7. Theory of Computation

> "In theory there is no difference between theory and practice. In practice there is."
>
> *-Yogi Berra*

Introduction to Theoretical CS

Two fundamental questions.

- What can a computer do?
- What can a computer do with limited resources?

General approach.

- Don't talk about specific machines or problems. (Pentium IV running Linux kernel 2.4.22)
- Consider minimal abstract machines.
- Consider general classes of problems.

Why Learn Theory

In theory . . .

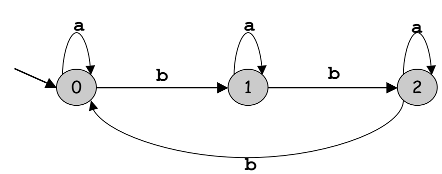
- Deeper understanding of what is a computer and computing.
- Foundation of all modern computers.
- Pure science.
- Philosophical implications.

In practice . . .

- Web search: theory of pattern matching.
- Sequential circuits: theory of finite state automata.
- Compilers: theory of context free grammars.
- Cryptography: theory of computational complexity.
- Data compression: theory of information.

Regular Expressions and DFAs

`a* | (a*ba*ba*ba*)*`



Pattern Matching Applications

Test if a string matches some pattern.

- Process natural language.
- Scan for virus signatures.
- Search for information using Google.
- Access information in digital libraries.
- Retrieve information from Lexis/Nexis.
- Search-and-replace in a word processors.
- Filter text (spam, NetNanny, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).
- Search for markers in human genome using PROSITE patterns.

Parse text files.

- Compile a Java program.
- Crawl and index the Web.
- Read in data stored in TOY input file format.
- Automatically create Java documentation from Javadoc comments.

Describing a Pattern

PROSITE. Huge database of protein families and domains.

Q. How to describe a protein motif?

Ex. [signature of the C2H2-type zinc finger domain]

- C
- Between 2 and 4 amino acids.
- C
- 3 more amino acids.
- One of the following amino acids: LIVMFYWCX.
- 8 more amino acids.
- H
- Between 3 and 5 more amino acids.
- H

CAASCGGPYACGGWAGYHAGWH

Regular Expressions: Basic Operations

Regular expression. Notation to specify a set of strings.

Operation	Regular Expression	Yes	No
Concatenation	`aabaab`	`aabaab`	every other string
Wildcard	`.u.u.u.`	`cumulus` `jugulum`	`succubus` `tumultuous`
Union	`aa | baab`	`aa` `baab`	every other string
Closure	`ab*a`	`aa` `abbba`	`ab` `ababa`
Parentheses	`a(a|b)aab`	`aaaab` `abaab`	every other string
	`(ab)*a`	`a` `ababababa`	ε `abbbaa`

Regular Expressions: Examples

Regular expression. Notation is surprisingly expressive.

Regular Expression	Yes	No
`.* spb .*` contains the trigraph `spb`	`raspberry` `crispbread`	`subspace` `subspecies`

`a* | (a*ba*ba*ba*)*`
multiple of three `b`'s
	`bbb`
	`aaa`
	`bbbaababbaa`
		`b`
		`bb`
		`baabbbaa`

`.*0....` fifth to last digit is `0`	`100011` `98701234`	`111111111` `403982772`

`gcg (cgg|agg)* ctg`
fragile X syndrome indicator
	`gcgctg`
	`gcgcggctg`
	`gcgcggaggctg`
		`gcgcgg`
		`cggcggcggctg`
		`gcgcaggctg`

Generalized Regular Expressions

Regular expressions are a standard programmer's tool.

- Built in to Java, Perl, Unix, Python, ...
- Additional operations typically added for convenience.
- Ex: `[a-e]+` is shorthand for `(a|b|c|d|e)(a|b|c|d|e)*`.

Operation	Regular Expression	Yes	No
One or more	`a(bc)+de`	abcde abcbcde	ade bcde
Character classes	`[A-Za-z][a-z]*`	capitalized Word	camelCase 4illegal
Exactly k	`[0-9]{5}-[0-9]{4}`	08540-1321 19072-5541	111111111 166-54-111
Negations	`[^aeiou]{6}`	rhythm	decade

Regular Expressions in Java

Validity checking. Is `input` in the set described by the `re`?

```
public class Validate {
   public static void main(String[] args) {
      String re    = args[0];
      String input = args[1];
      System.out.println(input.matches(re));
   }
}
```

powerful string library method

C2H2 type zinc finger domain

```
% java Validate "C.{2,4}C...[LIVMFYWC].{8}H.{3,5}H" CAASCGGPYACGGAAGYHAGAH
true
```

legal Java identifier

```
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true
```

valid email address (simplified)

```
% java Validate "[a-z]+@([a-z]+\\.)+(edu|com)" doug@princeton.edu
true
```

need quotes to "escape" the shell

PROSITE Patterns

PROSITE. Huge database of protein families and domains.
PROSITE patterns. Special notation to describe protein motif.

Ex. The signature of the C2H2-type zinc finger domain is:

```
C-x(2,4)-C-x(3)-[LIVMFYWC]-x(8)-H-x(3,5)-H
```

Cys wildcard concatenation any one of these amino acids 8 copies His

Regular expressions. Easy to translate PROSITE patterns to regular expressions and use rich RE libraries.

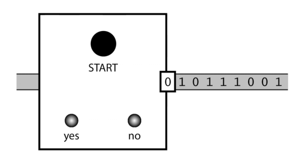
Solving the Pattern Match Problem

Regular expressions are a concise way to describe patterns.

- How would you implement `String.matches` ?
- Hardware: build a deterministic finite state automaton (DFA).
- Software: simulate a DFA.

DFA: simple machine that solves the pattern match problem.

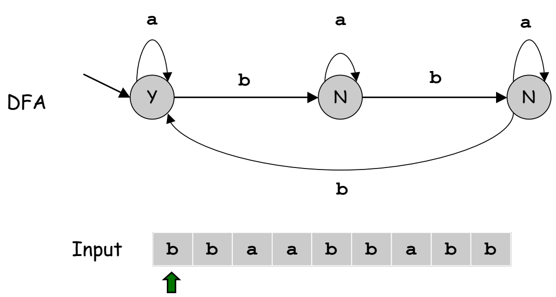
- Different machine for each pattern.
- Accepts or rejects string specified on input tape.
- Focus on `true` or `false` questions for simplicity.



Deterministic Finite State Automaton (DFA)

Simple machine with N states.

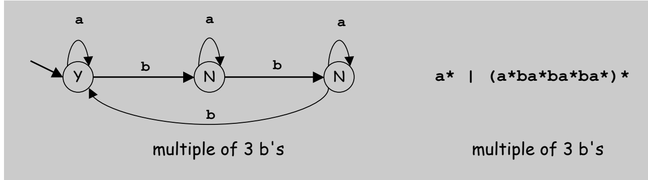
- Begin in *start state*.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- *Accept* or *reject* string depending on last state.



Theory of DFAs and REs

RE. Concise way to *describe* a set of strings.
DFA. Machine to *recognize* whether a given string is in a given set.

Duality: For any DFA, there exists a regular expression that describes the same set of strings; for any regular expression, there exists a DFA that recognizes the same set.



Practical consequence of duality proof: to match regular expression patterns, (i) build DFA and (ii) simulate DFA on input string.

Implementing a Pattern Matcher

Problem. Given a RE, create program that tests whether given input is in set of strings described.

Step 1. Build the DFA.

- A compiler!
- See COS 226 or COS 320.

Step 2. Simulate it with given input.

```
State state = start;
while (!StdIn.isEmpty()) {
   char c = StdIn.readChar();
   state = state.next(c);
}
System.out.println(state.accept());
```


Application: Harvester

Harvest information from input stream.

- Harvest patterns from DNA.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcggcggcggcggcggctg
gcgctg
gcgctg
gcgcggcggcggaggcggaggcggctg
```

- Harvest email addresses from web for spam campaign.

```
% java Harvester "[a-z]+@([a-z]+\\.)+(edu|com|net|tv)"
      http://www.princeton.edu/~cos126
doug@cs.princeton.edu
maia@cs.princeton.edu
wayne@cs.princeton.edu
```

↑ email validator (simplified)

Application: Harvester

Harvest information from input stream.

- Use `Pattern` data type to compile regular expression to NFA.
- Use `Matcher` data type to simulate NFA.
- (NFA is fancier but equivalent variety of DFA)

```java
import java.util.regex.Pattern;
import java.util.regex.Matcher;

public class Harvester {
   public static void main(String[] args) {
      String re        = args[0];
      In in            = new In(args[1]);
      String input     = in.readAll();
      Pattern pattern = Pattern.compile(re);
      Matcher matcher = pattern.matcher(input);
      while (matcher.find()) {
         System.out.println(matcher.group());
      }
   }
}
```


Application: Parsing a Data File

Ex: parsing an NCBI genome data file.

```
LOCUS AC146846 128142 bp DNA linear HTG 13-NOV-2003
DEFINITION Ornithorhynchus anatinus clone CLM1-393H9,
ACCESSION AC146846
VERSION AC146846.2 GI:38304214
KEYWORDS HTG; HTGS_PHASE2; HTGS_DRAFT.
SOURCE Ornithorhynchus anatinus
ORIGIN
     1 tgtatttcat ttgaccgtgc tgttttttcc cggtttttca gtacggtgtt agggagccac
    61 gtgattctgt ttgttttatg ctgccgaata gctgctcgat gaatctctgc atagacagct  // a comment
   121 gccgcaggga gaaatgacca gtttgtgatg acaaaatgta ggaaagctgt ttcttcataa
   ...
128101 ggaaatgcga cccccacgct aatgtacagc ttctttagat tg
//
```

```java
String re = "[ ]*[0-9]+([actg ]*).*";
Pattern pattern = Pattern.compile(re);
In in = new In(filename);
while (!in.isEmpty()) {
   String line = in.readLine();
   Matcher matcher = pattern.matcher(line);
   if (matcher.find()) {                                  // extract the RE part in parentheses
      String s = matcher.group(1).replaceAll(" ", "");   // replace this RE, with this string
      // do something with s
   }
}
```


Limitations of DFA

No DFA can recognize the language of all bit strings with an equal number of 0's and 1's.

- Suppose an N-state DFA *can* recognize this language.
- Consider following input: 0000000011111111

  (N+1 0's, N+1 1's)

- DFA must accept this string.
- Some state x is revisited during first N+1 0's since only N states.

  0000000011111111

  x x

- Machine would accept same string without intervening 0's.

  000011111111

- This string doesn't have an equal number of 0's and 1's.

Fundamental Questions

Which languages CANNOT be described by any RE?

- Bit strings with equal number of 0s and 1s.
- Decimal strings that represent prime numbers.
- Genomic strings that are Watson-Crick complemented palindromes.
- Many more. . . .

How can we extend REs to describe richer sets of strings?

- Context free grammar (e.g., Java).

Reference: http://java.sun.com/docs/books/jls/second_edition/html/syntax.doc.html

Q. How can we make simple machines more powerful?

Q. Are there any limits on what kinds of problems machines can solve?

Summary

Programmer.
- Regular expressions are a powerful pattern matching tool.
- Implement regular expressions with finite state machines.

Theoretician.
- Regular expression is a compact description of a set of strings.
- DFA is an abstract machine that solves pattern match problem for regular expressions.
- DFAs and regular expressions have limitations.

Variations
- Yes (accept) and No (reject) states sometimes drawn differently
- Terminology: Deterministic Finite State Automaton (DFA), Finite State Machine (FSM), Finite State Automaton (FSA) are the same
- DFA's can have output, specified on the arcs or in the states
  - These may not have explicit Yes and No states

Turing Machines

Challenge: Design simplest machine that is "as powerful" as conventional computers.

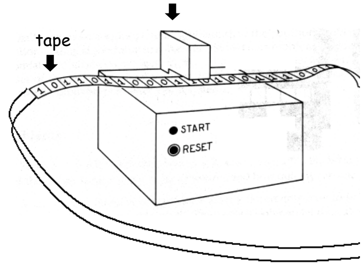
Alan Turing (1912-1954)

Turing Machine: Components

Alan Turing sought the most primitive model of a computing device.

Tape.
- Stores input, output, and intermediate results.
- One arbitrarily long strip, divided into cells.
- Finite alphabet of symbols.

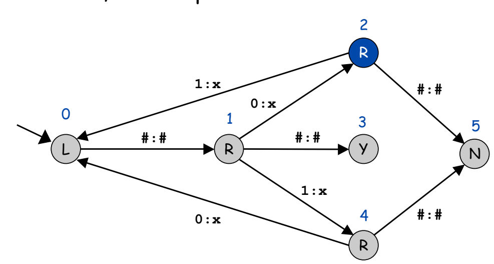
Tape head.
- Points to one cell of tape.
- Reads a symbol from active cell.
- Writes a symbol to active cell.
- Moves left or right one cell at a time.



Turing Machine: Fetch, Execute

States.
- Finite number of possible machine configurations.
- Determines what machine does and which way tape head moves.

State transition diagram.
- Ex. if in state 2 and input symbol is `1` then: overwrite the `1` with `x`, move to state 0, move tape head to left.



Before	...	#	#	x	x	x	1	1	0	#	#	...

Turing Machine: Fetch, Execute

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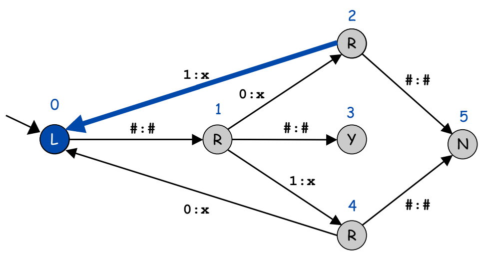


After

...	#	#	x	x	x	x	1	0	#	#	...

Turing Machine: Initialization and Termination

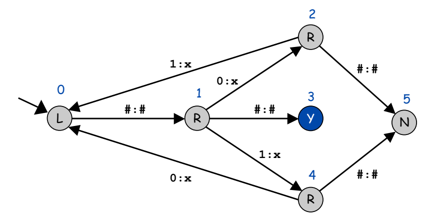
Initialization.

- Set input on some portion of tape.
- Set tape head.
- Set initial state.

...	#	#	0	0	1	1	1	0	#	#	...

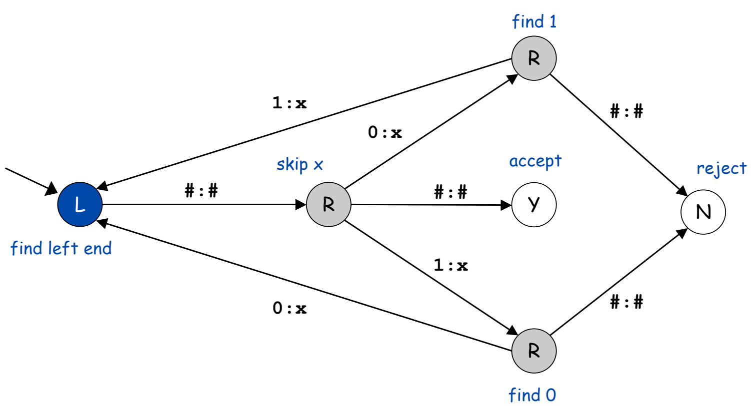
Termination.

- Stop if enter yes, no, or halt state.
- Infinite loop possible.



...	#	#	x	x	x	x	x	x	#	#	...

Example: Equal Number of 0's and 1's



...	#	#	0	0	1	1	1	0	#	#	...

Turing Machine Summary

Goal: simplest machine that is "as powerful" as conventional computers.

Surprising Fact 1. Such machines are very simple: TM is enough!

Surprising Fact 2. Some problems cannot be solved by ANY computer.

↑ next lecture

Consequences.

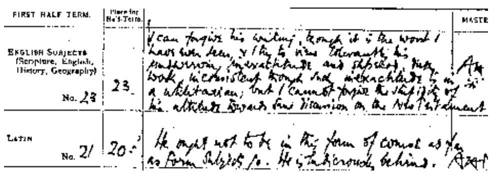
- Precursor to general purpose programmable machines.
- Exposes fundamental limitations of all computers.
- Enables us to study the physics and universality of computation.
- No need to seek more powerful machines!

Variations

- Instead of just recognizing strings, TM's can produce output: the contents of the tape
- Instead of Y and N states, TM's can have a plain Halt state

Alan Turing (1912-1954).

- Father of computer science.
- Computer Science's "Nobel Prize" is called the Turing Award.



FIRST HALF TERM.	Place for Half-Term.		MASTE
ENGLISH SUBJECTS (Scripture, English, History, Geography) No. 23	23	I can forgive his writing, though it is the worst I have ever seen, & I try to view tolerantly his unsatisfactory [illegible] and slipshod, dirty work, inconsistent though such [illegible] is in a utilitarian; but I cannot forgive the stupidity of his attitude towards sane discussion on the New Testament.	
LATIN No. 21	20	He ought not to be in this form of course as far as form subjects go. He is tediously behind.	

Alan's report card at 14.

Alan Turing and his elder brother.