The most beautiful helper sheet

* X, Y random variables

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y] \quad V[X] = \mathbb{E}[X^2] - (\mathbb{E}X)^2$$

$$P(X,Y) = P(X \mid Y)\,P(Y)$$

X, Y independent RV: $P(X,Y) = P(X)\,P(Y)$

* $$\nabla f(\theta) = \left(\frac{\partial f}{\partial \theta_1}, \frac{\partial f}{\partial \theta_2}, \ldots, \frac{\partial f}{\partial \theta_n}\right)$$

$$\nabla(f+g) = \nabla f + \nabla g$$

$$\nabla(fg) = f\,\nabla g + g\,\nabla f$$

* $$\mathbf{w} \cdot \mathbf{x} \equiv \mathbf{w}^T \mathbf{x} \equiv \langle \mathbf{w}, \mathbf{x} \rangle \equiv \sum_{j=1}^{d} w_j x_j$$

* $$\|w\|_p = \left(\sum_{j=1}^{d} |w_j|^p\right)^{\frac{1}{p}}$$

$$\|w\|_2 = \sqrt{\sum_j w_j^2}$$

* Set Ω is convex iff the line connecting any two points from Ω is in Ω

$V, u \in \Omega \Rightarrow \forall \alpha \in [0,1] \quad \alpha V + (1-\alpha) u \in \Omega$

* $f: \Omega \to \mathbb{R}$ is convex if $\forall u, v \in \Omega \ \alpha \in [0,1]$

$$f(\alpha u + (1-\alpha) v) \leq \alpha f(u) + (1-\alpha) f(v)$$

$$f(u) \geq f(v) + \nabla f(v) \cdot (u - v)$$

* $f: \Omega \to \mathbb{R}$ is L-Lipschitz

$\forall w_1 \, w_2 \in \Omega \quad |f(w_1) - f(w_2)| \leq L \|w_1 - w_2\|$

* $M \in \mathbb{R}^{d \times d}$, $M_{ij} = M_{ji}$, the following are equivalent:

(1) $M \succcurlyeq 0$ (2) $\forall w \in \mathbb{R}^d \quad w^T M w \geq 0$ (3) $f(w) = w^T M w$ convex

(4) $\exists A$ s.t. $M = A^T A$ (5) Eigen values of $M \geq 0$

* $\nabla^2 f = \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right)$

$A \succcurlyeq B \Leftrightarrow A - B \succcurlyeq 0$

$$\alpha I \preccurlyeq \nabla^2 f(\theta) \preccurlyeq \beta I$$

α strong convex

β smooth

* Sample complexity:

$$H = \{ h_j \mid h_j : \mathcal{X} \to \{-1, 1\} \}$$

$$\exists\, h^* \in H \text{ s.t. } \forall (x,y) \sim D,\ h^*(x) = y$$

examples needed for ε inaccuracy s.t w.p. $1-\delta$ $\mathcal{L}_D(ERM(S, H)) \leq \varepsilon$ is

$$m \geq \frac{\log(|H|) + \log(1/\delta)}{\varepsilon}$$

* Do the right thing

Good Luck!