Introduction to Machine Learning - COS 324

Written Homework Assignment 4

Due date: The minute after 11:59pm of one week from announcement in class.

Electronic submissions only.

Note:

- Consulting with other students from this course is allowed. If you do so, clearly state whom you consulted with for each problem separately.
- Searching the internet or literature for solutions is **prohibited**.

I For this exercise we restrict ourselves to one dimensional functions, $d = 1$. Prove the equivalence of the two definitions of convexity shown in class. That is, we defined that $f : K \mapsto \mathbb{R}^d$ is convex if and only if $f((1-\alpha)x+\alpha y) \leq (1-\alpha)f(x)+\alpha f(y)$ for all $x, y \in K$ and $\alpha \in [0, 1]$. Show that f (assuming it is differentiable) is convex if and only if

$$f(x) \geq f(y) + f'(y)(x - y)$$

II Prove:

(a) The sum of convex functions is convex.

(b) Let f be α_1-strongly convex and g be α_2-strongly convex. Then $f + g$ is $(\alpha_1 + \alpha_2)$-strongly convex.

(c) Let f be β_1-smooth and g be β_2-smooth. Then $f + g$ is $(\beta_1 + \beta_2)$-smooth.

III **BONUS** Let $f(x) : R^d \mapsto R$ be a convex differentiable function and $\mathcal{K} \subseteq R^d$ be a convex set. Prove that $x^* \in \mathcal{K}$ is a minimizer of f over $\mathcal{K}$ if and only if for any $y \in K$ it holds that $(y - x^*)^\top \nabla f(x^*) \geq 0$.