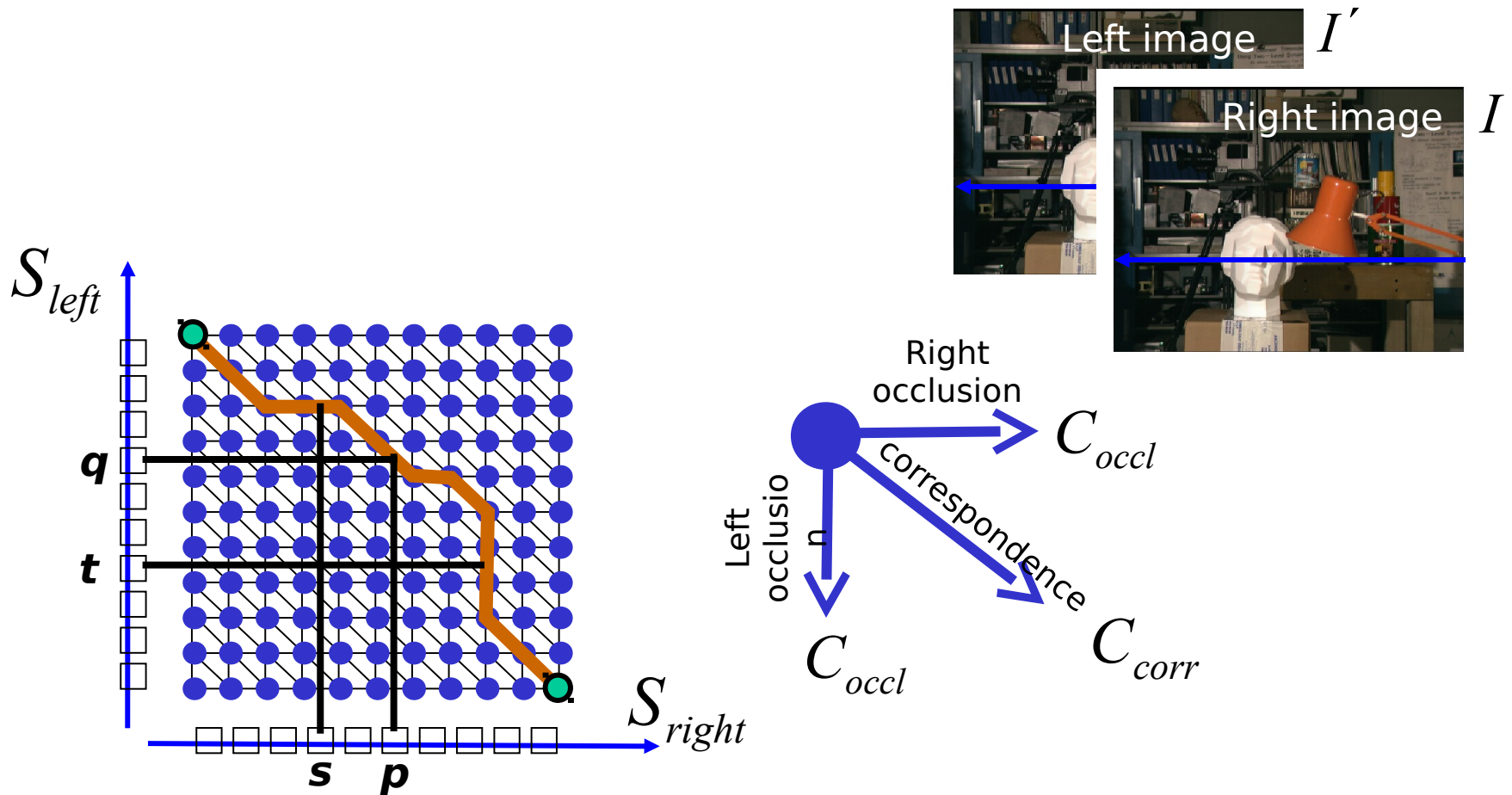
- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - We expect disparity values to change slowly (for the most part – with a small sparse set of discontinuities)

# Scanline stereo

- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently



# “Shortest paths” for scan-line stereo

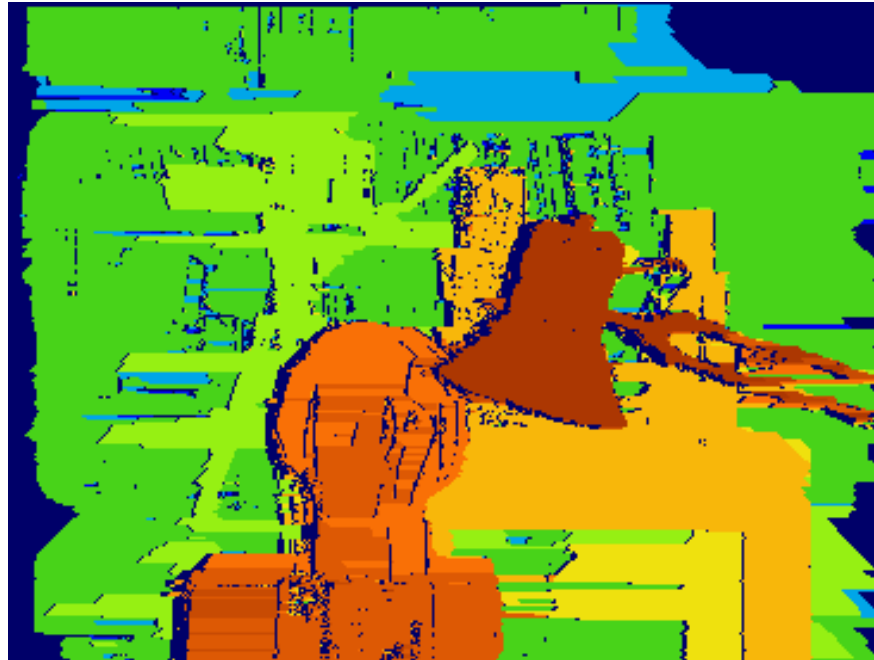


Can be implemented with dynamic programming

Ohta & Kanade '85, Cox et al. '96

## Coherent stereo on 2D grid

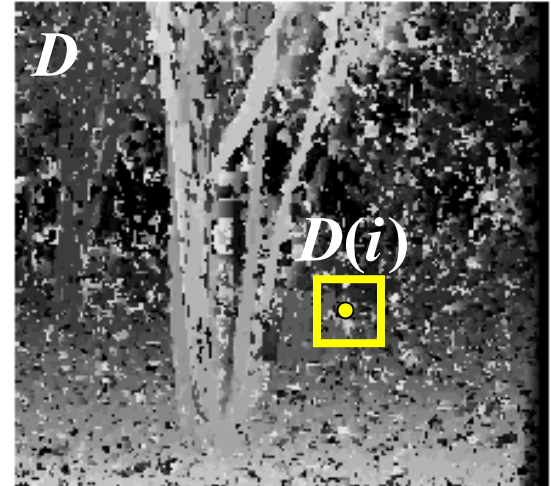
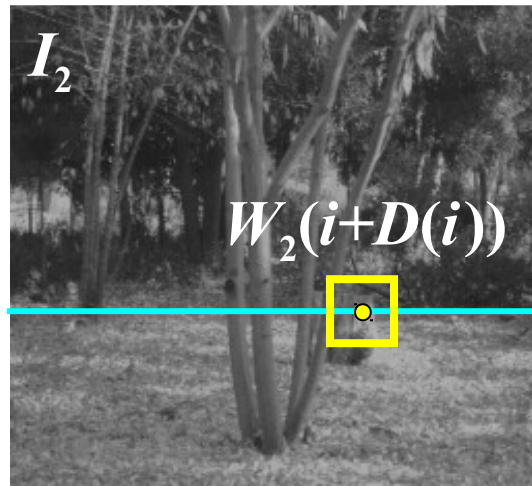
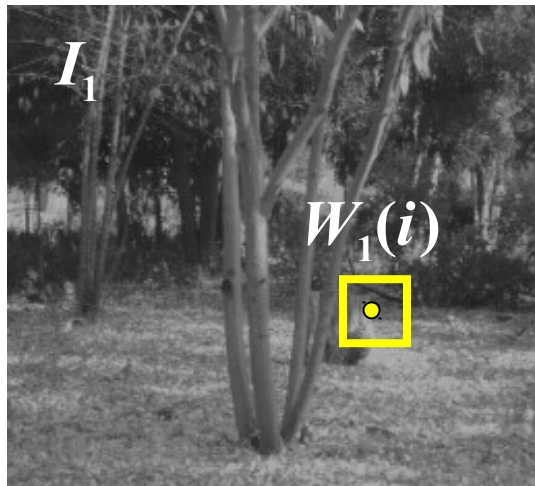
- Scanline stereo generates streaking artifacts



- Can't use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid



# Stereo matching as energy minimization



$$E(D) = \underbrace{\sum_i (W_1(i) - W_2(i + D(i)))^2}_{\text{data term}} + \lambda \underbrace{\sum_{\text{neighbors } i,j} \rho(D(i) - D(j))}_{\text{smoothness term}}$$

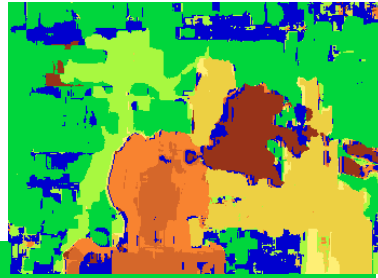
- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih,

[Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

# Many of these constraints can be encoded in an energy function and solved using graph cuts

Before



Graph cuts



Ground truth

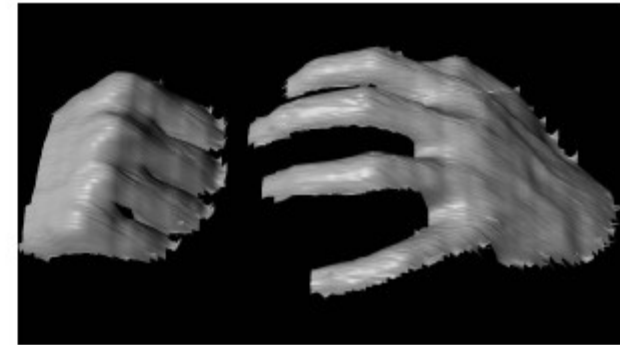
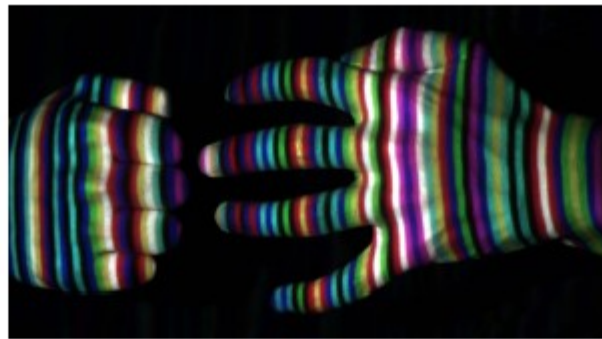
Y. Boykov, O. Veksler, and R. Zabih,  
**Fast Approximate Energy Minimization via Graph Cuts**, PAMI 2001

For the latest and greatest: <http://www.middlebury.edu/stereo/>

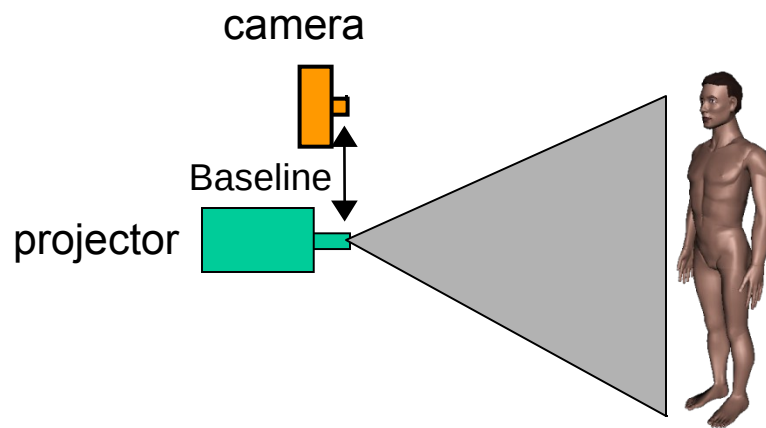
# Stereo

- Advantages:
  - Passive
  - Cheap hardware (2 cameras)
  - Easy to accommodate motion
  - Intuitive analogue to human vision
- Disadvantages:
  - Only acquire good data at “features”
  - Sparse, relatively noisy data (correspondence is hard)
  - Bad around silhouettes
  - Confused by non-diffuse surfaces
- Variant: multibaseline stereo to reduce ambiguity

# Active stereo with structured light



- Project “structured” light patterns onto the object
  - Simplifies the correspondence problem
  - Allows us to use only one camera



L. Zhang, B. Curless, and S. M. Seitz.

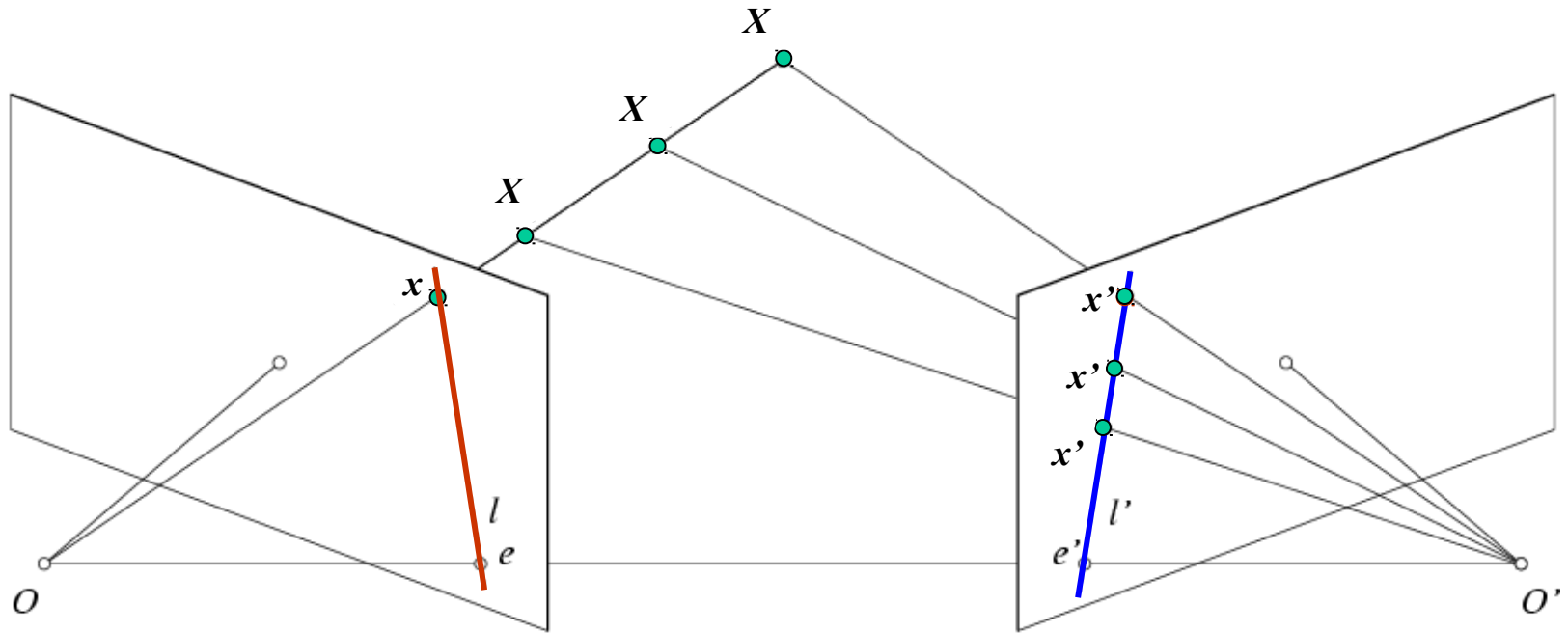
Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming

# Kinect: Structured infrared light



<http://bbzipo.wordpress.com/2010/11/28/kinect-in-infrared/>

# Summary: Key idea: Epipolar constraint



Potential matches for  $x$  have to lie on the corresponding line  $l'$ .

Potential matches for  $x'$  have to lie on the corresponding line  $l$ .